Mu Alpha Theta National Convention: Denver, 2001
Complex Numbers Topic Test – Alpha Division

1. Which of the following are subsets of the irrational numbers?
   I. The real numbers
   II. The complex numbers
   III. The imaginary numbers

   (A) I only (B) II only (C) III only (D) II & III only (E) NOTA

2. Which of the following are pure imaginary numbers?
   I. -4
   II. $-4+2i$
   III. $\sqrt{-7}$
   IV. $3-i\sqrt{7}$

   (A) II only (B) III only
   (C) II & III only (D) II, III, & IV only (E) NOTA

3. Fill in the blanks: $-1 + 3i$ graphed in the complex plane lies 3 units _____ the origin and 1 unit _____ the origin.

   (A) above, to the left of (B) to the right of, below
   (C) above, to the right of (D) to the left of, below (E) NOTA

4. Evaluate: $25+\frac{3i+2}{6}-\frac{i}{2}$

   (A) $i+26$ (B) $\frac{i+76}{3}$ (C) $\frac{76}{3}$ (D) 27 (E) NOTA

5. Evaluate: $\sum_{n=1}^{12}(-2i)^n$

   (A) $2098-1476i$ (B) $-2548+1586i$
   (C) $3276+1638i$ (D) $3688-1826i$ (E) NOTA

6. Evaluate: $\left(3\sqrt{2}-i\right)\left(-2+5i\sqrt{2}\right)$

   (A) $8\sqrt{2}+28i\sqrt{2}$ (B) $-8\sqrt{2}+28i$
   (C) $32i-\sqrt{2}$ (D) $28i-11\sqrt{2}$ (E) NOTA

7. Solve for b: $2-\frac{b}{4+i\sqrt{3}}+\frac{b}{4-i\sqrt{3}}=0$

(A) $\frac{4+16i\sqrt{3}}{5}$ (B) $\frac{16i+1}{19}$ (C) $\frac{19i\sqrt{3}}{3}$ (D) $\frac{4i\sqrt{3}}{13}$ (E) NOTA

8. Evaluate: $\overline{3+4i}-|i|$

(A) $-4-4i$ (B) 4 (C) $5-i$ (D) $2-4i$ (E) NOTA

9. Determine the sum of A and B in the system of equations.

$$4A-3B=4-3i$$
$$2A-5B=2+6i$$

(A) $\frac{9+5i}{14}$ (B) $1+4i$ (C) $\frac{2-9i}{2}$ (D) $6+3i$ (E) NOTA

10. Find the determinant of $\begin{bmatrix} i & -2 \\ 1-i & i+1 \end{bmatrix}$

(A) $-3+3i$ (B) $1-i$ (C) $-3-i$ (D) $1+3i$ (E) NOTA

11. The reciprocal of a number is squared and added to half of one half. If the result is zero, which of the following is a possible value for the number?

(A) $-i\sqrt{2}$ (B) -2 (C) $\sqrt{2}$ (D) $2i$ (E) NOTA

12. Convert $2i-1$ to polar form. Give your answer in the form (r,θ).

(A) $\left(\sqrt{5},-\text{Arctan}\left(\frac{1}{2}\right)\right)$ (B) $\left(\sqrt{5},\pi-\text{Arctan}(2)\right)$
(C) $\left(\sqrt{5},\text{Arctan}\left(\frac{1}{2}\right)\right)$ (D) $\left(\sqrt{5},\frac{\pi}{2}+\text{Arctan}(2)\right)$ (E) NOTA

13. Which of the following is not equal to $4e^{-\frac{5\pi i}{4}}$?

(A) $-2\sqrt{2}+2i\sqrt{2}$ (B) $4e^{\frac{3\pi i}{4}}$ (C) 4 cis 120° (D) $4e^{-\frac{13\pi i}{4}}$ (E) NOTA

