Round one

Algebra

- 10 Simplify: $3.375^{\frac{2}{3}}$. Give answer in decimal form.
- 20 Solve for x: $\log(\log x) = 2$

 10^{100}

(2.25)

A cubic polynomial function with leading coefficient 1 has roots 2, -1, and 3. What is the coefficient of quadratic term of the polynomial?
 (-4)

40
$$A = \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$$
 What is det $A - \det(A^{-1})$?
(1.5 or 3/2)

50 What is the fourth term of $(a+b)^{11}$?

 $(165a^8b^3)$

Trigonometry

10 Simplify: $\cos 45^\circ \sin 210^\circ - \sin 30^\circ \cos 135^\circ$

(0)

20 Express $\frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x}$ as a single trigonometric function of *x*.

(Tan x)

30 In triangle *ABC*, *AB*=7, *AC*=5, and the measure of angle *A* is 60 degrees. Find *BC*.

 $(\sqrt{39})$

40 A circle of area 25π is circumscribed about a regular hexagon. What is the exact area of the hexagon?

 $\left(\frac{75\sqrt{3}}{2}\right)$

50 Solve $\frac{2\tan x}{1-\tan^2 x} = \sqrt{3}$ on the interval $(0 \le x \le \pi)$.

 $\left(\frac{\pi}{6},\frac{2\pi}{3}\right)$

Calculus

10 At what values of x does the function $f(x) = x^3 + \frac{9}{2}x^2 + 6x + 5$ have local extrema (min and max values)? (x = -2 and -1)

20 Evaluate:
$$\int_{-\pi}^{\pi} |\sin x| dx$$

- 30 What is the average value of $f(x) = x^2 10$ on the closed interval [-1,2]? (-9)
- 40 Find the indefinite integral: $\int xe^x dx$ $((x-1)e^x + c)$
- 50 What is the arc length of $y = \frac{2}{3}\sqrt{(x-1)^3}$ from (1,0) to (9, $\frac{32\sqrt{2}}{3}$)? Answer may be given as a mixed number.

$$\left(17\frac{1}{3}\right)$$
 or $\left(\frac{52}{3}\right)$

(4)

Geometry

- 10 When pi is rounded to 7 places after the decimal, what is the final digit?
 - (7)
- 20 What is the area of a triangle with sides 6, 11, and 15? Give exact answer in simplified form.

 $(20\sqrt{2})$

 $(\frac{24\sqrt{13}}{13})$

- 30 The diagonals of a rhombus are 8 and 12. What is its altitude? Give exact answer in simplified form.
- 40 Each interior angle of a regular polygon measures 160 degrees. How many diagonals does it have?

(135)

50 A 16 x 12 rectangle is inscribed in a great circle of a sphere. What is the exact volume of the sphere?

 $\left(\frac{4000\pi}{3}\right)$

Probability and Statistics

- 10 A distribution with median 40 is skewed to the right. The mean is:
 - a. greater than 40.
 - b. In the interval [32, 48].
 - c. Less than 40.
 - d. Insufficient information.

(a)

- 20 Which of the following statements are equivalent?
 - a. A and B are independent. b. P(A | B) = P(A). c. A and B are disjoint. d. $P(A \cup B) = 1$. e. $P(A \cap B) = P(A | B)P(A)$. (a,b)
- 30 A distribution has mean μ and standard deviation σ . What theorem tells us that as n becomes large, the sampling distribution becomes normally distributed with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$?

(The Central Limit Theorem)

40 Each morning Curly flips two fair coins. If they both land on heads, he randomly selects a suit from closet A; otherwise he randomly selects a suit from closet B. Closet A has 10 green, 5 red and 5 blue suits; closet B has 4 green, 14 red and 2 blue suits. What is the probability Curly got his suit from closet A, given the suit is green? Give answer as a reduced fraction.

(5/11)

50 A **non-normal** distribution has mean 50 and standard deviation 10. At least what portion of the data points lie in the interval [30, 70]?

(3/4)

Round two with answers

Algebra

10 Solve the system:
$$\begin{cases} 3x - 2y = 8\\ x + 6y = 6 \end{cases}$$

- $\left(3,\frac{1}{2}\right)$
- 20 Convert 1011_2 to base three. Read the digits of the answer from left to right; do not use base ten terms.

