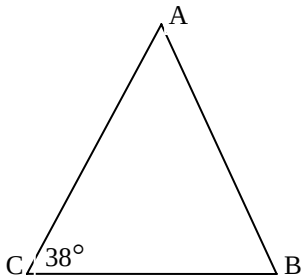
Mu Alpha Theta National Convention: Denver, 2001
Geometry Topic Test – Alpha Division

1. What is the measure of angle *A*, in degrees, given that *AB* = *AC*?

(A) 88° (B) 95°
(C) 98° (D) 104°
(E) NOTA



2. A cube made of green material is painted white, then cut into 512 congruent cubes. How many of the smaller cubes have exactly two white faces?

(A) 64 (B) 72 (C) 188 (D) 216 (E) NOTA

3. What is the circumference, in centimeters, of a circle with area 216π square centimeters?

(A) 12π cm (B) $12\pi\sqrt{3}$ cm (C) $12\pi\sqrt{6}$ cm (D) $24\pi\sqrt{2}$ cm (E) NOTA

4. Two vertices are chosen on a cube of edge length six centimeters. What is the greatest possible distance between these points, in centimeters?

(A) $6\sqrt{5}$ cm (B) 12 cm (C) $6\sqrt{2}$ cm (D) $6\sqrt{3}$ cm (E) NOTA

5. What is the perimeter of a regular polygon whose interior angles each measure 144°, and a side of which is 7 units long?

(A) 91 units (B) 84 units (C) 77 units (D) 70 units (E) NOTA

6. The points *A* (4, 9), *B* (2, -3), and *C* (-1, 7) form a triangle in the Cartesian plane. What are the coordinates of the centroid of this triangle?

(A) $\left(\frac{5}{3},\frac{13}{3}\right)$ (B) $\left(\frac{3}{2},\frac{9}{2}\right)$ (C) $(1,5)$ (D) $\left(\frac{5}{2},4\right)$ (E) NOTA

7. What is the surface area, in square centimeters, of a rectangular box with length 5 centimeters, width 8 centimeters, and height 13 centimeters?

(A) 396 cm^2 (B) 418 cm^2 (C) 444 cm^2 (D) 456 cm^2 (E) NOTA

8. What is the surface area, in square meters, of a sphere of radius 4 meters?

(A) $16\pi\sqrt{15}$ m^2 (B) $12\pi\sqrt{30}$ m^2 (C) $32\pi\sqrt{5}$ m^2 (D) 64π m^2 (E) NOTA

9. What is the maximum value of $f(x) = 14 - 3x - 2x^2$?

(A) $\frac{121}{8}$ (B) $\frac{243}{16}$ (C) 15 (D) $\frac{245}{16}$ (E) NOTA

10. How many half-meter by one-third-meter rectangular tiles would it take to completely tile the floor of a 22-meter by 33-meter room?

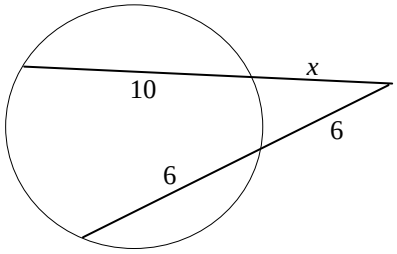
(A) 3989 (B) 4111 (C) 4356 (D) 4238 (E) NOTA

11. If the volume of a sphere is tripled, what is the ratio of the new diameter to the old diameter?

(A) 3 (B) $\sqrt{3}$ (C) $\sqrt[3]{3}$ (D) $\frac{1}{3}$ (E) NOTA

12. What is the value of *x* in the figure to the right?

(A) 4 (B) 5 (C) $-5+\sqrt{97}$
(D) $3+\sqrt{5}$ (E) NOTA



13. What is the area of the triangle with vertices at the points (-8, -3), (-12, -8), and (-6, 2)?

(A) 3 (B) $\frac{7}{2}$ (C) 4 (D) 5 (E) NOTA

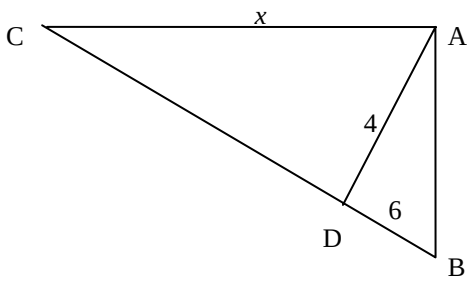
14. What is the ratio of the volume of a cone of height 25 millimeters and radius 6 millimeters to the volume of a sphere of radius 15 millimeters?