14. Simplify: $\dfrac{e^{\frac{2\pi}{3i}} \times 4e^{\frac{\pi i}{2}}}{6e^{\frac{3\pi i}{4}}}$

(A) $e^{\frac{13\pi i}{12}}$ (B) $\dfrac{2e^{-\frac{5\pi i}{6}}}{3}$ (C) $\dfrac{e^{-\frac{11\pi i}{12}}}{3}$ (D) $\dfrac{2e^{-\frac{11\pi i}{12}}}{3}$ (E) NOTA

15. Evaluate: $\left(-\dfrac{1}{2}-\dfrac{i\sqrt{3}}{2}\right)^{47}$

(A) $-\dfrac{1}{2}+\dfrac{i\sqrt{3}}{2}$ (B) 1 (C) $\dfrac{1}{2}-\dfrac{i\sqrt{3}}{2}$ (D) $\dfrac{1}{2}+\dfrac{i\sqrt{3}}{2}$ (E) NOTA

16. Find all solutions to $x^2+2x+8=0$

(A) $1\pm 3i$ (B) $-1\pm i\sqrt{7}$ (C) $-1\pm\sqrt{17}$ (D) $1\pm\sqrt{7}$ (E) NOTA

17. How many complex roots does the equation $4x^3-3x^2+2x-3=0$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

18. What is the sum of the real roots of $x^3+x^2+6x-8=0$?

(A) -1 (B) 1 (C) -8 (D) 8 (E) NOTA

19. What is the product of the complex roots of $3x^3-x^2+7x-4=0$?

(A) $-\dfrac{3}{4}$ (B) $\dfrac{3}{4}$ (C) $\dfrac{4}{3}$ (D) 3 (E) NOTA

20. What is the sum of the squares of the roots of $x^2-6x+11=0$?

(A) 11 (B) 14 (C) 17 (D) 36 (E) NOTA

21. Which of the following are complex fourth-roots of -1?

I. $1+i$
II. $1-i$
III. $-1+i$
IV. $-1-i$

(A) I & II only (B) I & III only (C) II & IV only (D) I, II, III, & IV (E) NOTA

22. Z_1 and Z_2 are distinct nth roots of 1, where n is an integer. Which of the following is/are <u>not</u> necessarily an nth root of 1?

I. $Z_1 Z_2$
II. $\frac{Z_1}{Z_2}$
III. $Z_1 + Z_2$
IV. $Z_1 - Z_2$

(A) II only (B) IV only (C) I & II only (D) III & IV only (E) NOTA

23. What is the product of the five fifth roots of 1-*i*?

(A) $i-1$ (B) $1-i$ (C) 1 (D) $-i\sqrt{2}$ (E) NOTA

24. Given that $x^2 = i$ and $y^2 = -i$, $x+y$ could equal which of the following?

I. $i\sqrt{2}$
II. $-\sqrt{2}$
III. $\sqrt{2}$

(A) III only (B) I & III only (C) II & III only (D) I, II, & III (E) NOTA

25. If the third term of a geometric sequence is 12 and the seventh term is 48, which of the following are possible values for the fourth term?

I. $24\sqrt{2}$
II. $24i$
III. $-12i\sqrt{2}$

(A) I only (B) II only (C) III only (D) I & II only (E) NOTA

26. Find the sum of the terms of the geometric sequence: $3i+1, \frac{7i-1}{5}, \frac{13i-9}{25}, \ldots$

(A) $5i$ (B) $\frac{2+i}{5}$ (C) $2i+1$ (D) diverges (E) NOTA

27. What is the limit of the real component of $y = e^{(-2+3i)x}$ as $x \in \Re$ increases to positive infinity?

(A) 0 (B) 1 (C) $e^{-\frac{3\pi i}{2}}$ (D) ∞ (E) NOTA

28. What real value(s) of b will ensure that $x^2 - bix - 2 = 0$ has a double root?

(A) 0 (B) $\pm 2\sqrt{2}$ (C) $\sqrt{2}$ (D) no real value (E) NOTA

29. A quadratic equation with integer coefficients and leading coefficient of 1 has a root at $4+3i$. What is the value of its constant term?

