Mu Alpha Theta National Convention: Denver, 2001

Denver Bowl Solutions – Theta Division

1. 6^x always ends in $\boxed{6}$

2. sum of roots $= \frac{-b}{a} = \frac{-2}{3} = \boxed{\frac{2}{3}}$

3. The major axis is the one with the larger denominator. $49 \Rightarrow$ semi-axis length $= 7 \Rightarrow$ major axis length $= \boxed{14}$

4. $\sqrt{2^2+4^2+4^2} = \sqrt{4+16+16} = \sqrt{36} = \boxed{6}$

5. $A = 625 \Rightarrow s = 25 \Rightarrow p = \boxed{100}$

6. $\frac{3^4 \cdot 2^6}{2^5} = 81 \cdot 2 = \boxed{162}$

7. $114 = 4 \cdot 25 + 2 \cdot 5 + 4 = \boxed{424_5}$

8. $\frac{DE}{2} = \frac{AB}{5} \Rightarrow DE = \frac{2 \cdot 2}{5} = \boxed{\frac{4}{5}}$

9. $(1 - i\sqrt{3})^6 = (2e^{-i\frac{\pi}{3}})^6 = 2^6 e^{-2i\pi} = \boxed{64}$

10. $\frac{148}{11} = 13 \quad \frac{13}{11} = 1 \quad 13+1 = \boxed{14}$

11. $8 = 8 \quad = 2 \cdot 4 \quad = 2 \cdot 2 \cdot 2$
    $2^7 = 128 \quad 2^3 \cdot 3^1 = \boxed{24} \quad 2^1 \cdot 3^1 \cdot 5^1 = 30$

12. Merchant lost: change: \$20
    shoes: \$12
    $\boxed{32}$

13. $x = a^2 = b^5 \Rightarrow x = c^{10} \quad 2^{10} = \boxed{1024}$

14. summing & dividing by $2 \Rightarrow x+y+z = \frac{15}{2}$
    $y+z = 3 = \frac{6}{2} \Rightarrow x = \boxed{\frac{9}{2}}$

15. 0 is attainable (far apart)
    1 is attainable (vertex on ellipse)
    2 is attainable (" inside ")
    3 is " (" on other side of ellipse)
    4 " " (" barely outside ")
    $\boxed{5}$ unattainable

16. $d = \frac{100-37}{13-4} = \frac{63}{9} = 7$
    20th term $= 100 + (20-13)7$
    $= 100 + 49 = \boxed{149}$

17. $3x+y = 7 \quad ①$
    $x-2y = 18 \quad ②$
    $① - 3 \cdot ② \Rightarrow 7y = -47$
    $y = \boxed{\frac{-47}{7}}$

18. $n(q) = 2 = \sqrt{2q+3}$
    $4 = 2q+3 \Rightarrow q = \frac{1}{2}$
    $p(n(\frac{1}{2})) = 4 \cdot \frac{1}{4} + 12 \cdot \frac{1}{2} + 86 = \boxed{93}$

19. $\vec{A} \cdot \vec{B} = 3 \cdot -2 + -2 \cdot 1 = \sqrt{3^2+2^2}\sqrt{2^2+1^2}\cos\theta$
    $\cos\theta = \frac{-8}{\sqrt{65}} = \boxed{\frac{-8\sqrt{65}}{65}}$

20. $\frac{(1+3i)(2-4i)}{(2+4i)(2-4i)} = \frac{2+6i-4i+12}{4+8i-8i+16} = \frac{14+2i}{20}$
    $= \boxed{\frac{7+i}{10}}$

21. $12m-13=-22$
$12m=-9$
$m=\boxed{\frac{-3}{4}}$

22. $3\cdot 1 \to 3.33$
$3\cdot\sum_{1}^{33} n = 3\cdot\frac{33\cdot 34}{2} = 3\cdot 17\cdot 33$
$=99\cdot 17 = 1700-17$
$=\boxed{1683}$

23. $(x+2)^2+(y-4)^2=6^2+4^2+16=81$
$r=\sqrt{81}=\boxed{9}$

24. 7 letters, one triple(i), one double(l)
$\frac{7!}{3!2!}=\frac{7\cdot 6\cdot 5\cdot 4}{2}=21\cdot 20=\boxed{420}$

25. $(3\log_b a)(\log_a b^{1/2})$
$(3\log_b a)(\frac{1}{2}\log_a b)=\boxed{\frac{3}{2}}$

