Mu Alpha Theta National Convention: Denver, 2001
Gemini Test – Theta Division

1. A poker hand is defined as drawing five cards at random without replacement from a standard deck of 52 playing cards. How many different three-of-a-kind hands are possible? Note: three-of-a-kind means three cards of the same rank and one each of two other ranks (e.g. 7779K).

   (A) 20,592 (B) 54,912 (C) 64,896 (D) 10,296 (E) NOTA

2. Mmm! Someone dropped off a box of chocolates outside the room. Opening the box, you see twenty-five pieces of delicious candy, but they are scattered randomly in the box. You see from the nutritional information that there are five coconut cremes, seven strawberry cremes, three walnut-covered chocolates, six raspberry cremes, and four caramels. If each piece looks the same, what is the probability of getting exactly three pieces of coconut cremes out of the seven you pick?

   (A) $\frac{204}{1265}$ (B) $\frac{51}{506}$ (C) $\frac{63}{1265}$ (D) $\frac{51}{5060}$ (E) NOTA

3. Simplify: $\dfrac{4-2i}{-3+5i}$

   (A) $\dfrac{-1-7i}{24}$ (B) $\dfrac{-1+13i}{17}$ (C) $\dfrac{-11-7i}{17}$ (D) $\dfrac{-3-14i}{34}$ (E) NOTA

4. Which of the following is an equation of a circle which passes through the points $(1, 0)$, $\left(\frac{1}{2},\frac{1}{2}\right)$, and $\left(\frac{1}{2},-\frac{1}{2}\right)$?

   (A) $x^2+\left(y+\frac{1}{2}\right)^2=\frac{1}{4}$ (B) $x^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{4}$

   (C) $\left(x+\frac{1}{2}\right)^2+y^2=\frac{1}{4}$ (D) $\left(x-\frac{1}{2}\right)^2+y^2=\frac{1}{4}$ (E) NOTA

5. Evaluate: $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 0 \end{vmatrix}$

   (A) 30 (B) 24 (C) 15 (D) 6 (E) NOTA

6. What is the distance between the foci of the conic section with equation $13x^2 - 78x - 5y^2 - 20y = 33$?

(A) 4 (B) 6 (C) 8 (D) 12 (E) NOTA

7. What is the product of the volume of a circular cylinder with radius 1 and height h and the volume of a cube with edge length equal to the cylinder's height?

(A) $6\pi h^4$ (B) πh^4 (C) $6\pi h^3$ (D) πh^3 (E) NOTA

8. Find the distance between the points $5+4i$ and $-2-i$ in the Complex plane.

(A) $5\sqrt{3}$ (B) $3\sqrt{5}$ (C) $5\sqrt{2}$ (D) $\sqrt{74}$ (E) NOTA

9. Solve for s: $4^{2s-1} = 1024$

(A) 4 (B) 2 (C) 1 (D) 3 (E) NOTA

10. Five coins are flipped behind a screen, and you are told (truthfully) that at least two heads appeared. What is the probability that exactly four heads appeared?

(A) $\frac{5}{32}$ (B) $\frac{3}{16}$ (C) $\frac{5}{26}$ (D) $\frac{7}{32}$ (E) NOTA

11. When the Cartesian point (-3, 4) is expressed in polar form using radians rounded to the nearest hundredth, which of the following could it be?

(A) (5, 1.59) (B) (1.59, 5) (C) (5, 2.21) (D) (2.21, 5) (E) NOTA

12. Evaluate: $\frac{1}{2}\begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix} * \frac{1}{4}\begin{bmatrix} 0 & 5 \\ 4 & -2 \end{bmatrix}$

(A) $\begin{bmatrix} 8 & 1 \\ 8 & -14 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & \frac{1}{8} \\ 1 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & \frac{1}{8} \\ 1 & -\frac{7}{4} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{2}{3} & \frac{1}{6} \\ \frac{2}{3} & -\frac{7}{3} \end{bmatrix}$ (E) NOTA

