Mu Alpha Theta National Convention: Denver, 2001

Denver Bowl Solutions – Alpha Division

1. This is a line in 3-space. If z increases by 1, y also does, & x decreases by 3.

2. $\log(2^{10}) + \log(5^{10}) = \log(2^{10} \cdot 5^{10})$
$= \log(10^{10}) = 10$

3. 9 w/ side = 1
3 w/ side = 2
1 w/ side = 3
13

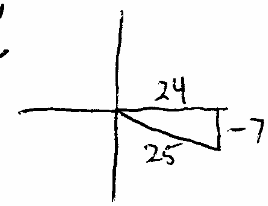
4. $41 = a + 9d$
$S = (a + (a+18d))\frac{19}{2} = 19(a+9d)$
$= 19 \cdot 41 = 779$

5. First card doesn't matter: p = 1
2nd card: $\frac{3}{51} = \frac{1}{17}$

6. sum equations
$3z = 21$
$z = 7$

7. last 3 digits div. by 8
$\Rightarrow x = 1, 3, 5, 7, 9 \Rightarrow 25$

8. $432 = 2 \cdot 216 = 2 \cdot 6^3 = 2^4 \cdot 3^3 \Rightarrow 7$

9.


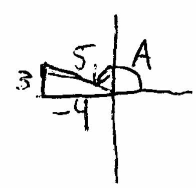
$\Rightarrow \frac{24}{25}$

10. $[4\cdot2 + 3\cdot4 + {-3}\cdot6 \quad 4\cdot1 + 3\cdot0 + {-3}\cdot{-3}]$
$[-14 \quad 13]$

11. $A \cdot B = 1\cdot3 + {-2}\cdot0 + 4\cdot1 = \sqrt{21}\sqrt{10}\cos\theta$
$\cos\theta = \frac{1}{\sqrt{210}} = \frac{\sqrt{210}}{210}$

12. $\frac{\binom{8}{3}}{2^8} = \frac{\cancel{8}\cdot7\cdot\cancel{6}}{\cancel{3}\cdot\cancel{2}\cdot2^{\cancel{8}5}} = \frac{7}{32}$

13. $\frac{\frac{\ln 27}{\ln 4}}{\frac{\ln 81}{\ln 8}} = \frac{3\cancel{\ln 3}}{2\cancel{\ln 2}} \cdot \frac{3\cancel{\ln 2}}{4\cancel{\ln 3}} = \frac{9}{8}$

14.


$\sin(2A) = 2\sin A\cos A$
$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \frac{-24}{25}$

15. $S = \frac{a}{1-r} = \frac{\frac{4}{3}S}{1-r}$
$\cancel{S} - Sr = \frac{4}{3}S - S$
$r = \frac{\frac{1}{3}S}{-S} = -\frac{1}{3}$

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16. $\frac{1}{r}+\frac{1}{s}+\frac{1}{t} = \frac{st+rt+rs}{rst} = \frac{\frac{-3}{2}}{-\frac{-3}{2}} = -1$

17. $2(22-13)+3\left(\frac{22\cdot 23-13\cdot 14}{2}\right)$

$2\cdot 9 + 3\cdot 162$

$18+486 = 504$

18. Answer is probably $a+b\sqrt{2}$

$\Rightarrow 59+30\sqrt{2} = (a^2+2b^2)+2ab\sqrt{2}$

$\Rightarrow a^2+2b^2=59$

$ab=15 \quad \Rightarrow a=3,\ b=5$

$3+5\sqrt{2}$

19. There are $\underline{4}$ ways to pick the suit.

The straight could be $2,3,4,5,6 \rightarrow 10,J,Q,K,A$

$\Rightarrow \underline{9}$ possibilities

$4\cdot 9 = 36$

20. $\cancel{\sqrt{3}+\frac{1}{2}+\frac{1}{2}+\frac{1}{\sqrt{3}}+\frac{2\sqrt{3}}{3}}$

$\sqrt{3}+\frac{1}{2}+\frac{1}{2}+-\frac{1}{\sqrt{3}} = 1+\frac{2\sqrt{3}}{3}$

21. $m+e=\frac{1}{2}$

$m-e=\frac{1}{3} \xrightarrow{\text{sum}} 2m=\frac{5}{6} \Rightarrow m=\frac{5}{12}$

rates

$\frac{12}{5}$

22. C T

6 1

5 2 $\Rightarrow \frac{4}{4\cdot 6} = \frac{1}{6}$

4 3

3 4

23. $4x+50 = 9(2+x)+2x(2+x)$

$a^2n+b^2m = t^2c+mcn$

$0=2x^2+9x-32$

$x=\frac{-9\pm\sqrt{81+256}}{4}$

$=\frac{-9\pm\sqrt{337}}{4}$ can't be negative.

