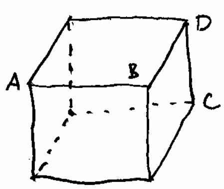
1. $AB = AC \Rightarrow \angle B = \angle C = 38°$
$180° = \angle A + \angle B + \angle C \Rightarrow \angle A = 180° - \angle B - \angle C = 180° - 2 \cdot 38°$
$\angle A = 104°$

2. For a cube to have exactly 2 white faces, it must be along an edge, but not at a corner, so:
#cubes w/ exactly 2 white faces
$= (\#\text{edges}) \cdot (\#\text{ such cubes / edge})$
$= 12 \cdot (\sqrt[3]{512} - 2) = 12 \cdot 6 = 72$

3. $A = 216\pi = \pi r^2 \Rightarrow r = \sqrt{216} = 6\sqrt{6}$
$C = 2\pi r = 12\pi\sqrt{6}$

4. Greatest possible distance is the diagonal of the cube, ie, AC. By Pythagoras,



$AC^2 = AB^2 + BC^2 = AB^2 + (BD^2 + DC^2)$
$= 6^2 + 6^2 + 6^2 = 3 \cdot 6^2 \text{ cm}^2$
$AC = 6\sqrt{3} \text{ cm}$

5. exterior $\angle = 180° -$ interior $\angle = 180° - 144° = 36°$, also,
ext. $\angle = 360°/n$ for a regular n-gon, so
$n = 10$. $P = n \cdot \ell = 10 \cdot 7 \text{ units} = 70 \text{ units}$.

6. Centroid of a $\triangle$ is at $(\bar{x}, \bar{y})$
$(\bar{x}, \bar{y}) = \left(\frac{4+2-1}{3}, \frac{9-3+7}{3}\right) = \left(\frac{5}{3}, \frac{13}{3}\right)$

7. $SA_{Box} = \sum_{Side} SA_{side} = 2(l \cdot w + w \cdot h + l \cdot h)$
$= 2(40 + 104 + 65)\ cm^2 = 418\ cm^2$

8. $SA = 4\pi r^2 = 4 \cdot \pi (4m)^2 = 64\pi\ m^2$

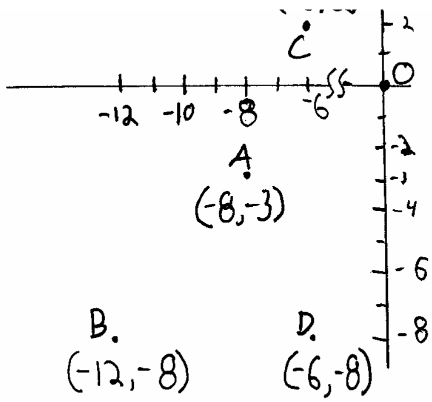
9. $\lim_{x \to \pm\infty} f(x) = -\infty$, so any max will have $f'(x_0) = 0$
$f'(x) = -4x - 3 \Rightarrow x_0 = -\frac{3}{4}$
$f(x_0) = f(-\frac{3}{4}) = 14 - 3 \cdot (-\frac{3}{4}) - 2 \cdot (-\frac{3}{4})^2 = \frac{121}{8}$
or put f into standard form: $f(x) = -2(x + \frac{3}{4})^2 + \frac{121}{8}$

10. The floor is either 44×99 tiles or 66×66 tiles.
Both yield 4356 tiles.
or notice $6\ \text{tiles}/m^2 \cdot 22 \cdot 33\ m^2$ floor gives
4356 tiles.

11. $\frac{4}{3}\pi r_{new}^3 = 3 \cdot \frac{4}{3}\pi r_{old}^3 \Longrightarrow r_{new}^3 = 3 r_{old}^3$, $r_{new} = \sqrt[3]{3}\, r_{old}$
$r_{new}/r_{old} = \sqrt[3]{3}/1 = \sqrt[3]{3}$
or V is a cubic measure, radius is linear
$3:1$ cubic $\Longrightarrow \sqrt[3]{3}:1$ linear

12. $6 \cdot (6+6) = x \cdot (10 + x)$ gives $x^2 + 10x - 72 = 0$
which has roots $x = -5 \pm \sqrt{97}$. Since x is a distance,
we must have $x \geq 0$, so $x = -5 + \sqrt{97}$

