Mu Alpha Theta National Convention: Denver, 2001
Colorado Bowl – Mu Division

1. Determine $\frac{dy}{dx}$ of the equation $x^2 + y^2 = 625$ at the point $(24, -7)$.

   (A) $\frac{25}{7}$ (B) $-\frac{25}{7}$ (C) $\frac{24}{7}$ (D) $-\frac{24}{7}$ (E) NOTA

2. Evaluate: $\lim_{h \to 0} \frac{5x^2 - 5(x+2h)^2}{h}$

   (A) $-10x$ (B) $10x$ (C) $-20x$ (D) $20x$ (E) NOTA

3. For what values of x is $f(x) = 4x^3 - 2x + 1$ decreasing as x increases?

   (A) $x < -\frac{\sqrt{3}}{2}$ (B) $|x| < \frac{\sqrt{3}}{2}$ (C) $x > \frac{\sqrt{6}}{6}$ (D) $|x| < \frac{\sqrt{6}}{6}$ (E) NOTA

4. Evaluate: $\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{1 - 2x}$

   (A) -1 (B) 1 (C) -2 (D) 2 (E) NOTA

5. What is the shape of the locus of points which are twice as far from the point $(3, 4)$ as they are from the line $3x + 4y = 5$?

   (A) ellipse (B) line (C) parabola (D) hyperbola (E) NOTA

6. Find the largest possible value of $\sqrt{a^2 + b^2}$ if $a + bi$ is a root of $x^3 - 10x^2 + 34x - 25 = 0$ and a and b are real numbers.

   (A) 5 (B) 11 (C) 1 (D) 25 (E) NOTA

7. Let *A* be a complex cube root of 1, and *B* be a complex cube root of -1. How many of the following could be the sum of *A* and *B*?

I. 0
II. 1
III. $i\sqrt{3}$
IV. $-i\sqrt{3}$

(A) I only (B) III only
(C) I, II, & III only (D) I, III, & IV only (E) NOTA

8. What real value(s) of *b* will ensure that $2x^2 + bx + 6 = 0$ has at least one real root?

(A) $|b| \le 4$ (B) $|b| \ge 4\sqrt{3}$ (C) $b \ge 4$ (D) no real value (E) NOTA

9. The equation $x^3 + ax^2 + bx + c = 0$, where *a*, *b*, and *c* are integers, has roots of 2 and $i - 3$. What is the sum of *a*, *b*, and *c*?

(A) -15 (B) -16 (C) -17 (D) -18 (E) NOTA

10. What is the measure of the smaller angle between the vectors $[-3, 2, 5]$ and $[1, 0, -4]$, to the nearest tenth of a degree?

(A) 146.6° (B) 149.7° (C) 150.1° (D) 154.8° (E) NOTA

11. Evaluate: $\int_1^2 \left(x^4 + x^2 + 1 + \frac{1}{x^2} \right) dx$

(A) $\frac{301}{30}$ (B) 10 (C) $\frac{29}{3}$ (D) $\frac{289}{30}$ (E) NOTA

12. Evaluate: $\begin{vmatrix} 3 & -1 & 2 \\ -2 & 0 & 4 \\ 5 & -3 & -4 \end{vmatrix}$

(A) 25 (B) 27 (C) 32 (D) 36 (E) NOTA

13. Solve the system of equations for *z*.

$$y + z = 4$$
$$x - 2z = 7$$
$$2x - 4y = 3$$

(A) $\frac{1}{2}$ (B) $\frac{5}{8}$ (C) $\frac{3}{4}$ (D) $\frac{7}{8}$ (E) NOTA

14. Let *K* be a positive integer such that $K \equiv 3 (\mathrm{mod}\, 5)$, $K \equiv 2 (\mathrm{mod}\, 6)$, and $K \equiv 12 (\mathrm{mod}\, 22)$. What is the smallest possible value for *K*?

(A) 122 (B) 128 (C) 166 (D) 188 (E) NOTA

15. What is the product of 214_{12} and 928_{12}, expressed in base 12? Use the digits *A* and *B* to express digit values of ten and eleven, if necessary.