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{5}$ (D) $\frac{1}{6}$ (E) NOTA

15. What is the length, in centimeters, of a 75° arc in a circle of radius 12 centimeters?

(A) $\frac{9\pi}{2}$ cm (B) 5π cm (C) $\frac{19\pi}{3}$ cm (D) $\frac{20\pi}{3}$ cm (E) NOTA

16. What is x in the triangle shown if $\overline{AD}$ is the altitude to the hypotenuse of right triangle ABC?



(A) $\frac{3\sqrt{89}}{2}$ (B) $\sqrt{119}$ (C) $6\sqrt{3}$
(D) $\frac{4\sqrt{13}}{3}$ (E) NOTA

17. In circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at right angles at E. If $AE = 12$, $BE = 24$, and $CE = 16$, what is the area of the circle?

(A) 325π (B) $80\pi\sqrt{20}$ (C) 361π (D) 380π (E) NOTA

18. Two circles have radii of 8 and 28 centimeters. The larger circle is fixed in place, while the smaller circle rolls around the outside of the larger circle. In their initial position, point A on the larger circle coincides with point B on the smaller circle. How many times will the smaller circle have rotated about its center by the first time these points coincide again?

(A) 7 (B) 8 (C) 9 (D) 10 (E) NOTA

19. The points A (2, 1), B (5, 5), and C (x, 6) form a right triangle in the Cartesian plane. How many values are possible for x?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

20. The dimensions of the outside of a framed rectangular picture are 34 centimeters by 39 centimeters. The area of the picture itself is 864 square centimeters. What is the width of the framing material, in centimeters, given that the frame is rectangular and of uniform thickness around the entire picture?

(A) $\frac{49+6\sqrt{21}}{2}$ cm (B) $\frac{35+5\sqrt{29}}{2}$ cm
(C) $\frac{45+4\sqrt{39}}{2}$ cm (D) $22+6\sqrt{5}$ cm (E) NOTA

21. A circle is completely divided into n sectors. The angles of these sectors form an arithmetic progression. If the smallest angle is 8°, and the largest is 52°, determine n.

(A) 11 (B) 12 (C) 13 (D) 14 (E) NOTA

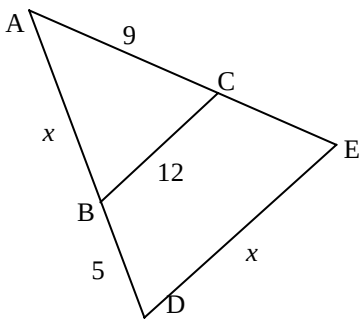
22. A pioneer is headed home from town, but must stop at the river along the way to get water. If the river runs east-west 24 kilometers north of the town, and the pioneer's home is 12 kilometers west and 8 kilometers north of the town, what is the shortest distance, in kilometers, the pioneer can travel?

(A) $4\sqrt{109}$ km (B) $15\sqrt{7}$ km (C) $5\sqrt{95}$ km (D) $4\sqrt{13}$ km (E) NOTA

23. A regular octahedron (a solid with eight faces that are all equilateral triangles; the shape of most 8-sided dice; the shape you get if you connect the centers of the faces of a cube) is painted white on three sides and red on the other five. How many distinguishable patterns can be produced in this way?

(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

24. Given that $\overline{BC}$ is parallel to $\overline{DE}$, what is x?



(A) $6+4\sqrt{6}$ (B) 16 (C) 17
(D) 18 (E) NOTA

25. What is the radius of the following circle? $4x^2+4y^2-16x-64y=52$

(A) $\frac{15}{2}$ (B) $\frac{17}{2}$ (C) 9 (D) $\frac{19}{2}$ (E) NOTA

26. What is the length of the common internal tangent of two circles of radii 10 and 16 units whose centers are 39 units apart?

(A) $24\sqrt{2}$ (B) 30 (C) 32 (D) $13\sqrt{5}$ (E) NOTA

27. Find the sum of all values of b such that the minimum distance from the point (2, -1) to the line $y=-\frac{3}{4}x+b$ is 10.

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

28. What is the surface area, in square millimeters, of a pyramid with a regular hexagonal base of edge length six millimeters and a height of eight millimeters?

(A) $180+54\sqrt{3}$ mm^2 (B) $54\sqrt{3}+70\sqrt{6}$ mm^2
(C) $140\sqrt{3}$ mm^2 (D) $54\sqrt{3}+18\sqrt{91}$ mm^2 (E) NOTA

29. What is the surface area of a right circular cone with base radius 5 centimeters and height 12 centimeters?

(A) 85π (B) 90π (C) 95π (D) 100π (E) NOTA

30. A coffeepot is in the shape of a truncated cone. It has a circular base that tapers uniformly to a circular top parallel to the base and centered over the base, having a radius equal to half the radius of the base. Whatever spout this coffeepot has is of negligible dimensions. There is a mark halfway up the coffeepot (by height) which says "2 cups". If the pot were filled up to the very top, how many cups of coffee would it then hold?