13. Simplify: $\dfrac{(a^4b^{-3})^2}{(a^{-2}b^5)^{-2}}$

(A) $a^{10}b^{-8}$ (B) $a^{10}b^{-9}$ (C) a^4b^4 (D) a^2b^{-2} (E) NOTA

14. If $f(x)=2x^2+3x-4$ and $g(x)=-3x^2-x+2$, evaluate $\frac{1}{2}g(f(-1))$.

(A) -34 (B) -68 (C) -4 (D) $-\frac{5}{2}$ (E) NOTA

15. Beatrice is given x dollars to spend at the mall by her generous grandmother. She gets a pretzel and cheese for \$5. Shopping at Claire's, she sees earrings on sale for $\frac{1}{6}$ of how much she was originally given. After buying those, she heads across the hall to the Gap, where she then purchases jeans for \$15. An ice cream cone then costs her \$4, leaving her with $\frac{3}{10}$ of what she was originally given. How much was she originally given?

(A) \$35 (B) \$60 (C) \$65 (D) \$45 (E) NOTA

16. Which of the following pairs of lines contains one line parallel to and one line perpendicular to the line through the points (2, 1) and (0, -3)?

(A) $\begin{array}{l} y=2x+3 \\ y=-\frac{x}{2}+2 \end{array}$ (B) $\begin{array}{l} y=\frac{x}{2}-3 \\ y=-2x+1 \end{array}$ (C) $\begin{array}{l} y=\frac{x}{2}-3 \\ y=-\frac{x}{2}-1 \end{array}$ (D) $\begin{array}{l} y=2x-3 \\ y=-2x+1 \end{array}$ (E) NOTA

17. A line has the equation: $3x+4y=-12$. Find the length of the distance between its x and y intercepts.

(A) 4 (B) 4.5 (C) 5 (D) 5.5 (E) NOTA

18. Which of the following is/are NOT necessarily true?

I. $\log(AB)=\log A+\log B$

II. $\log(\frac{A}{B})=\log A-\log B$

III. $\log(A^p)=\log A*\log p$

IV. $10^{\log p}=10^p$

(A) None (B) IV & II (C) III & IV (D) I, II (E) NOTA

19. Evaluate: $e^{2\ln A-2\ln B}$

(A) A^2-B^2 (B) A^2+B^2 (C) $A^2+\frac{1}{B^2}$ (D) $A^2-\frac{1}{B^2}$ (E) NOTA

20. Find the remainder when 2001 is divided by w if: $\sqrt{3+\sqrt{-2+\sqrt{w}}}=2$

(A) 1 (B) 2 (C) 3 (D) 6 (E) NOTA

21. Which of the following is equal to 1230_4?

(A) 2210_3 (B) 123_{10} (C) 234_8 (D) 213_7 (E) NOTA

22. Determine the sum of the three smallest natural numbers which each have exactly nine positive integral factors.

(A) 292 (B) 361 (C) 392 (D) 517 (E) NOTA

23. In circle O, chords $\overline{PQ}$ and $\overline{RS}$ intersect at T. If $PS=8$, $TR=5$, $QR=4$, and TS is an integer, what is the largest possible value of PQ?

(A) 18 (B) 19 (C) 20 (D) 21 (E) NOTA

24. Evaluate for $|t|<2$: $\sum_{x=0}^{\infty}\left(\frac{t}{2}\right)^x$

(A) $\frac{2}{2-t}$ (B) $\frac{2}{t}$ (C) $\frac{1}{2-t}$ (D) $\frac{t}{2}$ (E) NOTA

25. What is the coefficient of the quadratic term of the polynomial of lowest order that satisfies the following conditions?

Its leading coefficient is 1.
All of its coefficients are rational.
Two of its roots are $3+\sqrt{5}$ and $3-i\sqrt{5}$.

(A) $\frac{29}{3}$ (B) 11 (C) $9+\sqrt{5}$ (D) 54 (E) NOTA

26. Workout William was dared by his older brother to toss a grapefruit straight up at an initial velocity of 100 ft/sec. Its height h (in feet) at time t (in seconds) is given by $h(t)=-16t^2+100t+6$. How high does the grapefruit go before returning to William?