24. $21=7\cdot 3$

$2^6\cdot 3^2 = 64\cdot 9 = 576$

25. $1837 < \text{sum of all} < 1837+180 = 2017$

$\parallel$

$180(n-2) \Rightarrow n = 13$

26. B&G are tied at 85.

B loses by $\frac{15}{100}\cdot 15 = \frac{225}{100} = \frac{9}{4}$

27. $\sin\theta = 2\sin\theta\cos\theta$

$\sin\theta(1-2\cos\theta)=0$

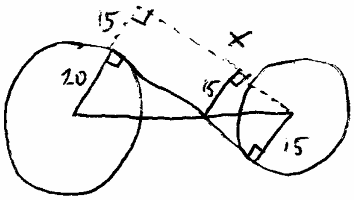
$\sin\theta=0$ or $\cos\theta=\frac{1}{2}$

$\theta=0$ or π $\quad \theta=\frac{\pi}{3}$ or $\frac{5\pi}{3}$

$0+\pi+\frac{\pi}{3}+\frac{5\pi}{3} = 3\pi$

28. $\frac{8+12i}{5-12i} \cdot \frac{5+12i}{5+12i} = \frac{40-144+156i}{25+144}$
$= \frac{-104+156i}{169} = \frac{-8+12i}{13}$

29. $a^2=25,\ b^2=169$
$c^2 = b^2 - a^2 = 144 \Rightarrow c = 12$
center is $(4,-4)$ so foci are
$(4,8)$ & $(4,-16)$ (y is major axis)

30.


$35^2 + x^2 = 40^2$
$x^2 = 40^2 - 35^2$
$x = 5\sqrt{8^2-7^2} = 5\sqrt{15}$

31. $n(q)=4 \Rightarrow q=\frac{5}{4}$
$p(4) = p(n(\frac{5}{4})) = \frac{32 \cdot 25}{16} - \frac{16 \cdot 5}{4} + 6$
$= 50 - 20 + 6 = 36$

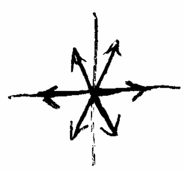
32. $397-257 = 140 = 2^2 \cdot 5 \cdot 7$
$565 - 397 = 168 = 2^3 \cdot 3 \cdot 7$
$GCF = 2^2 \cdot 7 = 28$

33. $ar^3 - ar^2 = 28$
$ar - a = 7$
$\Rightarrow r^2 = 4$
$r = \pm 2$
$a = \frac{7}{r-1} = \frac{7}{2-1} = 7$

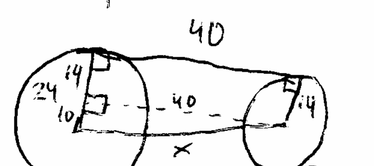
34. "$\frac{21}{2}$th term" $= \frac{1600}{20} = 80$
"$\frac{31}{2}$th term" $= \frac{1800}{30} = 60$
$d = \frac{80-60}{\frac{21}{2}-\frac{31}{2}} = \frac{20}{-5} = -4$

35. $\frac{11+1}{2} = 6$th term $= \frac{88}{11} = 8$
$\frac{21+1}{2} = 11$th term $= \frac{483}{21} = 23$
$\Rightarrow 16$th term $= 23+15 = 38$

36. 1 is a 6th root of 1,
& ~~the~~ roots are symmetric.



$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$
sum $= 5\pi$

37.


$x = \sqrt{10^2+40^2} = 10\sqrt{17}$

38. $\dfrac{9!}{3!3!3!} = \dfrac{3\cdot\cancel{9}\cdot\cancel{8}^{4}\cdot 7\cdot\cancel{6}\cdot 5\cdot 4\cdot\cancel{3}\cdot\cancel{2}}{\cancel{3\cdot2}\cdot\cancel{3\cdot2}\cdot\cancel{6}\cdot\cancel{2}}$

$= 12\cdot 7\cdot 20 = 1680$

39. $z = 2^y$ $\quad 4z^2 - 129z + 32 = 0$

$(4z-1)(z-32)$

$z = \frac{1}{4} \quad z = 32$

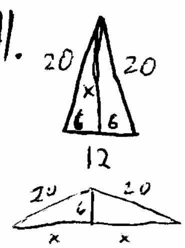
$y = -2 \quad y = 5 \quad -2, 5$

40. $1-3i,\ 1+3i,\ r$

$b = 0 \Rightarrow r = -2$

$z = -(-2)(1-3i)(1+3i) = 20$

41.