(A) $\frac{112}{37}$ cups (B) $\frac{25}{8}$ cups (C) 3 cups (D) $\frac{80}{27}$ cups (E) NOTA

31. What is the area, in square centimeters, of a regular octagon with sides of length 8 centimeters?

(A) $128+128\sqrt{2}$ cm^2 (B) $256+12\sqrt{30}$ cm^2
(C) $144+64\sqrt{3}$ cm^2 (D) $128+128\sqrt{3}$ cm^2 (E) NOTA

32. In triangle *ABC*, points *D*, *E*, and *F* are chosen on $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ respectively so that $\overline{DE}$ and $\overline{DF}$ are parallel to $\overline{AC}$ and $\overline{BC}$, respectively. If *BE* is 5 units and *EC* is 3 units, what is the ratio of the area of triangle *AFD* to the area of parallelogram *DFCE*?

(A) $\frac{3}{10}$ (B) $\frac{7}{15}$ (C) $\frac{5}{8}$ (D) $\frac{3}{2}$ (E) NOTA

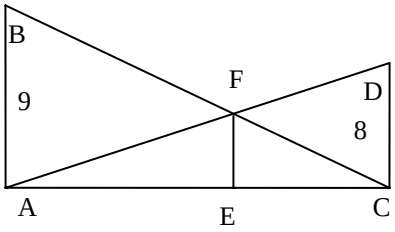
33. A equilateral triangle and a circle have the same center. The area of the triangle that is not inside the circle is equal to the area of the circle that is not inside the triangle. If the radius of the circle is one meter, find the length of a side of the triangle, to the nearest centimeter.

(A) 225 cm (B) 240 cm (C) 269 cm (D) 283 cm (E) NOTA

34. A right, circular cone has a base radius of 9 centimeters. Let point *A* be the apex of the cone, and $\overline{BC}$ be a diameter of the base of the cone. What is the volume of the cone, to the nearest hundredth of a cubic centimeter, if the measure of angle *BAC* is 40°?

(A) 909.79 cm^3 (B) 1099.32 cm^3 (C) 2097.44 cm^3 (D) 2351.91 cm^3 (E) NOTA

35. Find the length of $\overline{EF}$ if $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are all perpendicular to $\overline{AC}$ and the length of $\overline{AC}$ is 31.



(A) 5 (B) $\frac{11}{2}$ (C) $\frac{72}{17}$

(D) $\frac{365}{72}$ (E) NOTA

36. A cylinder of radius 3 centimeters is inscribed in a cone with base radius 9 centimeters and height 12 centimeters. If their axes lie on the same line, what is the volume of the cylinder, in cubic centimeters?

(A) $\frac{145\pi}{4}$ cm^3 (B) 54π cm^3 (C) 63π cm^3 (D) 72π cm^3 (E) NOTA

37. A circle with area 192π square millimeters is inscribed in a triangle with perimeter 200 millimeters. What is the area of the triangle, in square millimeters?

(A) 1400 mm^2 (B) $800\sqrt{3}$ mm^2 (C) $400\sqrt{5}$ mm^2 (D) $700\sqrt{2}$ mm^2 (E) NOTA

38. In triangle *ABC*, *AB* = 7 units, *BC* = 9 units, and *AC* = 10 units. Line segments $\overline{DE}$ and $\overline{FG}$ are drawn parallel to side $\overline{AC}$, with *D* and *F* on $\overline{AB}$ and *E* and *G* on $\overline{BC}$, with $\overline{DE}$ closer to $\overline{AC}$ such that the triangle is divided into three equal areas. What is the length of $\overline{DE}$?

(A) $\frac{10}{3}$ units (B) $\frac{20\sqrt{3}}{3}$ units (C) $\frac{20}{3}$ units (D) $\frac{10\sqrt{6}}{3}$ units (E) NOTA

39. What is the length of the bisector of angle *C* of triangle *ABC* with *a* = 6 units, *b* = 9 units, and *c* = 10 units?

(A) $\sqrt{30}$ units (B) $5+\sqrt{2}$ units (C) $8-2\sqrt{2}$ units (D) $\frac{10\sqrt{2}}{3}$ units (E) NOTA

40. A basketball court is made up of a grid of square parquet tiles. It is 80 tiles long and 55 tiles wide. If a straight line is drawn diagonally from one corner of the court to the opposite corner, how many tiles will it intersect? Note: "intersect" means that the line passes through the interior of the tile, not just along an edge or through a corner of the tile.

(A) 125 (B) 129 (C) 134 (D) 135 (E) NOTA