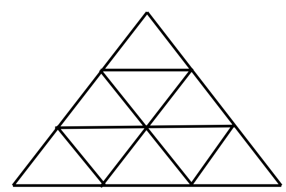
Mu Alpha Theta National Convention: Denver, 2001

State Bowl – Alpha Division

1. What is the shape of the graph of the equation $\frac{4-x}{3}=\frac{2y+1}{2}=z$?

2. Evaluate: $\log\left(4^5\right)+\log\left(5^{10}\right)$

3. How many triangles appear in the figure to the right?



4. Determine the sum of the first 19 terms of an arithmetic series whose tenth term is 41.

5. What is the probability that when two cards are drawn from a standard 52-card deck, they have the same denomination (e.g. both are aces, both are twos, … , both are kings)?

6. Determine the value of z in the following set of equations.

$$\begin{aligned} 3x + 2y - z &= 5 \\ x - 3y + 2z &= 12 \\ -4x + y + 2z &= 4 \end{aligned}$$

7. The four-digit base ten number $4X84$ is divisible by 8. Find the sum of all possible values of X.

8. Given that 432 can be written as $2^a \cdot 3^b$, where a, and b are rational numbers, evaluate $a + b$.

9. Evaluate: $\cos\left(\text{Arcsin}\left(-\frac{7}{25}\right)\right)$

10. Evaluate: $\begin{bmatrix} 4 & 3 & -3 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 4 & 0 \\ 6 & -3 \end{bmatrix}$

11. What is the cosine of the angle between the vectors $[1, -2, 4]$ and $[-3, 0, 1]$?

12. When eight coins are flipped, what is the probability that tails occurs exactly three times?

13. Evaluate: $\dfrac{\log_4 27}{\log_8 81}$

14. Angle A is in quadrant II such that $\sin A = \dfrac{3}{5}$. Evaluate $\sin(2A)$.

15. If the first term of an infinite geometric series is four-thirds of the sum of all the terms in the series, what is the common ratio of the series?

16. What is the sum of the reciprocals of the roots of the equation $2x^3 + 4x^2 - 3x - 3 = 0$?

17. Evaluate: $\sum_{n=14}^{22}(2+3n)$

18. Simplify $\sqrt{59+30\sqrt{2}}$

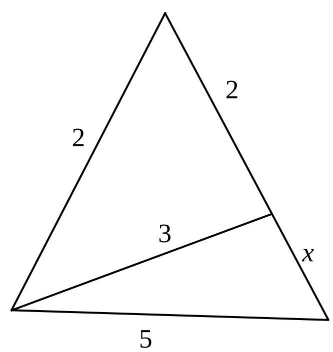
19. In how many distinct ways can a five-card poker hand be a straight flush? Note, a straight flush is five cards of adjacent denominations in a single suit (e.g. the 4, 5, 6, 7, and 8 of hearts). For the purposes of this problem assume that aces are always the highest cards in a suit, directly above the king.

20. Evaluate: $\tan\left(\frac{\pi}{3}\right)+\sin\left(\frac{5\pi}{6}\right)+\cos\left(-\frac{\pi}{3}\right)+\cot\left(\frac{5\pi}{3}\right)$

21. A man can walk up a moving "up" escalator in two minutes. He can walk down the same escalator in three minutes. How many minutes would it take him to climb the escalator if it were not moving?

22. A cubical die and a tetrahedral die have faces numbered one through six and one through four, respectively. What is the probability that when both dice are rolled, the sum of their results is seven? Note: the result for a tetrahedral die is the number that is lying face down.

23. What is the length of x in the triangle shown?



24. Find the smallest natural number with exactly 21 positive integral factors.

25. The sum of all the angles except one in a convex polygon is 1837°. How many sides does the polygon have?

26. A boy and a girl race 100 meters, and the girl wins when the boy has only traveled 85 meters. To make the next race more even, the girl starts 15 meters behind the starting line of the 100-meter stretch. How far behind the winner of the second race will the loser be?

27. What is the sum of all values of θ, $0 \le \theta < 2\pi$, for which $\sin(\theta) = \sin(2\theta)$?

28. Simplify: $\dfrac{8+12i}{5-12i}$

29. What are the coordinates of the foci of the ellipse with equation $\frac{(x-4)^2}{25}+\frac{(y+4)^2}{169}=1$?

30. What is the length of the common internal tangent of two circles of radii 20 and 15 whose centers are 40 units apart?

31. If $n(q)=4q-1$ and $p(n(q))=32q^2-16q+6$, what is the value of $p(4)$?

32. When divided by a natural number *r*, the numbers 257, 397, and 565 all give the same remainder. What is the largest possible value of *r*?