13.



notice $\triangle BCD = \triangle ABC + \triangle ABD + \triangle ACD$

so, in area,

$\frac{1}{2}\cdot 6 \cdot 10 = \triangle ABC + \frac{1}{2}\cdot 6 \cdot 5 + \frac{1}{2}\cdot 2 \cdot 10$

$30 - 15 - 10 = \triangle ABC$

$5 = \triangle ABC$'s area

alternately, use Hero of Alexandria, where the side lengths are $2\sqrt{34}$, $\sqrt{29}$, $\sqrt{41}$

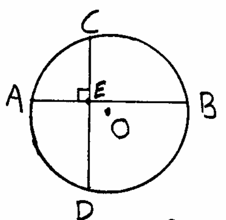
14 $$\frac{V_{cone}}{V_{sphere}} = \frac{\frac{1}{3}\pi\, 25\cdot 6^2}{\frac{4}{3}\pi\, 15^3} = \frac{25\cdot 6^2}{4\cdot 15^3} = \frac{2^2\cdot 3^2\cdot 5^2}{2^2\cdot 3^3\cdot 5^3} = \frac{1}{15}$$

15. $$\text{arc length} = \frac{\text{arc measure}}{360^\circ}\cdot 2\pi r = \frac{75^\circ}{360^\circ}\cdot 2\cdot\pi\cdot 12\text{ cm} = 5\pi\text{ cm}$$

16. $\triangle ABD \sim \triangle CAD$, $AB = \sqrt{AD^2 + BD^2} = \sqrt{16+36} = 2\sqrt{13}$

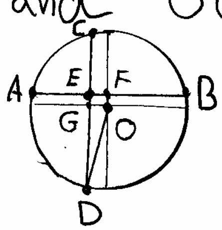
$$\frac{X}{AD} = \frac{AB}{BD} \quad \text{so} \quad X = \frac{AD\cdot AB}{BD} = \frac{4\cdot 2\sqrt{13}}{6} = \frac{4\sqrt{13}}{3}$$

17.



$AE\cdot EB = CE\cdot ED$, so $ED = 18$

Next add points F, G along $\overline{AB}$ and $\overline{CD}$ respectively such that $\overline{OF}\perp\overline{AB}$ and $\overline{OG}\perp\overline{CD}$. We have the following diagram:

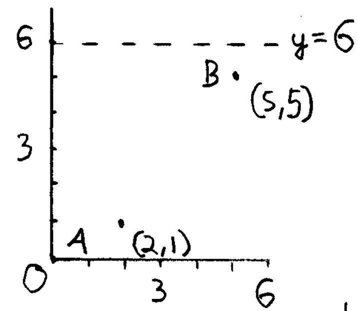


where $GD = \frac{1}{2}CD = 17$, $OG = EF = \frac{1}{2}AB - AE = 6$

so $r^2 = OD^2 = GD^2 + OG^2 = 325$

$A = 325\pi$

18. $\text{lcm}(8,28) = 56 = 2 \cdot 28 = 7 \cdot 8$, so the smaller circle must roll twice around the larger circle. For each complete time around the larger circle, the smaller circle makes an additional revolution, giving a total of $7+2$ rotations.

19. 

Either $\angle A = 90°$, $\angle B = 90°$ or $\angle C = 90°$

case 1, $\angle A = 90°$: $\overline{AB} \perp \overline{AC}$, so

$$\text{slope}(\overrightarrow{AC}) = -\frac{1}{\text{slope}(\overrightarrow{AB})} = -\frac{5-2}{5-1} = -\frac{3}{4}$$

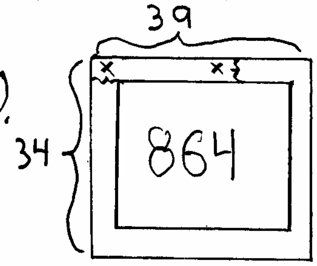
so C lies at the intersection of $y=6$, $y-1 = -\frac{3}{4}(x-2)$ which exists and is unique.

case 2, $\angle B = 90°$: similar to case 1, giving the point at the intersection of $y=6$, $y-5 = -\frac{3}{4}(x-5)$.

case 3, $\angle C = 90°$: we have $\overline{BC} \perp \overline{AC}$, so

$\frac{6-5}{x-5} = -\frac{x-2}{6-1}$ giving $-5 = x^2 - 7x + 10$, which has no real roots.

