Mu Alpha Theta National Convention: Denver, 2001
Equations and Inequalities Topic Test – Alpha Division

1. Arthur buys a piece of land. Bart buys the same land from Arthur for at least one hundred percent more than what Arthur paid for it. Chris buys the same land from Bart for at least three hundred percent more than what Bart paid for it. Donald buys the same land from Chris for at least fifty percent more than what Chris paid for it. If Donald paid 4.2 million dollars for the land, what is the largest amount that Arthur could have paid for the land?

   (A) \$350,000 (B) \$466,667 (C) \$700,000 (D) \$2,800,000 (E) NOTA

2. For what values of x does $x^2 - 7x + 17 > 5$?

   (A) $x > -4$ (B) $x < -3$ or $x > 4$
   (C) $x < 3$ (D) $x > 4$ or $x < 3$ (E) NOTA

3. How many of the following are roots of $x^3 + 25 = -2$?

   I: 3
   II: -3
   III: $\frac{3}{2} + i\frac{3\sqrt{3}}{2}$
   IV: $\frac{-3}{2} + i\frac{3\sqrt{3}}{2}$

   (A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

4. For what values of x, $0° \le x < 360°$, does $\cot x = 1$?

   (A) $45^\circ, 225^\circ$ (B) $90^\circ, 270^\circ$ (C) $135^\circ, 315^\circ$ (D) $0^\circ, 180^\circ$ (E) NOTA

5. For what values of x, $0° \le x < 360°$, does $|\sec x| = 1$?

   (A) $45^\circ, 225^\circ$ (B) $90^\circ, 270^\circ$ (C) $135^\circ, 315^\circ$ (D) $0^\circ, 180^\circ$ (E) NOTA

6. Given that $f(x) = \frac{8}{3+5x}$, what is $f^{-1}(x)$?

   (A) $\frac{8-3x}{5x}$ (B) $\frac{8-5x}{5x}$ (C) $\frac{8}{5-3x}$ (D) $\frac{8x}{3x+5}$ (E) NOTA

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7. Given that $g(d) = 3d + 2$ and $f(d) = \frac{2d^2}{3}$, evaluate $g(f(14))$.

(A) $\frac{790}{3}$ (B) $\frac{793}{3}$ (C) 394 (D) $\frac{17248}{3}$ (E) NOTA

8. Solve for *f*: $2^{f-4}16^{f+4} = 8^{3f-3}$

(A) $\frac{-3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{7}{3}$ (D) $\frac{21}{4}$ (E) NOTA

9. Solve for *y* given that *x* and *y* are both integers. $6^{x-2}24^{y+2} = 3{,}779{,}136$

(A) -6 (B) -4 (C) 4 (D) 6 (E) NOTA

10. What is the sum of the non-real roots of $3x^3 + 6x^2 + 4x + 1 = 0$?

(A) -4 (B) -3 (C) -2 (D) -1 (E) NOTA

11. What is the sum of the reciprocals of the roots of $f(x) = x^5 - 2x^2 + 3x + 5$?

(A) $\frac{-3}{5}$ (B) $\frac{-5}{3}$ (C) $\frac{3}{5}$ (D) $\frac{2}{3}$ (E) NOTA

12. Given that $f(x-3) = 3x^2 - 5x - 2$, what is the constant term of $\frac{f(x)}{5}$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

13. Given that $\frac{3x+9}{x^2+7x+10} = \frac{A}{x+2} + \frac{B}{x+5}$, find *A* + *B*, where *A* and *B* are real numbers.

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

14. Given that $\frac{25x-11}{x^2-2x+1} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$, find $A \cdot B$, where A and B are real numbers.

(A) 250 (B) 350 (C) 450 (D) 550 (E) NOTA

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15. Given that $x + y + z = 1$ and $3x - 2y + 3z = 5$, solve for x in terms of z.

