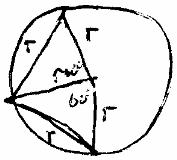
Mu Alpha Theta National Convention: Denver, 2001

School Bowl Solutions – Mu Division

1. $A = 4$, $T = \frac{2\pi}{3\pi} = \frac{2}{3}$

$4 + \frac{2}{3} = \frac{14}{3}$

2. 

$\frac{360-120}{360} = \frac{240}{360} = \frac{2}{3}$

3. $6 \cdot 5 = 30 = 20_x \Rightarrow x = 15$

$15_{15} = 20_{10}$ $\quad \frac{20}{4} = 5 = 5_{15}$

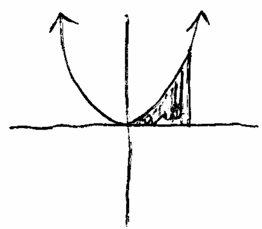
4. 2 of the same color $\Rightarrow$ 4 marbles (because there are 3 colors).

Because there are only three of each color, this also guarantees a different color.

5. To be divisible by 3, the digits must sum to a multiple of 3, and thus the number of 2's must be a multiple of 3. To be divisible by 2, the last digit must be a 2. 222

6. # ways to get $GX = \binom{4}{1}\binom{10}{1} = 40$

ways to avoid XX
$\binom{14}{2} - \binom{3}{2} - \binom{2}{2} - \binom{5}{2} - \binom{4}{2} = 71$ $\quad \frac{40}{71}$

7. 

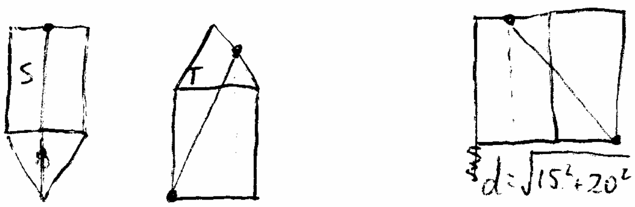
$\int_0^4 \pi (x^2)^2 \, dx = \pi \frac{x^5}{5}\Big|_0^4 = \frac{1024\pi}{5}$

8. $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = 27\pi^3 - 9\pi$

9. This is almost exactly half of a row of Pascal's Triangle.

$\frac{1}{2}(2^{21}) - 1 = 2^{20} - 1 = 1024^2 - 1 = 1048575$

10. "unfold" the terrarium



$d = 20 + 5\sqrt{3}$ $\quad d = \sqrt{(7.5)^2 + (20 + 2.5\sqrt{3})^2}$ $\quad d = \sqrt{15^2 + 20^2} = 25$

11. $3, a, b, c \Rightarrow \binom{6}{3} = 20$ "good" ways

$\binom{9}{4} = 126$ total ways $\frac{20}{126} = \frac{10}{63}$

12. $\left.\frac{\tan x(1-\cos x)}{\sin^2 x}\right|_{x=0} = \frac{0}{0}$

$\left.\frac{\sec^2 x(1-\cos x) + \tan x \sin x}{2\sin x \cos x}\right|_{x=0} = \frac{0}{0}$

$\left.\frac{\sec^2 x \sin x + 2\sec^2 x \tan x(1-\cos x) + \sec^2 x \sin x + \tan x \cos x}{2\cos^2 x - 2\sin^2 x}\right|_{x=0} = \frac{0}{1}$

13. $9, 15, 12, \frac{27}{2}, \ldots$

$+6 \quad -3 \quad +\frac{3}{2} \quad \ldots$

$9 + \frac{6}{1 - -\frac{1}{2}} = 9 + 4 = 13$

14. 1 cup: 10 2: 5, 5 3: 7, 2, 1

4: 2, 2, 2, 4 5: 2, 2, 2, 2, 2 6-10: use 1's & 2's.

10

15. $P(C) = 1 - P(T)$

$\binom{4}{2} T^2 (1-T)^2 = \binom{4}{3} T (1-T)^3$

$6T = 4(1-T)$

$10T = 4$

$T = \frac{2}{5}$

16. $f(0) = 8, \ f(4) = 192$

$f'(x) = 9x^2 - 2 \Rightarrow x = \pm\sqrt{\frac{2}{3}}$

$f''(x) = 18x \Rightarrow x = \frac{+\sqrt{2}}{3}$ is a local minimum.

