Mu Alpha Theta National Convention: Denver, 2001
School Bowl – Alpha Division

1. Two dice are rolled behind a screen. A friend of yours looks at the dice and tells you (honestly) that there are no sixes. What is the probability there is at least one five?

2. Starting with a black cube, I pick two faces at random and paint each of them either white or red (they do not have to be the same color). How many visually distinguishable patterns can I produce with this method?

3. A solid concrete block in the shape of a cube with edge lengths of one meter rests on the bottom of the ocean. A starfish is on the cube on one of the upper corners. A mussel is at the base of the cube at the farthest corner from the starfish. What is the minimum distance, in meters, the starfish must travel to reach the mussel?

4. What is the probability that when a slug types each of the letters A-Z exactly once (this takes a while), either of the strings "SLIME" or "CRAWL" appears? You may express your answer in terms of factorials.

5. Evaluate: $\sum_{n=0}^{5} \cos\left(\frac{\pi}{15} + \frac{\pi n}{3}\right)$

6. Evaluate: $\log_8 81 \cdot \log_9 125 \cdot \log_{25} 16$

7. Simplify: $\left(\log_c b^{\log_a b}\right)\left(\log_a c^{\log_b a}\right)$

8. Simplify: $\sqrt{259 + 72\sqrt{3}}$

9. What is the sum of the squares of the roots of the equation $4x^3 - 2x + 1 = 0$?

10. In circle O, there are two parallel chords on the same side of the center and five centimeters apart, whose lengths are twenty and eight centimeters. How far is the twenty-centimeter chord from the center of the circle?

11. What is the equation of the plane through the points $(1, 3, -3)$, $(-2, 1, 5)$, and $(-4, -2, -2)$?

12. How many non-degenerate & non-congruent triangles have integer side lengths and perimeters equal to twenty?

13. What is the value of the constant term in the expansion of $\left(2x^2+1-\frac{3}{x}\right)^3$?

14. For how many real numbers n is $(n+\pi i)^4$ a pure imaginary number?

15. How many natural numbers are factors of 7560 and multiples of 14?

16. Two complex numbers, A and B, sum to $-8+2i$. Their product is twice the result of dividing B by the square of A. Determine the sum of all possible values of A.

17. Factor completely over the rational numbers: $x^{12} - y^{12}$.

18. What is the smallest integer greater than $\left(\sqrt{3}+\sqrt{7}\right)^2$?

19. Determine $vwxyz$ from the following system of equations.

$$
\begin{aligned}
v+w+y+z &= 21 \\
w+x+y+z &= 11 \\
v+w+x+y &= 13 \\
v+w+x+z &= 9 \\
v+x+y+z &= 14
\end{aligned}
$$

20. Lori likes her chocolate milk to be 12% chocolate syrup. Tom accidentally makes it with 8% chocolate syrup, filling a glass with a volume of one-half of a liter. How many liters of chocolate milk must Lori pour out, to be replaced by an equal volume of chocolate syrup, to result in her ideal 12% mix?

21. Express, in base nine, the result when 41201_9 is divided by 152_9.

22. Each person in a group of 38 people shakes hands with exactly three other people in the group. How many total handshakes take place?

23. The sum of the first three terms of a real-valued geometric sequence is 30, while the sum of the first six terms of the same sequence is 270. What is the common ratio of the sequence?

24. If $\sin A = \frac{12}{13}$, where $-\frac{\pi}{2} < A < \frac{\pi}{2}$, and $\cos B = -\frac{3}{5}$ where $0 < B < \pi$, what is the value of $\tan(A+B)$?

25. If $N \equiv 2 \bmod 3$ and $N!$ ends in 221 zeros, what is the largest possible value of N?

26. In the base six addition problem shown, each letter stands for a unique digit in base six. What is the largest possible value of T?

$$\begin{array}{rccccc} & & A & A & C & S \\ + & & & T & O & O \\ \hline & S & C & O & O & T \end{array}$$

27. Given that A is a six-digit number with a units digit of 1, and B is a natural number which is the fourth-root of A, what is the largest possible value of B?

28. Two circles have radii that differ by five centimeters. If the length of the common internal tangent is 10 centimeters, and the length of the common external tangent is 12 centimeters, how many centimeters apart are the centers of the circles?

29. How many sets of two or more consecutive integers exist such that the product of their elements is equal to 360?

30. What is the smallest natural number which is twice a perfect cube and one-third of a perfect square?

31. Determine the sum of all values of *x* for which $4\left(\log_9 x^{\log_3 x}\right) - 6\log_{27} x - 1 = 11$.

32. In regular tetrahedron *ABCD*, *E* is the midpoint of $\overline{BC}$. What is the cosine of angle *ADE*?

33. A group of friends splits the cost of a cabin at a ski resort evenly. If one fewer person had come, the cost per person would have gone up by five dollars. If two more people had come, the cost would have gone down by four dollars. What was the cost of the cabin, in dollars?

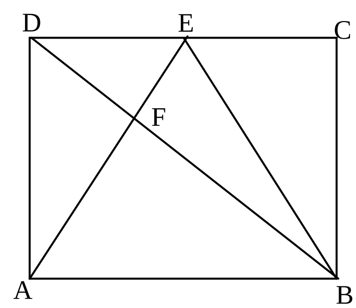
34. Determine the sum of all values of x ($0 < x \le 2\pi$) for which $2\left(\cos^2(2x)\right) + 3\sin(2x) = 0$.

35. Find the sum of the three smallest natural numbers with at least nine positive integral factors.

36. How many values of x satisfy $1 - \frac{x}{48\pi} = -\sin x$?

37. Evaluate: $\sum_{n=5}^{32}\left(2n^2-3n-4\right)$

38. In the figure shown, *ABCD* is a rectangle, *AEB* is an equilateral triangle, and diagonal $\overline{DB}$ intersects $\overline{AE}$ at *F*. What is the ratio of the area of triangle *AFB* to the area of rectangle *ABCD*?



39. Consider a triangle with sides of lengths 12, 6, and 15 centimeters. How long, in centimeters, is the angle bisector to the 15-centimeter side?

40. Two dice are rolled. What is the probability that the sum of the numbers on their top faces is a prime number?