(A) $-z - \frac{7}{5}$ (B) $-z + \frac{7}{5}$ (C) $z - \frac{7}{5}$ (D) $z + \frac{7}{5}$ (E) NOTA

16. Given that $x + 3y + 4z = -5$, $-x + y - 2z = 5$, and $-5x - y - z = 1$, evaluate $x + y + z$.

(A) -2 (B) -1 (C) 0 (D) 1 (E) NOTA

17. Given that $\log_8(4x) + \log_2(2x^4) = \frac{-47}{3}$, evaluate $\sqrt{x} - x - 10x^2$.

(A) $\frac{29}{128}$ (B) $\frac{\sqrt{2}}{4} - \frac{9}{32}$ (C) $\frac{-3}{8}$ (D) $\frac{\sqrt{2}}{2} - 3$ (E) NOTA

18. Given that $\log_{10}(x) = \log_9(27)$, evaluate $x^3 - 50x$.

(A) $9500\sqrt{10}$ (B) $175\sqrt{15}$ (C) 14375 (D) 122500 (E) NOTA

19. Assuming that N is a natural number less than or equal to 1000, what is the least possible value of the expression $\lfloor \log_3 N \rfloor - \log_3 N$. Note: $\lfloor x \rfloor$ stands for the greatest integer less than or equal to x.

(A) 0 (B) $5 - \log_3 728$ (C) $6 - \log_3 730$ (D) $6 - \log_3 1000$ (E) NOTA

20. Solve for x: $\frac{x-2}{-3x-2} \geq -3$

A) $(-\infty, -1]$ B) $(-\infty, -1)$ C) $\left(\frac{-2}{3}, \infty\right)$ D) $[\frac{-2}{3}, \infty)$ E) NOTA

21. Bill is five years older than Mary, and in three years will be seven times Mary's age ten years ago. How old is Bill now?

(A) 18 (B) 19 (C) 20 (D) 21 (E) NOTA

22. If $r = 4$ is a solution to $6r - 7qr^3 + 11 = 5$, what is the value of q?

(A) $\frac{-5}{32}$ (B) $\frac{5}{32}$ (C) $\frac{-5}{16}$ (D) $\frac{5}{16}$ (E) NOTA

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23. One of the roots of $4x^2 + 30x + c = 0$ is twice the other. What is the value of c?

(A) 30 (B) 40 (C) 50 (D) 60 (E) NOTA

24. In the Mu Alpha Theta Survivor Contest, five contestants named A, B, C, D, and E are placed in a room. They cast one vote each, and each of the five votes is received by a contestant. A and B receive a total of 3 votes. D and E receive a total of 2 votes. A and D receive a total of 2 votes. A and E receive a total of 2 votes. Who received the most votes?

(A) D (B) C (C) B (D) A (E) NOTA

25. Given that $k(m) = 4m - 1$, what is $k^{-1}(m)$?

(A) $\frac{m-1}{4}$ (B) $\frac{m+1}{4}$ (C) $\frac{1}{4m-1}$ (D) $\frac{-1}{4m-1}$ (E) NOTA

26. What is the minimal set of quadrants which contains the set of points satisfying $-3y - x < 5$ and $-3x < -4$?

(A) I (B) I, IV (C) I, II, IV (D) I, III, IV (E) NOTA

27. The equation, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is the general form for what shape centered at the origin and aligned with the coordinate axes?

(A) parabola (B) circle (C) ellipse (D) hyperbola (E) NOTA

28. Given that $g(d) = 9d - 2$ and $f(d) = \frac{5d}{4d-3}$, evaluate $g(f(7))$.

(A) $\frac{305}{241}$ (B) $\frac{53}{5}$ (C) $\frac{427}{5}$ (D) $\frac{429}{5}$ (E) NOTA

29. How many natural numbers are solutions of $9w - 39 < 94$?

(A) 0 (B) 5 (C) 13 (D) 14 (E) NOTA

30. Solve for t: $t+1<-t-4<3t+17$

(A) $t<\frac{-5}{2}$ (B) $\frac{-21}{4}<t<\frac{-3}{2}$
(C) $\frac{-21}{4}<t<\frac{3}{2}$ (D) $\frac{-21}{4}<t<\frac{-5}{2}$ (E) NOTA

31. Given that for |r| < 1, $a-ar+ar^2-ar^3+\ldots=x$, $a+ar^2+ar^4+\ldots=y$ and $a+ar^3+ar^6+\ldots=z$, determine $\frac{a}{r^2+r+1}$ in terms of *x*, *y*, and *z*.

