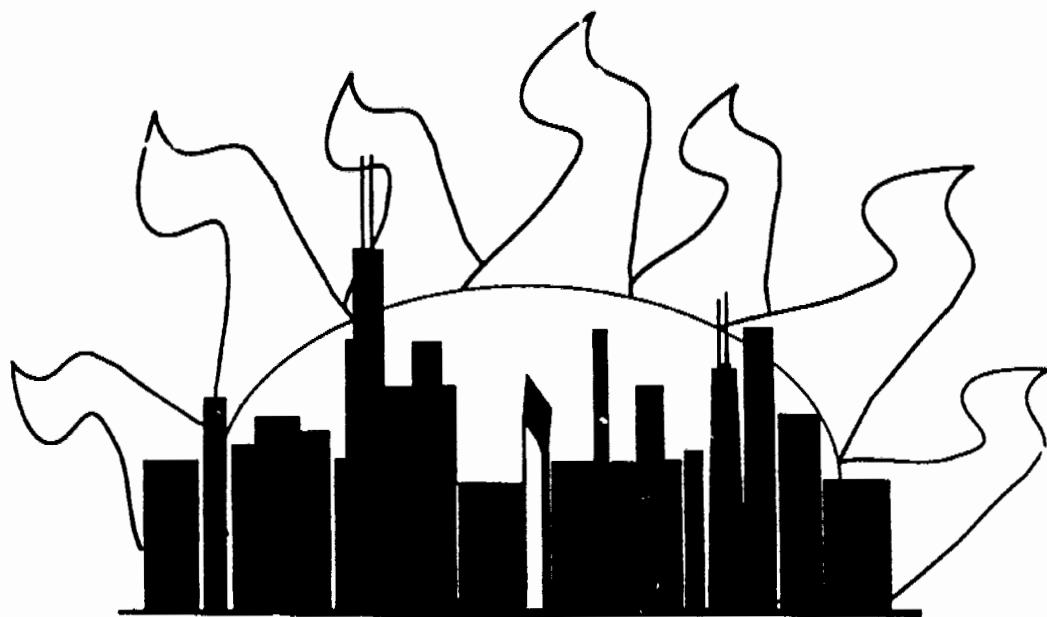


Theta Division

Topic Test 2

Matrices & Determinants



Mu Alpha Theta National Convention
Chicago 1998

General Instructions:

Unless otherwise stated all answers should be written as decimals.

If you are asked to give your answer as a fraction, please give your answer in a/b form where a and b are relatively prime.

Questions

1. Find the sum of the entries of $4A + BC$

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 4 & 0 \\ 1 & -1 & 6 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & -2 \\ -7 & 6 \\ 3 & 1 \end{bmatrix}$$

2. If $\begin{vmatrix} 5 & 2 \\ y & x \end{vmatrix} = x + y$, find the value of $\frac{x}{y}$. Express your answer as a reduced fraction.

3. The determinant of $\begin{vmatrix} -3 & 2 & -2 \\ 4 & x & 0 \\ 5 & -2 & 7 \end{vmatrix}$ is equal to the number of real solutions of the

equation $2y^3 - 17y^2 + 42y = 27$. Find the value of x . Give your answer rounded to the nearest hundredth.

4. For what positive value of x will the determinant of A be zero?

$$A = \begin{bmatrix} 3 & -2 & x \\ -1 & 5 & 2 \\ x & -3 & 1 \end{bmatrix}$$

Give your answer rounded to three decimal places.

5. Find the sum of the values of x that satisfy

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & \frac{3}{4} - x & 1 & 1 \\ 1 & 1 & 2 - x & 1 \\ 1 & 1 & 1 & x - \frac{7}{2} \end{vmatrix} = 0$$

6. Find the sum of a and b . $\begin{bmatrix} a & 1 \\ b & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 6 & 6 \end{bmatrix}$

7. Solve for x : $\begin{vmatrix} 4 & 2 \\ 3 & x \end{vmatrix} - 7 = \begin{vmatrix} 3 & 0 & 1 \\ 2 & x & 0 \\ -1 & 1 & 0 \end{vmatrix}$

8. Find the sum of the values for x and y which satisfy the following matrix equation:

$$\begin{bmatrix} 3y & 12 \\ 4 & x-1 \end{bmatrix} = \begin{bmatrix} x & 12 \\ 4 & 8 \end{bmatrix}$$

9. What is the value of a_{23} in the transpose of $A = \begin{bmatrix} 6 & 2 & -4 \\ 3 & -1 & 2 \\ 0 & 4 & 3 \end{bmatrix}$

10. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find k such that $A^2 + kA = \begin{bmatrix} 4 & 4 \\ 6 & 10 \end{bmatrix}$

11. Let A be the matrix $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Let $B = A^3 - 3A^2 + 3A - I$ where I is the 4×4

identity matrix. Find the sum of the entries of B .

12. Let A and B be matrices given by

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 3 & 2 \end{bmatrix}$$

Find the determinant of the product AB .

13. If a, b, c, d satisfy the following matrix equation, what is the value of $ab - cd$?

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

14. Let $[a] = [1 \ -3 \ -2] \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$ and $[b] = \begin{vmatrix} a & 4 \\ -1 & 3 \end{vmatrix}$. Find the value of $a + \sqrt{b}$.

15. If A^T is the transpose of matrix A , then solve for m .

$$2 \begin{bmatrix} 3 & m \\ 4 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ x & -4 \end{bmatrix}^T \begin{bmatrix} 2 & 4 \\ -1 & 2y \end{bmatrix}$$

16. What is the difference in the sum of the entries in $([B] + [C])[A]$ and $[B][A] + [C][A]$?

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 2 \\ 4 & 1 & 0 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 4 & 0 \\ 3 & -1 & 2 \\ -2 & 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$$

17. For what value of a is

$$\begin{vmatrix} 2 & 1 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & a \end{vmatrix} + \begin{vmatrix} 0 & a & 1 \\ 1 & 3a & 0 \\ -2 & a & 2 \end{vmatrix} = 14?$$

Give your answer as a reduced fraction in lowest terms.

18. The United States has diplomatic relations with Russia and the Mexico, but not with Cuba. Mexico has diplomatic relations with the United States and Russia, but not with Cuba. Russia has diplomatic relations with the United States, Mexico and Cuba. Cuba has diplomatic relations with Russia but not with the United States or Mexico. A country is not considered to have diplomatic relations with itself. We can set up the following communication matrix:

		TO			
		U.S.	Russia	Cuba	Mexico
FROM	U.S.	0	1	0	1
	Russia	1	0	1	1
	Cuba	0	1	0	0
	Mexico	1	1	0	0

How many ways can Cuba get a message to Mexico using two intermediaries?

19. Find the sum of all values of x for which $\begin{vmatrix} x-2 & 2 \\ 3 & x-3 \end{vmatrix} = 0$

20. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $x = \begin{bmatrix} x \\ y \end{bmatrix}$. The geometric transformations defined by $x' = Ax$, $x' = Bx$ and $x' = Cx$ are reflections across the lines $y = x$, $x = 0$, and $y = 0$, respectively. The matrix $D = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$ also represents a reflection across some line through the origin. What is the slope of that line?