Mu Alpha Theta National Convention: Denver, 2001
School Bowl – Mu Division

1. What is the sum of the amplitude and period of the graph of $y = 4\cos(3\pi x)$?

2. Two points are chosen on the circumference of a circle. What is the probability that the chord connecting these two points is longer than the radius of the circle?

3. Given that all numbers in this problem and the answers are expressed in the same base, if one fifth of 20 is 6, what is one fourth of 15?

4. A bag contains three blue marbles, three green marbles, and three red marbles. What is the minimum number of marbles I must withdraw (without looking) to ensure that I will have removed at least two marbles of the same color and at least one marble of a different color from the matched pair?

5. What is the smallest natural number composed of no digits other than 2's and 3's when written in base ten, which is divisible by both two and three?

6. A bag contains three white marbles, two red marbles, five blue marbles, and four green marbles. Two marbles are drawn at random from the bag by an impartial judge who examines them and states truthfully that they are not the same color. What is the probability that one of the marbles is green?

7. What is the volume of the solid generated when the region satisfying $y < x^2$, $y > 0$, $x > 0$, and $x < 4$ is rotated about the x-axis?

8. If $x + \frac{1}{x} = 3\pi$, what is the value of $x^3 + \frac{1}{x^3}$?

9. Evaluate: $\sum_{n=11}^{20} \binom{21}{n}$

10. A terrarium is in the shape of a right triangular prism, the base and top of which are equilateral triangles with sides of ten centimeters, and the height of which is twenty centimeters. A millipede at the center of one edge of the top of the terrarium. His food is at the base of the terrarium, in the corner furthest from him. What is the minimum distance, in centimeters, he must crawl to get his food?

11. Four distinct numbers are chosen from the first nine natural numbers. What is the probability that 3 is the smallest of the numbers chosen?

12. Evaluate: $\lim_{x \to 0} \frac{\tan x(1-\cos x)}{\sin^2 x}$

13. A sequence exists for which $S_1 = 9$, $S_2 = 15$, and $S_n = \dfrac{S_{n-1} + S_{n-2}}{2}$. Evaluate: $\lim\limits_{n\to\infty} S_n$.

14. For how many values of *n* from one to ten inclusive is it possible to distribute ten pennies in *n* Styrofoam cups so that no cup contains a number of pennies that is a multiple of three and so that no cup is empty?

15. When Chuck and Tom play pool, the probability that Chuck wins exactly two games out of four is equal to the probability that he wins exactly three games out of four. What is the probability that Tom wins any given game? Assume that the outcome of a game cannot affect the probabilities in the other games, and that both players have a non-zero chance of winning a game.

16. What is the maximum value of the function $f(x) = 3x^3 - 2x + 8$ for $0 \le x \le 4$?

17. What is the average value of $y = x\sin x$ for $0 < x < \pi$?

18. What is the smallest natural number which is twice a perfect square and one-fifth of a perfect cube?

19. A group of friends evenly divides the cost of a cabin at a ski resort. If three fewer people had come, the cost per person would have gone up by four dollars. If six more people had come, the cost would have gone down by five dollars. What was the cost of the cabin, in dollars?

20. The number of terms in a finite arithmetic sequence is odd. The sum of the even-numbered terms is equal to 129, while the sum of the odd-numbered terms is equal to 172. The first term is considered an odd-numbered term. If the last term minus the first term is 42, what is the value of the first term?

21. The beginning portion of Tom's hike is level, followed by an uphill section leading to his destination, a hilltop. Tom hikes at a rate of six kilometers an hour on the level section, then trudges uphill at half that speed. Coming downhill, he trips at the top and slides downhill at a rate of twelve kilometers per hour, and is so sore from this that he limps home along the level stretch at four kilometers per hour. If Tom takes six hours to complete his hike, what is the total distance, in kilometers, he traveled?

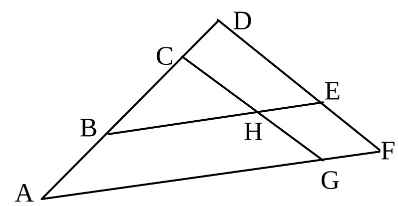
22. What is the probability that when a snake types each of the letters A-Z exactly once, either of the strings "SLITHER" or "FANG" appears? Express your answer as the sum of terms of the form $\pm\frac{a!}{b!}$, where a and b are natural numbers.

23. Evaluate: $\sum_{n=7}^{21}\left(3n^3+2n+5\right)$

24. Evaluate: $\int_{-3}^{3}\left(-12x^{11}+6x^7+3x^3+9x\right)dx$

25. The probability that R is less than x is $P_R(x) = \frac{x^2}{16}$ for $0 \le x \le 4$, is 1 for $x > 4$, and 0 for $x < 0$. What is the expected value of the square root of R?

26. In triangle ADF, line segment $\overline{BE}$ is drawn parallel to $\overline{AF}$, such that $4AB = BD$. Line segment $\overline{CG}$ is drawn parallel to $\overline{DF}$, intersecting $\overline{BE}$ at H, such that $3CG = 2DF$. What is the ratio of the area of trapezoid $ABHG$ to that of triangle ADF?



27. What is the smallest number greater than one-thousand which is both a perfect square and a triangular number?

28. A data set has 9 elements, all of which are integers between 0 and 100, inclusive. If the mode of the data set is 43, the median is 71, and the arithmetic mean is 65, what is the largest possible value of the range?

29. Determine $y'(2)$ if $y = 3x^{x^2}$.

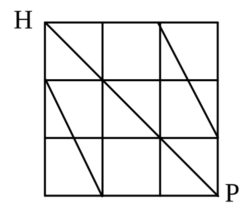
30. What is the value of the constant term in the expansion of $\left(x + 2 - \frac{2}{x^2}\right)^7$?

31. Consider a conical sponge, which when exposed to water, increases its volume at a rate of sixteen cubic centimeters per second, subject to the constraint that the height and base radius increase at the same rate when measured in centimeters per second (i.e. it will remain conical, but its aspect ratio may change). How quickly (in centimeters per second) is the height of the sponge changing when it has a base radius of eight centimeters and a height of twelve centimeters?

32. If $\tan A = \frac{12}{5}$, where $-\frac{\pi}{2} < A < \frac{\pi}{2}$, and $\tan B = -\frac{3}{4}$ where $0 < B < \pi$, what is the value of $\tan(A+B)$?

33. A is a ten-digit number a units digit of 9. B is a natural number which is the sixth-root of A. What is the largest possible value of B?

34. A man wishes to walk from his house (H) to the park (P), which is three blocks south and three blocks east of his house (see map to right). How many possible routes could the man take if he must be decreasing his distance (as the crow flies) from the park at all times and may turn (or not) any time two streets cross?



35. Find the sum of the four smallest natural numbers with at least ten positive integral factors.

36. Evaluate: $\sum_{n=1}^{23}\left[\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)\right]^n$

37. Find the sum of all real values of x that satisfy $3^{2x+1}+9=28\cdot 3^{x}$.

38. Evaluate $\sum_{n=2}^{\infty}\frac{n-1}{5^{n}}$.

39. Line A passes through the points $(5,4,-11)$ and $(17,7,-17)$. Line B passes through the points $(-15,14,-11)$ and $(6,-7,-4)$. At what point, if any, do these points intersect?

40. The sides of a cube are each painted either black or white. How many visually distinguishable paintings are possible?