$\angle C = 90°$ cannot occure, so there are only 2 possible values for x.

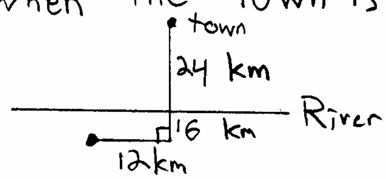
20. 

$(34-x)(39-x) = 864$ cm^2 gives $x = 33$ or $x = \frac{7}{2}$ cm, where 33 cm is invalid, so $x = \frac{7}{2}$cm

21. With an arithmetic progression,
$\sum \text{total} = n \cdot \text{average} = n \cdot \frac{\text{first} + \text{last}}{2}$, so

$n = \frac{360° \cdot 2}{8° + 52°} = 12$

22. By symmetry, we can consider when the town is on the other side of the river giving $D_{min} = \sqrt{40^2 + 12^2}$ km $= 4\sqrt{109}$ km

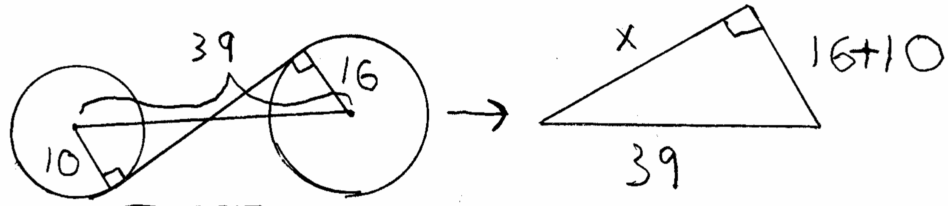


23. We need only consider the white faces.
There is only 1 way that the faces may share 2 edges (all have a common vertex) – case 1.
case 2: If only one edge is shared, then the third face must have exactly one vertex in common with one of the two other faces.
case 3: no edge is shared: The 3 white faces must each be adjacent to a common white face.

There are 24 case 1's, 24 case 2's, 8 case 3's giving the total $\binom{8}{3}$ patterns with 3 distinguishable patterns.

24. $\frac{x}{12} = \frac{x+5}{x}$ so $x^2 - 12x - 60 = 0$, $x = 6 \pm \sqrt{96}$ $x = 6 + 4\sqrt{6}$

25. $x^2 + y^2 - 4x - 16y = 13$ complete the squares
$x^2 - 4x + 4 + y^2 - 16y + 64 = 13 + 4 + 64 = 9^2$

26. 

$x = \sqrt{39^2 - 26^2} = \sqrt{845} = 13\sqrt{5}$

27. The line $\perp$ to $y = -\frac{3}{4}x + b$ passing through $(2,-1)$ is $y-(-1) = \frac{4}{3}(x-2)$; $y = \frac{4}{3}x - \frac{11}{3}$. The points 10 away from $(2,-1)$ on this line are $(8,7)$ and $(-4,-9)$ giving $7 = -\frac{3}{4}\cdot 8 + b$, $b = 13$ and $-9 = -\frac{3}{4}\cdot(-4) + b$, $b = -12$: $\sum b = 1$

28. Slant height is $\sqrt{(3\sqrt{3})^2 + 8^2}$ mm $= \sqrt{91}$ mm

$SA_{Base} = 6\cdot\left(\frac{1}{2}\cdot 6\cdot 3\sqrt{3}\right) mm^2 = 54\sqrt{3}\ mm^2$

$SA_{Side} = 6\cdot\left(\frac{1}{2}\cdot 6\cdot\sqrt{91}\right) mm^2 = 18\sqrt{91}\ mm^2$

$SA = 54\sqrt{3} + 18\sqrt{91}\ mm^2$

29. Slant height is $\sqrt{5^2+12^2}$ cm $= 13$ cm

$SA_{Base} = \pi\cdot 5^2 cm^2 = 25\pi cm^2$

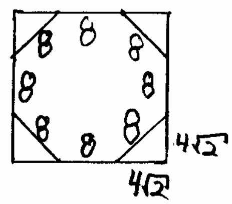
$SA_{Side} = \frac{5}{13}\cdot\pi\cdot 13^2 cm^2 = 65\ cm^2$

