Mu Alpha Theta National Convention: Denver, 2001
Colorado Bowl – Alpha Division

1. Evaluate: $\sin 30° + \cos\left(-\frac{\pi}{4}\right) - \tan 120° + \sec\left(\frac{2\pi}{3}\right)$

   (A) $\frac{-2+3\sqrt{2}}{2}$ (B) 1

   (C) $\frac{-3+\sqrt{2}+2\sqrt{3}}{2}$ (D) $\frac{1}{2}$ (E) NOTA

2. What is the tenth term of the geometric sequence with first term 12288 and common ratio $\frac{1}{4}$?

   (A) $\frac{3}{32}$ (B) $\frac{3}{64}$ (C) $\frac{3}{128}$ (D) $\frac{3}{256}$ (E) NOTA

3. The probability that contestant A answers this question correctly is $\frac{5}{6}$, while the probability that contestant B answers this question correctly is $\frac{9}{10}$. What is the probability that contestant A gets the problem wrong, and contestant B gets the problem right?

   (A) $\frac{3}{20}$ (B) $\frac{1}{12}$ (C) $\frac{1}{60}$ (D) $\frac{3}{4}$ (E) NOTA

4. Evaluate: $\log_{125}\frac{1}{625}$

   (A) $-\ln 5$ (B) $-\frac{4}{3}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{3}{4}$ (E) NOTA

5. Which of the following is <u>*not*</u> equal to $4-4i$?

   (A) $4\sqrt{2}e^{-\frac{9\pi i}{4}}$ (B) $4\sqrt{2}e^{\frac{\pi i}{4}}$ (C) $4\sqrt{2}e^{-\frac{\pi i}{4}}$ (D) $4\sqrt{2}e^{\frac{7\pi i}{4}}$ (E) NOTA

6. What is the magnitude of the vector $[-4, 18, -12]$?

   (A) 20 (B) $7\sqrt{10}$ (C) 22 (D) $10\sqrt{5}$ (E) NOTA

7. Simplify: $\dfrac{\tan^2\theta\csc\theta}{\sec\theta\sin\theta}$

(A) $\sin\theta$ (B) $\sec\theta$ (C) $\csc\theta\tan\theta$ (D) $\cos\theta\cot\theta$ (E) NOTA

8. Which of the following equals 1 for every value of θ between $\dfrac{\pi}{4}$ and $\dfrac{\pi}{3}$?

I. $\theta^2 - \dfrac{7\pi}{12}\theta + \dfrac{\pi^2}{12}$
II. $\csc^2\theta - \cot^2\theta$
III. $\sin^2\theta - \cos^2\theta$

(A) I only (B) II only (C) I & III only (D) II & III only (E) NOTA

9. What is the sum of the mean, mode, and median of the following data set?

15, 3, 9, 20, 14, 8, 11, 9, 10

(A) 29 (B) 30 (C) 30.5 (D) 31 (E) NOTA

10. The sum of the first 33 terms of an arithmetic sequence is 121. What is the 17th term of the sequence?

(A) 3 (B) 4
(C) $\dfrac{11}{3}$ (D) cannot be determined (E) NOTA

11. Evaluate: $\displaystyle\sum_{h=1}^{100}\left[\sqrt{h}\right]$. Note: [$x$] equals the greatest integer less than or equal to x.

(A) 615 (B) 625 (C) 815 (D) 825 (E) NOTA

12. The probability the Mariners win any baseball game is $\dfrac{4}{5}$. What is the probability they win at least four of their first five games this season?

(A) $\dfrac{462}{625}$ (B) $\dfrac{2304}{3125}$ (C) $\dfrac{1024}{3125}$ (D) $\dfrac{256}{3125}$ (E) NOTA

13. A game of poker is played in which five cards are dealt from a standard 52-card deck. What is the probability of receiving exactly three-of-a-kind? (Three-of-a-kind means three of a particular denomination and one each of two other denominations, e.g. KKK37 or 77725.)