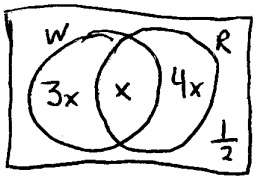
26. $42-30+1=13$ distinct quantities
$\frac{40}{13}=3.x \Rightarrow$ 5 rotates with same #

28. $252=\frac{n(n-3)}{2}$
$n^2-3n-504=0$ $\quad 504=21\cdot 24$
$(n-24)(n+21)=0$
$n=\boxed{24}$

27. even # of factors $\Rightarrow$ not perfect squares.
$\sum_{1}^{24} n - \sum_{1}^{4} n^2 = \frac{24\cdot 25}{2} - \frac{4\cdot 5\cdot 9}{6} = 300-30=\boxed{270}$

29. $4\cdot 5 \to 4.25$
$4\cdot\sum_{5}^{25} n = 4\left(\frac{25\cdot 26-4\cdot 5}{2}\right)=4(325-10)$
$=4\cdot 315=\boxed{1260}$

30. 3rd term $=18 \Rightarrow$ 1st term $=9\cdot 18=162$
$S=\frac{a}{1-r}=\frac{162}{1-\frac{1}{3}}=162\cdot\frac{3}{2}=\boxed{243}$

31. 

$8x=\frac{1}{2}$
$x=\boxed{\frac{1}{16}}$

32. $\begin{bmatrix} 1\cdot 3+3\cdot 0 & 1\cdot -2+3\cdot 1 & 1\cdot 0+3\cdot 4 \\ 2\cdot 3+-1\cdot 0 & 2\cdot -2+-1\cdot 1 & 2\cdot 0+-1\cdot 4 \end{bmatrix}$
$\boxed{\begin{bmatrix} 3 & 1 & 12 \\ 6 & -5 & -4 \end{bmatrix}}$

33. $(x^2+3)(x+2)=0$
$x=-2, i\sqrt{3}, -i\sqrt{3}$ $\quad \boxed{2}$

34. $\log 2=\log\left(\frac{10}{5}\right)=\log 10-\log 5$
$=\boxed{1-a}$

35. $\frac{14}{63}=\boxed{\frac{2}{9}}$

36. She needs 3 of the same color. If she got 8, they could conceivably be 2-2-2-2, so she needs $\boxed{9}$.

37. They're tied when there are 8 meters left. The boy goes $\frac{92}{100}$ of that distance by the time the girl wins, so he loses by $\frac{8}{100}\cdot 8 = \boxed{\frac{16}{25}}$.

38. $2^{3f-6}\cdot 2^{4f+2} = 2^{12-8f}$

$3f-6+(4f+2) = 12-8f$

$15f = 16$

$f = \boxed{\frac{16}{15}}$

39. $\frac{15\cdot 16\cdot 31}{6} + 3(15)$

$1240+45 = \boxed{1285}$

40. $\sqrt{k^2+24} > 0$

$k^2 > -24$ $\boxed{\text{all real numbers}}$

41. $\frac{n(n+1)}{2} = 253$

$n^2+n-506 = 0$ $\quad 506 = 22\cdot 23$

$(n-22)(n+23) = 0 \Rightarrow n = \boxed{22}$

42. $\underline{0}\,\underline{0}\,\underline{0}\,\underline{x}\,\underline{x}\,\underline{x}\,\underline{x}$

$4x = 7\cdot 75 = 525$

$x = \frac{525}{4} > 100 \Rightarrow \boxed{100}$

(the zeros could have been higher)

43. $\frac{\ln 27}{\ln 4}\cdot\frac{\ln 32}{\ln 9} = \frac{3\ln 3}{2\ln 2}\cdot\frac{5\ln 2}{2\ln 3} = \boxed{\frac{15}{4}}$

44. $P(\text{full}) = \frac{1}{8} \Rightarrow P(\text{not full}) = \frac{7}{8}$

$\frac{7}{8}\cdot\frac{1}{2} + \frac{1}{8}\cdot 0 = \boxed{\frac{7}{16}}$

45. 200

$200/12 = 16 \text{ r } 8$

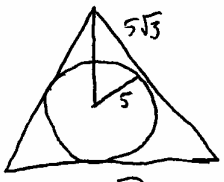
$16+8/12 = 2 \text{ r } 0$

$200+16+2 = \boxed{218}$

46. $5t = 3t+2000$

$2t = 2000$

$t = \boxed{1000}$

47. 