 (102_{3})

30 How many subsets may be formed from a set of 12 things?

(4096)

40 What is the sum of the roots of $3x^3 - 6x^2 - 39x - 30 = 0$?

(2)

- 50 Let x = A and y = B be the vertical and horizontal asymptotes of $f(x) = \frac{2x-5}{6-3x}$. What are the Cartesian coordinates of the vertex of $y = Ax^2 + Bx$?
 - $\left(\frac{1}{6}, -\frac{1}{18}\right)$

Calculus

10 If
$$f(x) = 2x^{5} - 4 \ln x$$
, find $f''(1)$.
(44)
20 Evaluate: $\int_{0}^{1} x \sin(\pi x^{2}) dx$
 $(\frac{1}{\pi})$
30 Evaluate: $\lim_{x \to 0} \frac{\cos x - 1}{x^{2}}$
 $(-\frac{1}{2})$

40 Evaluate:
$$\int_{0}^{\frac{1}{2}} xe^{2x} dx$$
 $(\frac{1}{4})$

50 If
$$y = \frac{4x-5}{3x+2}$$
, What is $y'(2)$?

Geometry

10 The volume of cube A is $\frac{1}{64}$ the volume of cube B. If the length of an edge of cube B is 128, what is the length of an edge of cube A?

(32)

20 How many diagonals are there in a 13-sided polygon?

(65)

30 Ray AC and ray AB are tangent to circle H at C and B. AB = AC, BC = 10, and $m \angle BAC = 60^{\circ}$. What is the area of circle H?

$$(\frac{100\pi}{3})$$

40 A circle has area 100. Square A is inscribed in the circle and square B is circumscribed about the circle. What is the area between squares A and B?

$$(\frac{200}{\pi})$$

50 The coordinates of the vertices of a triangle are (15, 3), (23, 11) and (7, 22). What are the coordinates of the centroid of the triangle?

 $\{(15, 12)\}$

Probability and Statistics

10 In a group of 150 people, 30 are dumb, 90 are silly and 50 are neither. How many are dumb, but not silly?

(10)

20 The odds against Q are 7:5 What is P(Q)?

 $(\frac{5}{12})$

30 The number of rat hairs in a small container of yogurt is normally distributed with mean 8.3 and standard deviation 1.6. Approximately what percent of the population will be in the interval [5.1, 11.5]?

(95%)

40 My trick coin has $P(heads) = \frac{1}{5}$. What is the probability that I get exactly 3 heads when flipping the coin 4 times?

 $(.0256 \text{ or } \frac{16}{625})$

50 Evaluate: $\int_{-1.645}^{1.645} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$. Answers within .0001 of the actual value will be accepted.

(.9)

Trigonometry

10 Evaluate: $\cot \frac{17\pi}{6} - \sqrt{6} \csc \frac{-9\pi}{4}$

 $(\sqrt{3})$

20 In triangle ABC, AB = 10, BC = 12 and $m \angle B = 45^{\circ}$. What is the area of triangle ABC?

 $(30\sqrt{2})$

 $(15\sqrt{3})$

30 A circle with radius 5 is circumscribed about an equilateral triangle. What is the perimeter of the triangle?

40 In triangle ABC, AB = 5, BC = 7 and AC = 8. What is $m \angle A$?

 $(60^{\circ} \text{ or } \frac{\pi}{6})$

50 Which one of the following is not true?

a.
$$\frac{1}{1 + \cos A} + \frac{1}{1 - \cos A} = 2\csc^2 A$$

b. $\frac{1}{\tan A} + \tan A = \cos A \sin A$
c. $\frac{\sec^2 A - \tan^2 A}{1 + \cot^2 A} = \sin^2 A$

(b)

Final Jeopardy

Colorado Vacation

- 10 An old west cookout is a great fun. At Alfred Packer's Mule Train you get a salad, two main dishes and two deserts. If there are 8 salads, 10 main dishes and 6 deserts available, how many times can you go to Alfred's without having the exact same meal? (5400)
- 20 Pat and Chris went hiking up Pikes Peak. Pat walks at a rate of 2640 inches per minute. How many miles per hour must Chris walk to keep up with Pat?

(2.5)

30 Eddie and Paula are lost in South Park. Eddie begins walking due east, while Paula walks north. After one hour they are 5 miles apart. If Paula walks twice as fast as Eddie, how many miles per hour does Eddie walk? Give answer to the nearest quarter mile per hour.

(2.25)

40 Fly by Night Airlines loses 50% of all the luggage they handle at Denver International Airport. Fred is using Fly by Night for his dream vacation to Commerce City, Colorado. If he checks 4 bags, what is the probability that exactly 2 get lost?

(.375 or 3/8)

50 Sister Scholastica spent the winter camping in the mountains. One day she noticed that the Fahrenheit and Celsius thermometers were registering the same temperature. What was the temperature?

(-40)

Love is a many splintered thing

10 Will's love is described by 2x+4y=5 and Grace's love is described by -4x+2y=-20. At what point will they meet and fall in love?