(A) $\frac{3}{119}$ (B) $\frac{19}{833}$ (C) $\frac{18}{833}$ (D) $\frac{88}{4165}$ (E) NOTA

14. Given that $A=\sum_{q=1}^{28}((2q)!)$, what is the remainder when A is divided by 28?

(A) 14 (B) 16 (C) 18 (D) 20 (E) NOTA

15. Solve for b: $\frac{b}{4+2i}+\frac{b}{3-i}=5$

(A) $\frac{5}{2}$ (B) 5 (C) 10 (D) 15 (E) NOTA

16. How many positive integral factors of 1422 are greater than 20?

(A) 9 (B) 8 (C) 7 (D) 6 (E) NOTA

17. Evaluate: $\begin{bmatrix} 3 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ 1 & 2 \end{bmatrix}$

(A) $\begin{bmatrix} 13 & -4 \\ -11 & -2 \end{bmatrix}$ (B) $\begin{bmatrix} 13 & -8 \\ -11 & -10 \end{bmatrix}$ (C) $\begin{bmatrix} 13 & -4 \\ -5 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} 13 & -8 \\ -5 & -2 \end{bmatrix}$ (E) NOTA

18. What is the sine of the smaller angle between the vectors $[2, 3]$ and $[-1, 5]$?

(A) 0 (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{3}}{3}$ (D) 1 (E) NOTA

19. Bag A contains 14 red jellybeans and 6 blue jellybeans. Bag B contains 14 red jellybeans and 25 blue jellybeans. A jellybean is chosen at random from bag A and placed in bag B. A jellybean is then chosen at random from bag B. What is the probability that the jellybean chosen from bag B is red?

(A) $\frac{37}{100}$ (B) $\frac{587}{1600}$ (C) $\frac{147}{400}$ (D) $\frac{3}{8}$ (E) NOTA

20. What is the area of the triangle whose vertices are $(1, 3, 4)$, $(2, 5, 7)$, and $(-1, 1, 1)$?

(A) $\frac{3}{2}$ (B) $\frac{\sqrt{10}}{2}$ (C) $\frac{2\sqrt{3}}{2}$ (D) $\frac{\sqrt{13}}{2}$ (E) NOTA

21. Which of the following is equivalent to: $\frac{\left(r^3 s^2\right)^3}{\left(r^{-3} s^4\right)^{-3}}$?

(A) $\frac{r^{18}}{s^{12}}$ (B) $\frac{r^{16}}{s^{18}}$ (C) s^{18} (D) $r^{16} s^{12}$ (E) NOTA

22. If $\sin A = \frac{7}{25}$, where $\frac{\pi}{2} < A < \frac{3\pi}{2}$, what is $\tan A$?

(A) $-\frac{7}{24}$ (B) $-\frac{24}{7}$ (C) $-\frac{24}{25}$ (D) $\frac{24}{25}$ (E) NOTA

23. What is the sum of all real values of x satisfying $\log_{12}(x+1) + \log_{144}(x-1) = 1$?

(A) -1 (B) 0 (C) 5 (D) 6 (E) NOTA

24. What is the sum of all the multiples of 18 from 300 to 1200, inclusive?

(A) 28692 (B) 29474 (C) 29880 (D) 31086 (E) NOTA

25. What is the sum of all values of x satisfying $25^x - 4 \cdot 5^x = 5$?

(A) 1 (B) 0 (C) -1 (D) -3 (E) NOTA

26. Two of the roots of a cubic equation with integer coefficients and leading coefficient of 1 are $3i + 1$ and 4. What is the coefficient of the linear term of the equation?

(A) 40 (B) 18 (C) 2 (D) -2 (E) NOTA

27. What is the length of the median, in centimeters, from vertex C in triangle ABC with $a = 4$, $b = 6$, and $c = 4$ centimeters, where a, b, and c are the lengths of the sides opposite vertices A, B, and C respectively?

