Mu Alpha Theta National Convention: Denver, 2001

Ciphering – Mu Division

0. What is the distance between the foci of the graph of $4x^2 + 9y^2 = 36$?

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1. Six identical right square pyramids with base edge lengths of 36 centimeters are glued to each of the six faces of a cube with edges of length 36 centimeters. For most pyramid heights, this produces a shape with 24 triangular faces. For what pyramid height, in centimeters, will these 24 triangular faces align to form 12 rhombus-shaped faces?

2. Given that $f(x) = 3x^2 - \frac{9}{\sqrt{x}}$, evaluate $f'\left(\frac{9}{4}\right)$.

3. In a regular 15-gon, each interior angle measures $X°$, and Y diagonals can be drawn. What is the sum of X and Y?

4. The vector from the origin to the point representing $-8+6i\sqrt{3}$ in the complex plane is rotated 150° clockwise about the origin. To what complex number does the vector now point?

5. Determine the value of B if $\frac{\log A}{\log B} = \frac{B}{A} = \frac{1}{2}$.

6. Determine the equation (in standard form) of the plane through the points $(1, 4, 22)$, $(4, 1, 7)$, and $(-3, -4, 6)$.

7. Evaluate: $\int 2(3a+4)^5\,da$

8. A positive integer has 5 digits when written in base 3. How many digits does it have when it's expressed in base 8?

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9. What is the third smallest natural number with exactly 14 positive integral factors?

10. Evaluate: $\int_{\pi/4}^{\pi/3} \sin(4\theta)\cos^2(4\theta)\,d\theta$

11. Evaluate $\sin^2 15° + \cos^2 75°$.

12. Bag A contains two white marbles and five red marbles. Bag B contains four white marbles and three red marbles. A marble is chosen at random from Bag A and placed in Bag B. A marble is then chosen at random from Bag B. If the marble chosen from Bag B is white, what is the probability that the marble chosen from Bag A was white?

13. Determine the area enclosed by the polar graph of $r = 3\cos\theta$.

14. Determine all values of *r* for which $9 \cdot 2^{2r+3} - 4^{2r} = 512$.

15. What values of x satisfy $\frac{x}{3} \le \frac{2x+1}{4} < 3 - \frac{x}{2}$?

16. In how many ways can four boys (Bob, Bill, Buck, and Beau) and three girls (Gail, Gertrude, and Glenda) be seated in nine desks?

17. Let X be a positive, two-digit integer. Let Y be X when its digits are reversed. What is the largest value of X such that $X = 3Y - 2$?