Mu Alpha Theta National Convention: Denver, 2001
School Bowl – Theta Division

1. If two men can carve seven statues in three days, how many days would it take three men to carve twelve statues at this rate?

2. If the mean of the degree measures of the interior angles of a convex polygon is 170°, what is the number of sides of the polygon?

3. Tony has a large marble collection, which he normally stores in five bags with equal numbers of marbles per bag. On Tuesdays, he likes to place them in three glass jars, with equal numbers of marbles per jar. This requires that he leave two marbles out of the jars, however. If Tony has over 100 marbles, what is the smallest number of marbles he can have?

4. Bill has two solutions in his chemistry lab. Solution A is 22% sodium, while Solution B is 48% sodium. Bill wishes to mix the two solutions to get a solution that is 30% sodium. What ratio of Solution A to Solution B will allow Bill to achieve his goal?

5. What is the probability that when a bird types each of the letters from A to Z exactly once, the string “SQUAWK” appears? You may express your answer in terms of factorials.

6. What is the volume, in cubic centimeters, of a pyramid with a 6-centimeter by 8-centimeter rectangular base, all other edges of which are 13 centimeters long?

7. What is the smallest natural number that is divisible by three and five and can be written using no digits other than threes and fives when expressed in base ten?

8. What is the coefficient of the x^2yz^2 term in the expansion of $(2x+y-3z)^5$?

9. Solve for *j*: $4^{32^j} = 16^{8^j}$

10. In sequence *S*, $S_0 = 2$, $S_1 = 3$, and $S_n = \dfrac{S_{n-1}}{S_{n-2}}$. Calculate the value of S_{20}.

11. A sector with an acute central angle of 120º is cut from a circle of radius *R*. A circle of radius *r* is then inscribed within that sector. What is the ratio of *r* to *R*?

12. If $\log_5 2 = b$, and $\log_5 3 = c$, express $\log_{36} 25$ in terms of *b* and/or *c*.

13. Simplify: $\sqrt{67+42\sqrt{2}}$

14. Express 10427_8 in base 16.

15. Three people are shown 2 red hats, a black hat, and a green hat that may be placed upon them. They are seated one behind the other, so that person A can see persons B and C, person B can see person C, and person C can see no one, and then three of the four the hats are placed upon them, one per person. What is the probability that person A can determine the color of her own hat?

16. Given that $N!$ ends in exactly 27 zeros, and N is a multiple of three, what is the largest possible value of N?

17. How many sets of two or more consecutive natural numbers exist, such that the sum of their elements is 36?

18. George has a coin collection consisting of only nickels, dimes, and quarters. The sum of the numbers of nickels and dimes is seven more than the number of quarters, the total amount of money in his coin collection (sadly, none of his coins are rare enough to be worth more than their original value) is $21.50, and the number of dimes is twenty-nine less than the sum of the numbers of nickels and quarters. What is the total number of nickels in George's coin collection?

19. A right-cylindrical water glass has a circumference of 24 centimeters and a height of 20 centimeters. An ant is on the outside of the glass two centimeters below the rim, and a speck of sugar is on the counter at the base of the opposite side of the glass. What is the minimum distance (in centimeters) the ant must travel to get the sugar?

20. How many distinct lines are there which pass through at least two points of the form (x, y), where both x and y are natural numbers between 3 and 5, inclusive?

21. What is the sum of all values of t for which: $3t-\sqrt{15t+4}=4$?

22. What is the fifth term of the expansion of $(x+3y)^{12}$, when the terms are arranged in decreasing powers of x?

23. Given five lines in the plane, at least two of which are parallel and at least two of which intersect in the point $(3,-4)$, what is the maximum possible number of points of intersection?

24. In the following base four addition problem, each of the letters represents a unique digit in base four. What is the value of A?

$$\begin{array}{rrr} & A & S \\ + & M & A \\ \hline M & M & M \end{array}$$

25. If a cube is to have each of its faces painted either black or white, how many distinguishable patterns can be produced?

26. When the digits of a two-digit number are reversed, the new value is five more than three times the original value. What is the smallest possible value of the original number?

27. If $f(x) = -\dfrac{1}{4x}$ and $g(x) = \sqrt{x^2 - 12x + 32}$, are both real-valued functions, what is the domain of $g(f(x))$?

28. Given that $k(m) = 4m^2 + 16m + 10$ for $m < -4$, evaluate $k^{-1}(43)$.

29. Paula canoed from the mouth of the river to her campsite four kilometers upstream in four hours. The next day, she canoed back to the mouth of the river in three hours. What was the speed of the river, in kilometers per hour, if both the river's speed and Paula's canoeing rate in still water are constants?

30. At Clymer University, only three majors are offered: Mathematics, Physics, and Chemistry. Every one of the 1024 students has at least one major, 666 of the students have at least two majors, and 314 of the students are triple-majors. If 512 students major in Math, what is the largest number of students who can major in both Math and Chemistry, but not Physics?

31. What is the value of the constant term in the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^{15}$?

32. If the roots of $x^2 + px + q = 0$ are each three greater than a different root of $2x^2 - 8x - 5 = 0$, what is the sum of p and q?

33. The sum of the first 12 terms of an arithmetic sequence is 54, while the sum of the first 30 terms of the sequence is 180. What is the first term of the sequence?

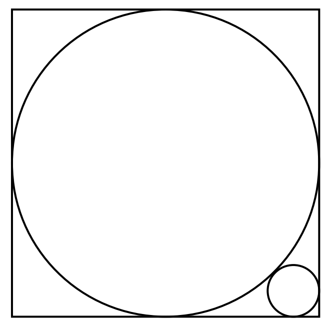
34. If $x \equiv a \bmod 12$ (where a is a natural number between 0 and 11 inclusive) and is a solution of the equation $3x \equiv 5 \bmod 16$, determine the sum of the possible values of a.

35. What is the sum of the digits in the integers from 250 to 750, inclusive?

36. A circle of radius eight intersects a circle with area 32π in such a way that the length of their common chord is 8. What is the area of the intersection of the two circles?

37. What is the area of the largest square that can be inscribed in a right triangle with legs of 4 and $4\sqrt{3}$?

38. Circle A is the largest circle that can be inscribed in square B. Circle C is a smaller circle which is then "inscribed" between circle A and square B (see figure – circle C is tangent to two adjacent sides of square B, and also tangent to circle A without overlapping circle A). What is the ratio of the radius of circle C to that of circle A?



39. A clock has been misconstructed in such a way that its hour hand moves clockwise at a rate of twelve degrees a minute and its minute hand moves counter-clockwise at a rate of thirty degrees a minute. If I set this clock to 12:00 (both hands pointing straight up), how many times will the two hands overlap by the time they first meet at their starting position (including their initial and final collocation)?

40. What is the measure of angle *ABC* in circle *O*, in degrees, if the measure of angle *OCE* is 20° and $\overline{DE}$ is a diameter of the circle and $\overline{EC}$ intersects $\overline{AD}$ at *B*?