(A) $1792A8_{12}$ (B) 171138_{12} (C) 174388_{12} (D) 175768_{12} (E) NOTA

16. Eight students take a test, receiving scores that are integers between 0 and 100, inclusive. If the mode of the scores is 95, and the mean of the scores is 90, what is the largest possible value for the range of the scores?

(A) 75 (B) 70 (C) 65 (D) 60 (E) NOTA

17. Evaluate: $\int_0^3 4(2t-3)^3\, dt$

(A) 1024 (B) $-\frac{3}{2}$ (C) $\frac{3}{2}$ (D) 0 (E) NOTA

18. A particle is at rest at the origin of the number line at time $t = 0$. If the particle's acceleration is $a(t) = 6t^2 - 8$, at what time $t > 0$ will the particle be at rest?

(A) 2 (B) 3 (C) $\frac{7}{2}$ (D) 5 (E) NOTA

19. What is the average value of $f(x) = 5 - 4x^2$ on the interval from $2 < x < 4$?

(A) -40 (B) $-\frac{79}{2}$ (C) -32 (D) $-\frac{97}{3}$ (E) NOTA

20. Simplify: $\int \frac{1}{\sqrt{4-x^2}}\,dx$

(A) $\frac{1}{2}\arctan(x)+C$ (B) $\frac{1}{2}\arcsin\left(\frac{x}{2}\right)+C$

(C) $\operatorname{arcsec}(x)+C$ (D) $\arccos\left(\frac{x}{2}\right)+C$ (E) NOTA

21. If event A occurs, the probability that event B also occurs is $\frac{2}{5}$. If event A does not occur, the probability that event B also does not occur is $\frac{1}{3}$. If the probability that either A or B (or both) occurs is $\frac{3}{4}$, what is the probability that both A and B occur?

(A) $\frac{1}{10}$ (B) $\frac{3}{40}$ (C) $\frac{1}{15}$ (D) $\frac{1}{20}$ (E) NOTA

22. Tom's fanny pack contains 18 quarters, 41 dimes, and 25 nickels. If he grabs three coins out of his fanny pack, what is the probability he gets exactly one of each type of coin?

(A) $\frac{225}{1162}$ (B) $\frac{449}{2324}$ (C) $\frac{677}{3486}$ (D) $\frac{1355}{6972}$ (E) NOTA

23. Three standard six-sided dice are rolled behind a screen. A friend of yours looks at the dice, and tells you there are no fives. What is the probability that the sum of the numbers shown on the dice is 12?

(A) $\frac{1}{24}$ (B) $\frac{2}{25}$ (C) $\frac{1}{25}$ (D) $\frac{1}{54}$ (E) NOTA

24. A real number *y* is chosen subject to the constraint that the probability that *y* is less than *x* is $P(x)=\begin{cases}0, & x\le 0\\ \frac{x^2}{16}, & 0<x<4\\ 1, & x\ge 4\end{cases}$. What is the probability that *y* is between 2 and 3, inclusive?

(A) $\frac{5}{16}$ (B) $\frac{3}{8}$ (C) $\frac{13}{32}$ (D) $\frac{1}{2}$ (E) NOTA

25. Evaluate: $\cfrac{2}{5+\cfrac{2}{5+\cfrac{2}{5+\cdots}}}$

(A) $\frac{\sqrt{5}-2}{2}$ (B) $\frac{\sqrt{33}-5}{2}$ (C) $\frac{3-\sqrt{5}}{2}$ (D) $\frac{4-\sqrt{10}}{2}$ (E) NOTA

26. Determine the sum of all the multiples of 12 between 140 and 380, inclusive.

(A) 4788 (B) 5016 (C) 5160 (D) 5532 (E) NOTA

27. A cubical sponge maintains its cubical shape at all times as it absorbs water at a rate of ten cubic centimeters per second (the cube's volume therefore increases at this rate). What is the rate of change of the edge length of the cube (in centimeters per second) at the moment it has a volume of 125 cubic centimeters?