17. $\frac{\int_0^\pi x \sin x \, dx}{\pi}$ $\quad u = x \quad v = -\cos x$
$\quad du = dx \quad dv = \sin x \, dx$

$\frac{1}{\pi}\left[-x\cos x\Big|_0^\pi - \int_0^\pi -\cos x \, dx\right]$

$\frac{1}{\pi}\left[\pi + \sin x\Big|_0^\pi\right] = 1$

18. $2^a 5^b$

$a = 2n+1 \quad b = 2o$

$a = 3m \quad b = 3p - 1$

$a = 3 \quad b = 2$

$2^3 \cdot 5^2 = 200$

19. $C = NP = (N-3)(P+4) = (N+6)(P-5)$

$$4N - 3P = 12$$
$$-5N + 6P = 30$$
$$3N = 54$$
$$N = 18 \Rightarrow P = 20$$
$$C = 360$$

20. $\Sigma e = 129$

$$\Sigma_o = \left(129 + \tfrac{n-1}{2}d\right) + a = 172$$
$$\text{last} - \text{first} = 42 = (n-1)d$$
$$129 + \tfrac{42}{2} + a = 172$$
$$a = 22$$

21. 

$$t = \frac{d_1}{6} + \frac{d_2}{3} + \frac{d_2}{12} + \frac{d_1}{4} = 6$$
$$\frac{5d_1 + 5d_2}{12} = 6$$
$$d_1 + d_2 = \frac{72}{5} \Rightarrow \text{total} = \frac{144}{5}$$

22. Its possible for both to occur.

$$P(\text{slither}) = \frac{20!}{26!}$$
$$P(\text{fang}) = \frac{23!}{26!}$$
$$P(\text{both}) = \frac{17!}{26!}$$
$$\frac{20!}{26!} + \frac{23!}{26!} - \frac{17!}{26!}$$

23. $3\left[\frac{21^2 \cdot 22^2 - 6^2 \cdot 7^2}{4}\right]$

$$+2\left[\frac{21 \cdot 22 - 6 \cdot 7}{2}\right]$$
$$+5[21-6]$$
$$= 158{,}760 + 420 + 75$$
$$= 159{,}255$$

24. Its an odd function so that any integral centered at zero = 0

25. $\frac{\partial P}{\partial x} = \frac{x}{8}$

$$\int_0^4 \frac{x}{8}\sqrt{x} = \frac{1}{8}\int_0^4 x^{3/2}$$
$$\frac{1}{8} \cdot \frac{2}{5} x^{5/2}\Big|_0^4 = \frac{1}{8} \cdot \frac{2}{5} \cdot 32 = \frac{8}{5}$$

26. $AB = \frac{1}{5}AD$, $CD = \frac{1}{3}AD \Rightarrow BC = \frac{7}{15}AD$

$$\frac{ABGH}{ABF} = \left(\frac{2}{3}\right)^2 - \left(\frac{7}{15}\right)^2 = \frac{100-49}{225} = \frac{51}{225} = \frac{17}{75}$$

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27. $\frac{n(n+1)}{2}$ #1

$45 = 3^2 \cdot 5$
$46 = 2 \cdot 23$
$47 = 47$
$48 = 2^4 \cdot 3$
$49 = 7^2$
$50 = 2 \cdot 5^2$] $\frac{49 \cdot 50}{2} = 1225 = 35^2$

28. $\underline{0}\ \underline{43}\ \underline{43}\ \underline{x}\ \underline{71}\ \underline{y}\ \underline{y}\ \underline{y}\ \underline{100}$

$257 + x + 3y = 9 \cdot 65 = 585$
$x + 3y = 328$
$y = 100,\ x = 28$ works 100

29. $\ln y = \cancel{x^2 \ln(2x)} = \ln 3 + x^2 \ln x$

$y' = y\left[2x \ln(x) + \frac{x^2}{x}\right]$
$y'(2) = y(2)\left[4 \ln 2 + 2\right]$
$= 48(4 \ln 2 + 2)$
$= 192 \ln 2 + 96$