$A = \frac{s^2\sqrt{3}}{4} = \frac{300\sqrt{3}}{4}$

$= \boxed{75\sqrt{3}}$

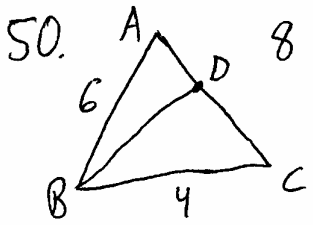
48. MIX = 7

MXI = 7

XMI = 7

$\frac{21^3}{9\cdot 8\cdot X} = \boxed{\frac{1}{24}}$

49. $\frac{\binom{4}{2}+\binom{8}{2}}{\binom{12}{2}} = \frac{4\cdot 3+8\cdot 7}{12\cdot 11} = \frac{68^{17}}{12\cdot 11_3} = \boxed{\frac{17}{33}}$

50. 

$\frac{AD}{DC} = \frac{6}{4}$

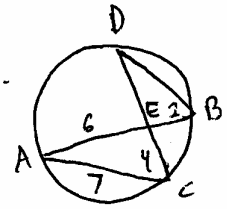
$DC = 8 - AD$

$\frac{AD}{8-AD} = \frac{6}{4}$

$4AD = 48 - 6AD \Rightarrow AD = \frac{48}{10} = \boxed{\frac{24}{5}}$

51. 1, (2,2,2) 3, 3, (4) 4, 5, 5, 6, 7, 8

mode = 2 median = 4 $2+4 = \boxed{6}$

52. 

$\triangle DEB$ & $\triangle AEC$ similar w/ ratio $\frac{1}{2}$

$\Rightarrow BD = \boxed{\frac{7}{2}}$

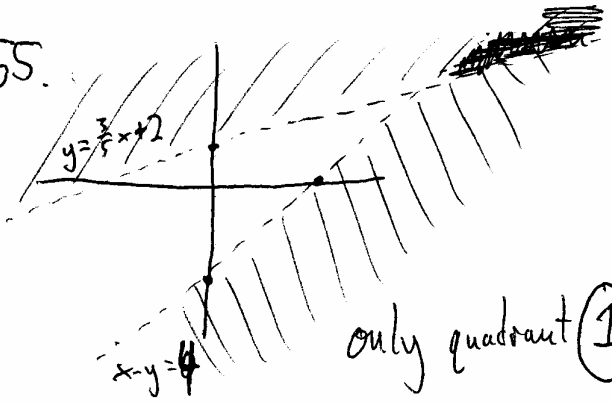
53. $b = \frac{1}{10}$

$b + s = \frac{1}{6} \Rightarrow s = \frac{1}{6} - \frac{1}{10} = \frac{1}{15} \Rightarrow \boxed{15}$

54. $33{,}600 = 2^3 \cdot 5^2 \cdot 168 = 2^5 \cdot 5^2 \cdot 3 \cdot 14$

$= 2^6 \cdot 3^1 \cdot 5^2 \cdot 7^1$

factors $= (6+1)(1+1)(2+1)(1+1) = \boxed{84}$

55. 

only quadrant $\boxed{1}$

56. $n + q = 32$ ①

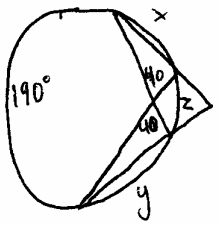
$5n + 25q = 460 \Rightarrow n + 5q = 92$ ②

② − ① $\Rightarrow 4q = 60$

$q = 15 \Rightarrow n = 17$

57. $\begin{vmatrix} k+1 & 1 \\ 4 & k-2 \end{vmatrix} = (k+1)(k-2) - 4 = k^2 - k - 6 = 0$

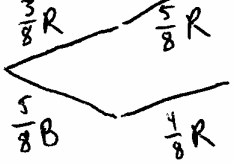
$(k-3)(k+2) = 0 \Rightarrow k = \boxed{3, -2}$

58. 

$x + y = 80 \Rightarrow z = 90$

$\angle BAC = \frac{180 - z}{2} = \boxed{50°}$

59. 3R 4R
5B 3G



$\frac{3}{8} \cdot \frac{5}{8} + \frac{5}{8} \cdot \frac{4}{8} = \boxed{\frac{35}{64}}$

60. $S = \frac{a(1-r^n)}{1-r} = \frac{144\left(1 - \left(\frac{1}{2}\right)^{10}\right)}{1 - \frac{1}{2}}$

$= 288\left(\frac{1023}{1024}\right) = 9 \cdot \frac{1023}{32} = \boxed{\frac{9207}{32}}$

55.