(A) $\frac{\sqrt{2}}{10}$ cm/s (B) $\frac{3}{25}$ cm/s (C) $\frac{2}{15}$ cm/s (D) $\frac{20-\sqrt{5}}{125}$ cm/s (E) NOTA

28. Determine the sum of the infinite geometric series with first term 324 and common ratio $\frac{1}{5}$.

(A) 395 (B) 400 (C) 405 (D) 450 (E) NOTA

29. Sally is surveying her hiking route before she starts. The path is completely straight, and Sally knows it begins with a level 1500 meter section and is followed by a section which rises at 20° to the horizontal. Sally can see the end point of the hike from the beginning, and has to look up at an angle of 15° from the horizontal. How long is the uphill section of Sally's hike, to the nearest meter?

(A) 4454 (B) 3749 (C) 2838 (D) 2017 (E) NOTA

30. What is the sum of all values of θ, for $0 < \theta \le 2\pi$, for which $\tan^2\theta - \sec\theta + 1 = 0$

(A) 2π (B) $\frac{5\pi}{2}$ (C) $\frac{8\pi}{3}$ (D) $\frac{19\pi}{6}$ (E) NOTA

31. Given that $\sin A = \frac{7}{8}$ and $\frac{\pi}{2} < A < \frac{3\pi}{2}$, and $\cos B = \frac{1}{4}$ and $0 < B < \pi$, evaluate $\cos(A+B)$.

(A) $\frac{-\sqrt{30}}{8}$ (B) $\frac{\sqrt{10}}{16}$ (C) $\frac{-\sqrt{15}}{4}$ (D) $\frac{3\sqrt{15}}{8}$ (E) NOTA

32. Evaluate: $\dfrac{\tan\frac{\pi}{3}}{\cos\frac{2\pi}{3}} + \dfrac{\csc\frac{\pi}{2}}{\sin\frac{3\pi}{2}}$

(A) 1 (B) $2\sqrt{3}-1$ (C) $2\sqrt{3}$ (D) $2\sqrt{3}+1$ (E) NOTA

33. What is the least common multiple of 135 and 1134?

(A) 4860 (B) 5130 (C) 5400 (D) 5670 (E) NOTA

34. What is the smallest positive integer multiple of 45 that has exactly 24 positive integral factors?

(A) 225 (B) 270 (C) 315 (D) 360 (E) NOTA

35. What is the largest integer *n* such that the ratio of 152! to 7^n is an integer?

(A) 21 (B) 22 (C) 23 (D) 24 (E) NOTA

36. Use first-order differentials about $x = 8$ to approximate $y = x^{\frac{1}{3}}$ at $x = 10$. Express your answer to the nearest hundredth.

(A) 2.15 (B) 2.16 (C) 2.17 (D) 2.18 (E) NOTA

37. Determine the volume of the solid generated when the region in the Cartesian plane satisfying $y < \cos(x)$, $y > \sin(x)$, $x > 0$, and $x < \frac{\pi}{4}$ is rotated about the x-axis.

(A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$ (C) $\frac{3\pi}{4}$ (D) π (E) NOTA

38. Evaluate: $\int e^{4x}dx$

(A) $\frac{(x-1)e^{4x}}{4}+C$ (B) $\frac{(16x-1)e^{4x}}{16}+C$

(C) $\frac{(x-4)e^{4x}}{4}+C$ (D) $\frac{(4x-1)e^{4x}}{16}+C$ (E) NOTA

39. The position vector in the complex plane of the number $-3+4i$ is rotated 45° clockwise about the origin. The new vector is the position vector of what complex number?

(A) $\sqrt{7}+3i\sqrt{2}$ (B) $\frac{\sqrt{2}}{2}+\frac{7\sqrt{2}}{2}i$ (C) $\sqrt{13}+2i\sqrt{3}$ (D) $\frac{\sqrt{23}}{2}+\frac{3\sqrt{3}}{2}i$ (E) NOTA

40. Evaluate: $\int_{\pi/6}^{\pi/2}\cos\theta\, d\theta$

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2-\sqrt{3}}{2}$ (D) $\frac{2+\sqrt{3}}{2}$ (E) NOTA