State Bowl Test

Mu Alpha Theta National Convention
Chicago 1998
1. Let $f$ be the function defined by $f(x) = x^3 + x^2 - 9x - 9$. Set $a = |f(0)|$ and $b$ equal to the largest root of $f(x) = 0$. Find the number of combinations of $a$ objects taken $b$ at a time.

2. If $27^{2x+1} = 9^{x/2}$ and $y$ is the area of a regular hexagon inscribed in a circle of radius 6, what is the value of $y/x$?

3. How much larger is $\sum_{k=0}^{\infty} 5\left(\frac{2}{3}\right)^k$ than $\sum_{j=0}^{10} 5\left(\frac{2}{3}\right)^j$. Give your answer correct to four significant digits.

4. In the following diagram, $\angle RTE$ is a right angle, the measure of $\angle TRE$ is $48^\circ$, and $M$ is the midpoint of $RE$. If $RE = 9$, what is the perimeter of $\triangle TEM$? Give your answer correct to four significant digits.

5. Let $a$ be the maximum distance from the origin to a point on the curve with the polar equation $r = 2 - \sin \theta$. Let $b$ be a positive number such that the period of $y = \sin(b \pi x) + 3$ is $4/9$. Give, as your answer, the larger value of $a$ or $b$.

6. Find the value of $S$ if $S = \sum_{n=1}^{\infty} \left(3^{-n} + \frac{2n-1}{5^n}\right)$.

7. Let $\log_2 2a \cdot \log_2 3 \cdot \log_5 5 = 3$. Let $b$ be the probability of picking an ace, queen, or a heart in one selection from a well-shuffled standard deck of 52 playing cards. Find the product of $a$ and $b$. Give your answer as a simplified fraction.

8. Find the sum of the coefficients of $x^6 y^4$ and $x^3 y^7$ in the expansion of $(2x - y)^{10}$.

9. If $a$ is the area of the triangle with vertices $(2, 1), (-3, 2), (4, -5)$ and $b$ is the sum of the zeros of the polynomial whose zeros are the reciprocals of the zeros of $x^4 - 3x^3 + 8x^2 - 7x + 2$, find $a/b$. 
10. Three cards are drawn, without replacement, from a well-shuffled standard deck of 52 cards. Find the probability that either three aces or three hearts are drawn. Give your answer as a simplified fraction.