$x = \sqrt{400-36}$

$= \sqrt{364} = 2\sqrt{91}$

$2x = 4\sqrt{91}$

42. $(x-9)^2 + (y+17)^2 = 22 + 81 + 289 = 392$

$r = \sqrt{392} = 14\sqrt{2}$

43. $4320 = 20\cdot 216 = 2^2\cdot 5\cdot 2^3\cdot 3^3$

$= 2^5\cdot 3^3\cdot 5 \Rightarrow a = 6$

$b = \frac{1}{2}$

$c = -3$

44. $\dfrac{\binom{11}{2}}{\binom{20}{3}} = \dfrac{11\cdot\cancel{5}\cdot\cancel{5}\cdot\cancel{2}}{\cancel{20}_{4}\cdot 19\cdot\cancel{18}\cdot 3} = \dfrac{11}{12\cdot 19} = \dfrac{11}{228}$

45. $c = 4\cdot 4 = 16$

$b = -(-2+-2) = 4$

$x^2 + 4x + 16 = 0$

$x = \dfrac{-4 \pm \sqrt{16-64}}{2} = -2 \pm 2i\sqrt{3}$

46. Put A & B on grill : 6

Remove A, Flip B, add C : 6

only $\frac{1}{2}$ done

Ditto for B, C, D 6

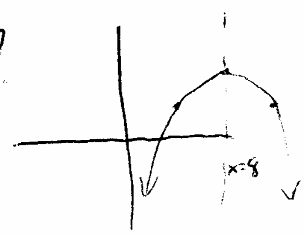
C, D, E 6

D, E, A 6

E, A 6

30

47.



$y = A(x-8)^2 + 6$

$3 = -9A + 6$

$-3 = -9A$

$\frac{1}{3} = A$

$y = -\frac{1}{3}(x-8)^2 + 6$

48. $x = \dfrac{19 \pm \sqrt{361+48}}{4} = \dfrac{19 \pm \sqrt{409}}{4}$

49. $\left(\frac{3}{4}(45)^2 + \frac{1}{4}(15)^2\right)\pi$

$\dfrac{6075 + 225}{4}\pi = \dfrac{6300\pi}{4} = 1575\pi$

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50. He's getting $\frac{13}{14}$ of the money he expects, so he's selling $\frac{14}{13}$ as much yarn as he intends.

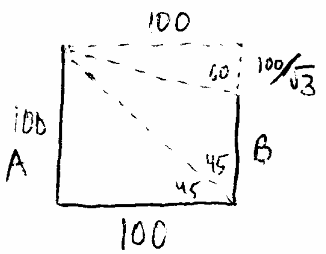
51. $9h+3c=81 \Rightarrow 3h+c=27$ ①

$2h+4c=38 \Rightarrow h+2c=19$ ②

$2\cdot① - ② \Rightarrow 5h=35$

$h=7 \Rightarrow c=6 \Rightarrow 13$

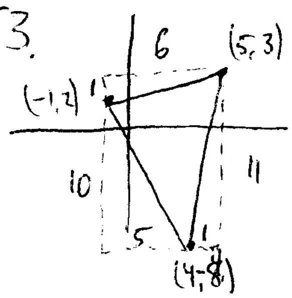
52.



$B = 100 - \frac{100}{\sqrt{3}}$

$= \frac{300-100\sqrt{3}}{3}$

53.



$A = 6\cdot 11 - \frac{1}{2}(6\cdot 1 + 10\cdot 5 + 11\cdot 1)$

$= 66 - \frac{67}{2} = \frac{65}{2}$

54. Notice $x+y+z=8$

OR $\vec{A} = [-2,-2,4]$

$\vec{B} = [-4,2,2]$

$\vec{A}\times\vec{B} = [-12,-12,-12]$

55. $\frac{t}{s} = 12$

$\frac{t+100}{s} = 42$

dividing $\Rightarrow \frac{t+100}{t} = \frac{42}{12} = \frac{7}{2}$

$2t+200 = 7t$

$200 = 5t$

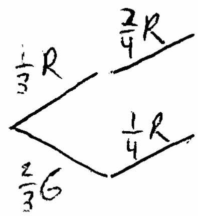
$40 = t$

56. $2^{3z+1}3^{3y}$

$2^{4x}3^{4w+2} \Rightarrow 2^4 3^6$

$\#\text{factors} = (4+1)(6+1) = 35$

57. 1R 1R
2G 2G



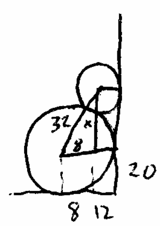
$\frac{1}{3}\cdot\frac{2}{4} + \frac{2}{3}\cdot\frac{1}{4} = \frac{4}{12} = \frac{1}{3}$

58. $\underline{x}$ $\underline{80}$ $\underline{80}$ $\underline{80}$ $\underline{80}$ $\underline{100}$ $\underline{100}$ $\underline{100}$ $\underline{100}$

$720 + x = 9\cdot 85 = 765$

$x = 45 \Rightarrow r = 55$

59.



$x = \sqrt{32^2 - 8^2}$

$= 8\sqrt{4^2-1^2}$

$= 8\sqrt{15}$

$20 + 8\sqrt{15}$

60. $72 = 8 \cdot 9 = 2^3 \cdot 3^2$

$384 = 3 \cdot 128 = 2^7 \cdot 3^1$

GCF $= \quad 2^3 \cdot 3^1 = 24$

LCM $= \quad 2^7 \cdot 3^2 = 1152$

Sum $= 1176$