$SA = 25\pi + 65\pi\ cm^2 = 90\pi cm^2$

30. Let the height of the 2 cups mark be 1 unit. Thus the height of the untruncated cone is 4 units. The volume below the mark is: 2 cups $= \frac{1}{3}\pi r^2\cdot 4 - \frac{1}{3}\pi\left(\frac{3}{4}r\right)^2\cdot 3$ $= \frac{1}{3}\pi r^2\left(4 - \frac{27}{16}\right) = \frac{1}{3}\pi r^2\left(\frac{37}{16}\right)$ units, so $\frac{1}{3}\pi r^2 = \frac{32 \text{ cups}}{37 \text{ units}}$

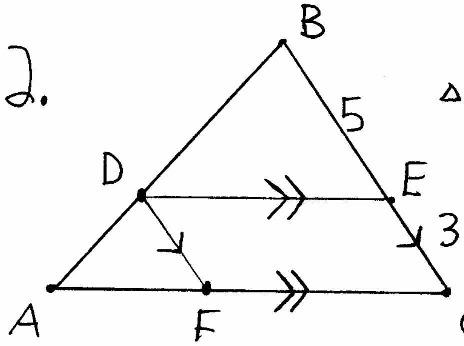
The volume of the pot is $\frac{1}{3}\pi r^2\cdot 4 - \frac{1}{3}\pi\left(\frac{1}{2}r\right)^2\cdot 2$ $= \frac{1}{3}\pi r^2\left(4 - \frac{1}{2}\right) = \frac{1}{3}\pi r^2\cdot\frac{7}{2}$ units $= \frac{7}{2}\cdot\frac{32 \text{ cups}}{37 \text{ units}}$ units $= \frac{112}{37}$ cups

31. Extend the octagon to a square:



$$A=(8+8\sqrt{2})^2-2\cdot(4\sqrt{2})^2=128+128\sqrt{2}\text{ cm}^2$$

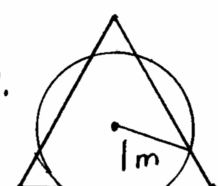
32.



$\triangle ABC \sim \triangle ADF \sim \triangle DBE$

The ratio of the area is the square of the ratio of the length, so the areas of the above $\triangle$'s are in $8^2:3^2:5^2$ ratio. Area $\square DFCE$ = Area($\triangle ABC - \triangle ADF - \triangle DBE$) so the ratio of the area of $\triangle AFD$ to the area of $\square DFCE$ is $3^2:(8^2-3^2-5^2)$; $\frac{9}{30}=\frac{3}{10}$

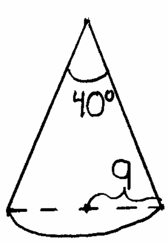
33.



$$\begin{aligned}\text{Area}\,\triangle &= \text{Area}(\triangle-\odot)+\text{Area}(\triangle\cap\odot)\\ &=\text{Area}(\odot-\triangle)+\text{Area}(\odot\cap\triangle)\\ &=\text{Area}\,\odot = \pi\text{ m}^2\end{aligned}$$

$$\text{Area}\,\triangle=\frac{s^2\sqrt{3}}{4}=\pi\text{ m}^2 \Rightarrow s=\sqrt{\frac{4\pi}{\sqrt{3}}}\text{ m}\approx 269\text{ cm}$$

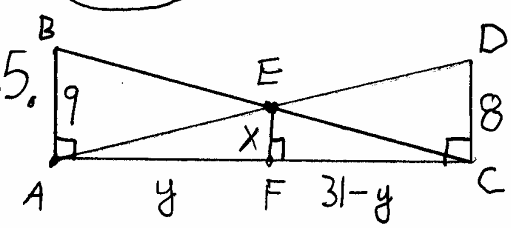
34.



$$\text{height}=\frac{9}{\tan 20°}$$

$$V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\cdot 9^2\cdot\frac{9}{\tan 20°}\text{ cm}^3\approx 2097.44\text{ cm}^3$$

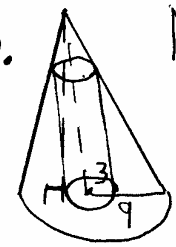
35.