(4.5, -1)

20 Barney is an equilateral triangle. A circle inscribed in Barney is filled with thoughts of Betty. If the circle has radius 10, what is Barney's area?

 $(300\sqrt{3})$

30 Ellen is planning her wedding. What is the minimum time for her to complete the tasks listed, given the amount of time they take and their prerequisites? Note that some events may be done concurrently.

Event	Time	Prerequisite(s)			
Find husband (A)	10	-			
Set date (B)	2	А			
Choose minister (C)	3	А			
Tell parents (D)	4	А			
Reserve church (E)	4	B, C			
Invite friends (F)	1	D, E			
		(18)			

40 Over time, four brothers' love for their girl friends grows as follows. Groucho: $\ln t^{t}$ Harpo: $t^{\sqrt{t}}$ Zeppo: 2^{t} Chico: t^{2} Rank the brothers from slowest to fastest in terms of the growth of their love as time becomes large.

(Groucho, Chico, Harpo, Zeppo)

50 Romeo is the function $y = x^2$ and Juliet is the function $y = \sqrt{x}$ What is the area between Romeo and Juliet, from zero to one?

(1/3)

Play ball!

10 The distance between bases is 90 feet. When Larry Walker hits a homerun, he runs along the baselines, making right turns at each base. How many inches does he run?

(4320)

20 John Elway has a small football field in his back yard. It is twice as long as it is wide, and has an area of 3200 square feet. What is the perimeter of his football field?

(240 ft.)

30 Antonio McDyse has a basketball team with 7 guards, 6 forwards, and 5 centers. How many ways can he pick a team of 2 guards, 2 forwards and 1 center. Note: Do not distinguish among guard or forward positions.

(1575)

40 Statistics show that goalies average 12 saves per game. A sample of 64 goalies had an average of 10 saves per game with a standard deviation of 4. To test the hypothesis that this group of goalies gets fewer saves than average ($H_A : \mu < 12$), you perform a one-sample z-test. What is the value of z?

(-4)

50 A golf ball travels in a path described by the function $f(t) = -10t^2 + 60t$, where *t* is time in seconds and f(t) is the height of the ball in feet at time *t*. What is the maximum height the ball will reach?

(90 ft.)

School Daze

10 Five Stoneman-Douglas students ditched five fifty minute classes per day for five weeks (five days in a week). How many total minutes of class did they ditch?

(31250)

20 Mr. Fukuhara's G.P.A. through 7 semesters of high school was 3.00. He got a bad case of senioritis and failed all his classes 2nd semester senior year (his *first* year as a senior). What was his exact G.P.A. after 8 semesters? Assume the same number of classes each semester, none weighted.

(2.625)

30 Each week of his senior year, Mr. Koski forgot 10% of what he knew the week before (and learned nothing new). How many weeks was it before he forgot more than 50% of all he knew before the year began?

40 Ms. Tipton slept through much of her senior year at Fibonacci High. The number of minutes she slept each day followed the pattern below. On what day did she sleep her 500th minute?

Day	1	2	3	4	5	6	7	8
Minutes	1	1	2	3	5	8	13	21

⁽⁷⁾

50 *C*arl *u*ndermines his *t*eachers by *c*laiming *l*ittle *a*ctual social sense. Each day Carl flips a coin that has probability of heads .4 and cuts class if the coin lands on tails. What is the probability of him cutting class exactly 3 out of 5 days?

(.3456 or 216/625)

There ain't no cure for the summer time blues

10 Madonna is working at Taco Bell this summer. They start her at \$7 per hour, then give her a 20% raise, followed by a \$.50 raise. New management comes in & gives her a 20% cut in pay. How much per hour is she now making?

(\$7.12)

20 Cher can move half a load of dirt in half the time that Britney can move two loads of dirt. Christina moves dirt at a rate equal to the sum of the rates of both Cher and Britney. If it takes Christina 6 hours to move a load of dirt, how long does it take Cher to move a load of dirt?

30 Your student president, David Smith, works on a road repair crew that has 3 student summer employees and one career employee. Regulations say only one person can work at a time - the others are watchers. The career employee works 7% of the time, and the students split the remaining time equally. Given David is not working, what is the probability the career employee is working? Round answer to the nearest percent.

(.10 or 10%)

⁽² hrs.)

40 Mr. Norris has a summer job as a spring. A force of 9 pounds is required to stretch him from his natural length of 6 feet to a length of 8 feet. What is the work that must be done to stretch Mr. Norris from his natural length to a length of 10 feet?

(36 in.-lb.)

50 Dr. Morris has a summer job being a point on a vector in three dimensions. If the magnitude and direction of a constant force are given by $5\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$, find the work done to move Dr. Morris from (1, -1, 2) to (4, 3, -1).

(5)