(A) $3\sqrt{3}$ cm (B) $3\sqrt{2}$ cm (C) $\sqrt{20}$ cm (D) $\sqrt{22}$ cm (E) NOTA

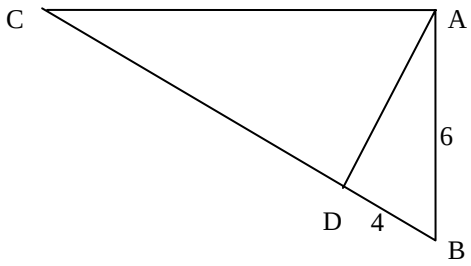
28. What is the volume, in cubic centimeters, of a right circular cone with base radius 6 centimeters and slant length 14 centimeters?

(A) $48\pi\sqrt{10}$ cm^3 (B) 168π cm^3 (C) $72\pi\sqrt{6}$ cm^3 (D) 144π cm^3 (E) NOTA

29. What is the y-intercept of the line perpendicular to $5x-2y=37$ and passing through the point $(5,-3)$?

(A) $(2,0)$ (B) $(0,-1)$ (C) $(0,-5)$ (D) $(0,2)$ (E) NOTA

30. What is *CD* in the triangle shown if $\overline{AD}$ is the altitude to the hypotenuse of right triangle *ABC*?



(A) $6-2\sqrt{7}$ (B) $\frac{40}{7}$
(C) $3+\sqrt{7}$ (D) 5 (E) NOTA

31. A rock climber plans to ascend a tricky cliff. It heads straight up for 100 meters, then has an overhang (the cliff pokes out into the air), where the climber must travel 30 meters of cliff face which makes an angle of 28° with the vertical. Before the climber begins his ascent, he stands at the base of the cliff, facing away, and looks up to the lip of the overhang to which he intends to climb. At what angle from the horizontal must he look up, to the nearest hundredth of a degree?

(A) 83.65° (B) 84.03° (C) 84.82° (D) 85.36° (E) NOTA

32. How many zeros does 45! end in, when expressed in base ten?

(A) 8 (B) 9 (C) 10 (D) 11 (E) NOTA

33. Find the area of the region that satisfies the following inequalities:

$$(x+3)^2+(y+4)^2\le 36$$
$$x<0$$

(A) $12\pi-9\sqrt{3}$ (B) 6π (C) $4\pi+3\sqrt{2}$ (D) $8\pi-3\sqrt{2}$ (E) NOTA

34. Find the period of the following equation: $y = \frac{x}{3} - \left[\frac{x}{3}\right]$, where [*z*] equals the greatest integer less than or equal to *z*.

(A) $\frac{1}{3}$ (B) $\frac{2\pi}{3}$ (C) 3 (D) 3π (E) NOTA

35. One of the roots of $3x^2 + cx + 36 = 0$ is three times the other. What is the value of *c*?

(A) 12 (B) ± 18 (C) 20 (D) ± 24 (E) NOTA

36. Given that $g(d) = 15 - 3d$ and $f(d) = 4d^2 - 3d - 8$, what is $g(f(6))$?

(A) -128 (B) -339 (C) 114 (D) 309 (E) NOTA

37. When a complex number, *z*, is multiplied by the complex number one less than itself, the result is equal to three less than the cube of *z*. What is the sum of all possible values of *z*?

(A) -6 (B) -2 (C) 0 (D) 1 (E) NOTA

38. Evaluate: $\begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix}^{-1}$

(A) $\frac{1}{6}\begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$ (B) $\frac{1}{6}\begin{bmatrix} 2 & 2 \\ -2 & 1 \end{bmatrix}$ (C) $\frac{1}{6}\begin{bmatrix} -1 & -2 \\ 2 & -2 \end{bmatrix}$ (D) $\frac{1}{6}\begin{bmatrix} 1 & -2 \\ 2 & 2 \end{bmatrix}$ (E) NOTA

39. Express 423_5 in base seven.

(A) 216_7 (B) 220_7 (C) 221_7 (D) 222_7 (E) NOTA

40. If $\sin^2\theta + \frac{3}{2}\cos\theta = \frac{3}{2}$, find the sum of all possible values of θ on the interval $0 \le \theta < 2\pi$.

(A) 2π (B) 3π (C) $\frac{7\pi}{2}$ (D) $\frac{11\pi}{3}$ (E) NOTA