(A) 25 (B) -9 (C) 7 (D) -1 (E) NOTA

30. What is the polynomial with integer coefficients and leading coefficient of 1 of smallest degree that has roots of $2-i$ and 4?

(A) $x^3 - 8x^2 - 11x - 20 = 0$ (B) $x^3 - 6x^2 + 4x - 20 = 0$
(C) $x^2 - 6x + 4 = 0$ (D) $x^3 + 6x^2 - 4x + 20 = 0$ (E) NOTA

31. A cubic equation with integer coefficients and leading coefficient of 1 has roots of $i-2$ and 7. What is the value of the coefficient of the quadratic term of the equation?

(A) 7 (B) 3 (C) 1 (D) -3 (E) NOTA

32. Let be $\vec{A}$ the position vector of the point $1-2i$ in the complex plane. When $\vec{A}$ is rotated 150° counter-clockwise about the origin, it is the position vector of a new complex number, b. What is the value of b?

(A) $\frac{2-\sqrt{3}}{2} + \left(\frac{1+2\sqrt{3}}{2}\right)i$ (B) $i\sqrt{5}$

(C) $1+2i$ (D) $\frac{5-2\sqrt{3}}{2} + \left(\frac{1+\sqrt{3}}{2}\right)i$ (E) NOTA

33. What is the measure, in radians, of the smaller angle between the position vectors of $2-4i$ and $1+3i$ in the complex plane?

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$ (E) NOTA

34. A teacher placed an equation of the form $x^2+bx+c=0$ on the board to be solved. Joe miscopied the value of *c* and got $-1\pm3i$ as the roots. Jim miscopied the value of *b*, resulting in roots of $-5\pm i$. Julie copied the problem down correctly... What will she get for roots?

(A) $-1\pm5i$ (B) $1\pm i\sqrt{5}$ (C) $\frac{3\pm3i}{2}$ (D) $1\pm\frac{3i}{2}$ (E) NOTA

35. The roots of the quadratic equation $z^2+pz+q=0$ are $1-i$ and $-2+2i$. Find the product of the complex numbers *p* and *q*.

(A) $4+4i$ (B) $4i$ (C) $2+6i$ (D) $4-i$ (E) NOTA

36. Solve the equation $z^2+(3+i)z+3i=0$ where *z* is an element of the complex numbers.

(A) $z\in\{\pm3i\}$ (B) $z\in\{-1,3i\}$ (C) $z\in\{-1,-3i\}$ (D) $z\in\{-3,-i\}$ (E) NOTA

37. A complex number *x* has the following property: when it is raised to the fourth power it is $(-11+3i)$ greater than when it is raised to the sixth power. Let *A* be the number of complex numbers with this property. Let *B* be the sum of all the complex numbers with this property. Let *C* be the product of all the complex numbers with this property. Determine the sum *A* + *B* + *C*.

(A) $-8-i$ (B) $-5+3i$ (C) $4-2i$ (D) $\frac{5-i}{3}$ (E) NOTA

38. Which of the following expressions is equivalent to Euler's number (e)?

(A) $\cos1+i\sin1$ (B) $e^{\pi i}+1$ (C) $(\cos1-i\sin1)^i$ (D) 4 arctan 1 (E) NOTA

39. Evaluate: $\frac{3+7i}{1+i}$

(A) $\frac{-3+7i}{2}$ (B) $5+2i$ (C) $\frac{3-7i}{2}$ (D) $-2+10i$ (E) NOTA

40. Which of the following is equal to $\ln\left(1+i\sqrt{3}\right)$?

(A) $2e^{\frac{\pi i}{2}}$ (B) $\frac{\pi i}{3}+\ln 2$ (C) $e^{\frac{\pi i}{2}}+\ln 2$ (D) $e^{\frac{\pi i}{3}}+e^{\frac{\pi i}{2}}$ (E) NOTA