(A) $\frac{xy}{z}$ (B) $\frac{xz}{y}$ (C) $\frac{yz}{x}$ (D) $\frac{y^2z}{x^2}$ (E) NOTA

32. The angle θ is defined by the following inequalities: $0\le\theta<\frac{\pi}{2}$, $\cot\theta\ge1$. Solve for θ.

(A) $\frac{\pi}{4}\le\theta\le\frac{\pi}{2}$ (B) $\frac{\pi}{4}<\theta\le\frac{\pi}{2}$ (C) $0\le\theta\le\frac{\pi}{4}$ (D) $0<\theta\le\frac{\pi}{4}$ (E) NOTA

33. Determine the period of the function $f(x)=\sin(x)+\sin(2x)+\sin(3x)+\sin(4x)+\sin(5x)$.

(A) $\frac{\pi}{30}$ (B) 2π (C) 30π (D) 60π (E) NOTA

34. Determine the area of the region enclosed by a circle with radius 1 centered at the origin that also satisfies the inequalities $y\le x$ and $y\ge-0.5$.

(A) $\frac{1}{4}+\frac{\sqrt{3}}{4}+\frac{5\pi}{27}$ (B) $\frac{1}{8}+\frac{\sqrt{3}}{8}+\frac{5\pi}{27}$ (C) $\frac{1}{8}+\frac{\sqrt{3}}{8}+\frac{5\pi}{24}$ (D) $\frac{1}{4}+\frac{\sqrt{3}}{4}+\frac{5\pi}{24}$ (E) NOTA

35. Find the domain of the function $y(x)=\text{Arcsin}(\ln(x))$.

(A) $(\frac{-\pi}{2},\frac{\pi}{2})$ (B) $(0,e)$ (C) $(\frac{1}{e},e)$ (D) $(-\infty,\infty)$ (E) NOTA

36. The variable *x* represents a real number randomly chosen from the continuous range zero to ten. The variable *y* represents a real number randomly chosen from the continuous range zero to ten. Determine the probability that $x + y \le 6$.

(A) $\frac{9}{50}$ (B) $\frac{3}{25}$ (C) $\frac{3}{10}$ (D) $\frac{3}{5}$ (E) NOTA

37. Determine the magnitude of $(4-3i)(5+12i)(8-15i)$.

(A) 1105 (B) $943i - 576$ (C) 1519 (D) 2737 (E) NOTA

38. Find the area of the region in the Cartesian plane that satisfies the following inequalities: $x \ge 0$, $y \ge 0$, $x - y \le 3$, $x + 2y \le 12$

(A) 15 (B) 20 (C) $\frac{45}{2}$ (D) 27 (E) NOTA

39. A college student wishes to have a diet consisting entirely of french fries and pepperoni pizza. The student must eat at least 1800 calories per day and at least 8 grams of protein per day. The calories per serving, protein per serving, and the cost per serving of the two foods are given in the following table:

	Calories per Serving	**Protein per Serving (in grams)**	**Cost per Serving**
French Fries	600	2	\$1.00
Pepperoni Pizza	600	4	\$1.50

What is the lowest possible cost per day for a diet that meets the above requirements? Fractional numbers of servings are allowed. Negative numbers of servings are not allowed.

(A) \$5.00 (B) \$4.50 (C) \$4.00 (D) \$3.50 (E) NOTA

40. A college student wishes to have a snack consisting entirely of Coca-Cola and coffee. The student needs to have at least 150 calories and at least 30 milligrams of caffeine. The calories per serving, amount of caffeine per serving and the cost per serving of the two foods are given in the following table:

	Calories per Serving	**Caffeine per Serving (in milligrams)**	**Cost per Serving**
Coffee	25	10	$1.00
Coca-Cola	50	5	$1.50

What is the lowest possible cost for a snack that meets the above requirements? Fractional numbers of servings are allowed. Negative numbers of servings are not allowed.

(A) $5.00 (B) $4.50 (C) $4.00 (D) $3.50 (E) NOTA