(A) 150.25 feet (B) 155.5 feet (C) 160.75 feet (D) 162.25 feet (E) NOTA

27. Find the area (to the nearest tenth of a square unit) of the circle that is circumscribed about the triangle formed by the points (-2, 1), (3, 5), and (5, -2).

(A) 18.2 (B) 21.5 (C) 24.3 (D) 26.8 (E) NOTA

28. What is the sum of the 100 smallest natural numbers that are divisible by either 2 or 3 (or both)?

(A) 3433 (B) 3450 (C) 7550 (D) 7575 (E) NOTA

29. How many integers satisfy $x^2 \le 25$?

(A) 5 (B) 6 (C) 10 (D) 11 (E) NOTA

30. The word "DIVIDE" is written on a slip of paper that is then cut up into six pieces, each with one letter on it. These six pieces of paper are then put in a hat, from which four are randomly drawn one at a time without replacement. How many different four-letter sequences can be produced in this manner?

(A) 54 (B) 120 (C) 180 (D) 360 (E) NOTA

31. Evaluate: $\log_6 \frac{1}{216} + \log_2 256 - \log_4 256$

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

32. Simplify: $-3 + 2t < 4t + 2 < -t + 12$

(A) $-5 < t < 10$ (B) $-\frac{5}{2} < t < 2$
(C) $-\frac{5}{2} < t$ and $t > 2$ (D) $-\frac{1}{2} < t < 2$ (E) NOTA

33. In the addition problem shown, let each of *A*, *B*, *C*, *D*, and *E* represent a unique digit in base ten. What is the largest possible value of *C*?

$$\begin{array}{r} AABC \\ +\,CDA \\ \hline AEDE \end{array}$$

(A) 6 (B) 7 (C) 8 (D) 9 (E) NOTA

34. How many diagonals exist in a convex decagon?

(A) 55 (B) 35 (C) 70 (D) 45 (E) NOTA

35. Herman received $100 from his humorous grandmother to "save for a rainy day." Herman lives in the desert, and it hasn't rained since before he was born. Nevertheless, he put the money into the bank with a rate of 8%. In two years, how much more interest (to the nearest cent) will he earn on his money if he chooses to compound continuously instead of quarterly?

(A) $0.18 (B) $4.56 (C) $67.74 (D) $72.48 (E) NOTA

36. Determine the sum of *x*, *y*, and *z* in the system of equations:

$$\begin{aligned} 3x+2y-z&=1\\ 5x+4y-2z&=2\\ -x+6y-z&=5 \end{aligned}$$

(A) 2 (B) 3 (C) -2 (D) -3 (E) NOTA

37. If $f(x)=ax^4-bx^3+cx+d$, what is the sum of the reciprocals of the roots?

(A) $-\frac{c}{d}$ (B) $ac+db$ (C) $\frac{c}{d}$ (D) $-\frac{bc}{ad}$ (E) NOTA

38. Wendy's has square prism hamburgers, and McDonald's has cylindrical ones. If we set the side length of Wendy's hamburgers equal to the diameter of McDonalds', what is the ratio of McDonald's hamburger volume to Wendy's, assuming equal depth?

(A) $\frac{\pi}{6}$ (B) $\frac{6}{\pi}$ (C) $\frac{\pi}{4}$ (D) $\frac{3}{\pi}$ (E) NOTA

39. Tony and Mark's project is due tomorrow morning. Their project is to fold 250 origami cranes. If Tony can fold 5 cranes in 28 minutes and Mark takes 42 minutes to fold ten cranes, how long does it take the two of them to finish their project (to the nearest minute)?

(A) 9 hours, 51 minutes (B) 10 hours, 0 minutes
(C) 10 hours, 3 minutes (D) 10 hours, 7 minutes (E) NOTA

40. The architecture class at a certain college made a model which is a 25cm:1m scaling of their classroom. A wall in the model is 40cm x 60cm. What is the largest number of posters of size .6m x 1m the students can fit on the wall of their classroom?

(A) 4 (B) 5 (C) 6 (D) 8 (E) NOTA