30. 3 cases: $\binom{7}{7} 2^7 + \binom{7}{2}(x)^2 \binom{5}{1}\left(\frac{-2}{x^2}\right)^1 \binom{4}{4}(2)^4 + \binom{7}{4}(x)^4 \binom{3}{2}\left(\frac{-2}{x^2}\right)^2 \binom{1}{1}(2)^1$

$= 128 + {-3360} + 840 = -2392$

31. $h' = r' = x$

$V = \frac{1}{3}\pi r^2 h$
$V' = \frac{1}{3}\pi(2r r' h + r^2 x)$
$16 = \frac{1}{3}\pi(2 \cdot 8 \cdot 12x + 64x)$
$\frac{48}{\pi} = (256x) \Rightarrow x = \frac{3}{16\pi}$

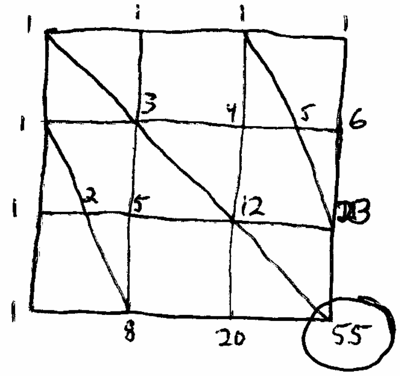
32. $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\frac{12}{5} + \frac{-3}{4}}{1 - \frac{12}{5} \cdot \frac{-3}{4}} = \frac{33}{56}$

33. A ends in 9 $\Rightarrow$ B ends in 3 or 7

$B < 10^{10/6} = 10 \cdot \sqrt[3]{100} \approx 46 \Rightarrow 43$

34. Calculate the number of ways to get to each vertex, moving down & right.



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35. $10 = 2\cdot 5$ $\quad 12 = 2\cdot 6 \quad = 3\cdot 4 \quad = 3\cdot 2\cdot 2$

$2^4\cdot 3 = 48 \quad 2^5\cdot 3 = \cancel{96} \quad 2^3\cdot 3^2 = 72 \quad 2^2\cdot 3\cdot 5 = 60$

$2^4\cdot 5 = 80 \quad 2^2\cdot 3\cdot 7 = \cancel{84}$

$2^4\cdot 7 = \cancel{112}$

$48+60+72+80 = 260$

36. $\sum_{1}^{23} \left(e^{\frac{-\pi i}{3}} e^{\frac{-\pi i}{4}}\right)^n$

$= \sum_{1}^{23} e^{\frac{-7\pi n i}{12}}$

↑ repeats every $n=24$

so $\sum_{1}^{24} = 0$

$\sum_{1}^{23} = \sum_{1}^{24} - 1 = 0 - 1 = -1$

38. $\frac{1}{25} + \frac{2}{125} + \frac{3}{625} + \cdots$

$= \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \cdots = \frac{1}{20}$

$+ \frac{1}{125} + \frac{1}{625} + \cdots = \frac{1}{100}$

$+ \frac{1}{625} + \cdots = \frac{1}{500}$

$\frac{1}{16}$

37. $y = 3^x \quad 3y^2 - 28y + 9 = 0$

$(3y-1)(y-9)$

$y = \frac{1}{3} \quad y = 9$

$x = -1 \quad x = 2$

$-1+2 = 1$

39. A: $x = 5+12s$, $y = 4+3s$, $z = -11-6s$

B: $x = -15+21t$, $y = 14-21t$, $z = -11+7t$

$5+12s = -15+21t \Rightarrow 12s - 21t = -20$

$4+3s = 14-21t \qquad 3s + 21t = 10$

$15s = -10$

$s = -\frac{2}{3} \Rightarrow t = \frac{4}{7}$

$x = -3, \quad x = -3$

$y = 2, \quad y = 2$

$z = -7, \quad z = -7$ ✓

40.

6B	0W	1
6W	0B	1
5B	1W	1
5W	1B	1
4B	2W	2
4W	2B	2
3B	3W	2

$\Rightarrow 10$