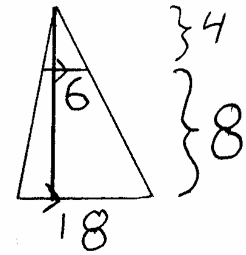
$$\frac{31}{8}=\frac{y}{x},\quad \frac{31}{9}=\frac{31-y}{x}$$

by similar $\triangle$'s

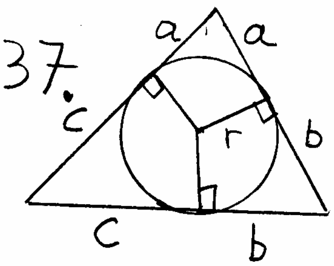
$$\frac{31}{9}=\frac{31-y}{x}=\frac{31}{x}-\frac{y}{x}=\frac{31}{x}-\frac{31}{8};\quad \frac{31}{x}=\frac{31}{9}+\frac{31}{8};\quad \frac{1}{x}=\frac{1}{9}+\frac{1}{8};\quad x=\frac{72}{17}$$

36. 

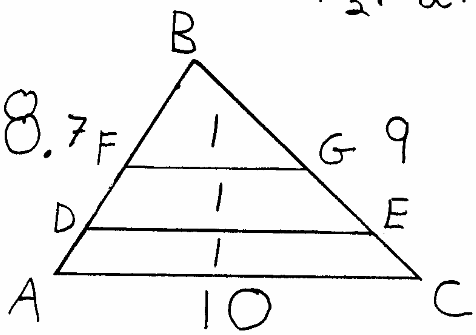
Notice the cross section:



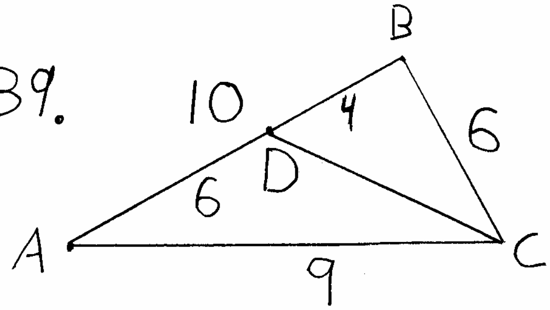
so the $\perp$ height of the cylinder is 8.
V cylinder is $\pi r^2 h = \pi \cdot (3\text{cm})^2 \cdot 8\text{cm} = 72\pi\,\text{cm}^3$

37. 

$\pi r^2 = 192\text{mm}^2 ; r = 8\sqrt{3}$ mm
$2(a+b+c) = 200\text{mm}$
Area of $\triangle = \frac{1}{2}r\cdot b + \frac{1}{2}r\cdot b + \frac{1}{2}r\cdot a + \frac{1}{2}r\cdot a + \frac{1}{2}r\cdot c + \frac{1}{2}r\cdot c = r(a+b+c) = 800\sqrt{3}\ \text{mm}^2$

38. 

$\triangle ABC \sim \triangle DBE$
area is in 3:2 ratio, so
length is $\sqrt{3} : \sqrt{2}$; $DE = \frac{10\sqrt{2}}{\sqrt{3}}$ units
$DE = \frac{10}{3}\sqrt{6}$ units

39. 

We know that the $\angle$ bisector divides the far side into segments proportional to the other sides of the triangle; hence $BD = 4$, $AD = 6$.
Using the law of cosines, we get:
$6^2 = 10^2 + 9^2 - 2\cdot 10 \cdot 9 \cdot \cos A$: $\cos A = \frac{29}{36}$, and
$CD^2 = 6^2 + 9^2 - 2\cdot 6 \cdot 9 \cdot \cos A = 30$, so
$CD = \sqrt{30}$ units

40. For notational purposes, put the following coordinate system on the basketball court: Let the length be along the x-axis, the width be along the y-axis with one corner being at (0,0) and the court extending in the positive direction. Further, let the side of a tile be 1 unit long.
Let the diagonal extend from (0,0) to (80,55).

The line will enter a new tile every time that either its x or y value obtains an integer value, except when both are simultaneously integer values.

Hence the solution is:

$$\text{length} + \text{width} - \gcd(\text{length}, \text{width})$$

$$80 + 55 - 5 = 130$$