33. In a geometric series with positive first term and common ratio, the difference between the fourth and third terms is 28 and the difference between the second and first terms is 7. What is the value of the first term?

34. The sum of the first twenty terms of an arithmetic sequence is 1600, while the sum of the first thirty terms of the sequence is 1800. What is the common difference of the sequence?

35. The sum of the first eleven terms of an arithmetic sequence is 88, while the sum of the first 21 terms of the sequence is 483. What is the 16th term of the sequence?

36. Each of the six sixth roots of 1 can be expressed as $e^{i\theta}$. Find the sum of all possible values of θ for $0 \le \theta < 2\pi$.

37. Two circles have radii of 14 and 24, and the length of their common external tangent is 40. How far apart are the centers of the circles?

38. A network of fire watchtowers in the forest use flags on a pole to communicate their observations to one another. If a watchtower has nine flags, three each of three different colors, and a message consists of all nine flags one above another on the pole, how many possible messages can be conveyed?

39. For what values of y does $2^{2y+2} - 129 \cdot 2^{y} + 32 = 0$?

40. A cubic equation with rational coefficients has $1-3i$ as one of its roots, 1 as its leading coefficient, and no quadratic term. What is the constant term?

41. An isosceles triangle has a 12-centimeter base and two 20-centimeter sides. What other length could the base have, keeping the other side lengths constant, and still yield a triangle with the same area?

42. What is the radius of the circle whose equation is $x^2 + y^2 - 18x + 34y = 22$?

43. If $4320 = 6^{a-1}15^{2b}3^c$, and a, b, and c are rational, what is a?

44. Three distinct numbers are chosen from the first twenty natural numbers. What is the probability that 9 is the smallest of those chosen?

45. When given an equation of the form $x^2 + bx + c = 0$ to solve, Joe miscopied the value of *b*, getting the single root 4 for his answer. Jim miscopied the value of *c*, and got a single root of -2. What are the roots of the original equation?

46. Tom is barbecuing some big burgers, which need six minutes of barbecuing on each side. If his grill only holds two of his burgers at a time, what is the minimum number of minutes in which he can fully grill five burgers?

47. What is the equation of the parabola that passes through the points $(5, 3)$, $(11, 3)$, and $(8, 6)$, the axis of symmetry of which is parallel to the y-axis?

48. For what values of *x* does $2x^2 - 19x - 6 = 0$?

49. A goat is tethered to an external corner of a 30 by 50-meter rectangular barn with a rope 45 meters long. What is the area (in square meters) of the region the goat can freely graze?

50. A yarn merchant makes 30% profit instead of 40% on his goods, because the meter-stick with which he measures his portions is too long. What is the actual length of his meter-stick, in meters?

51. Hercules is set upon by a pack of Hydras and Cerberuses. Hydras have nine heads and two legs each, while Cerberuses have three heads and four legs each. Hercules hastily counts 81 heads and 38 legs as he wades into battle… how many monsters does he have to fight?

52. There are two towers on opposite ends of a flat field. Tower A is 100 meters tall, and when a woman lies at the base of the tower B, she must look over at an angle of 45º from the vertical to see the top of tower A. When the woman ascends to the top of tower B and lies down, however, she must look over at an angle of 60º from the vertical to see the top of tower A. What is the height of tower B?

53. What is the area of the triangle in the Cartesian plane with vertices (5, 3), (-1, 2), and (4, -8)?

54. What is the equation of the plane through the points $(5,3,1)$, $(3,1,5)$, and $(1,5,3)$?

55. A woman standing near the railroad tracks noticed that a train passed her in twelve seconds, whereas it took the train 42 seconds to enter, travel through, and completely exit a one-hundred-meter tunnel. What was the length of the train, in meters?

56. Let A be the smallest natural number half of which is a perfect cube, and one-ninth of which is a perfect fourth power. How many positive integral factors does A have?

57. Keith has two bags, Bag A and Bag B, each of which contains one red and two green marbles. He takes a marble from Bag A and places it in Bag B, then draws a marble from Bag B. What is the probability that the marble drawn from Bag B is red?

58. A test is given to nine students in which their scores are integers between 0 and 100, inclusive. What is the greatest possible range of the test scores if the median of the scores is 80 and the mean is 85?

59. A cylinder of radius twenty is rolled along the floor until it touches the wall. A cylinder of radius twelve is then placed in the niche on top of the first cylinder, also touching the wall. How high above the floor does the second cylinder touch the wall, if the axes of the cylinders are parallel?

60. What is the sum of the greatest common factor and the least common multiple of 384 and 72?