

Mu Alpha Theta National Convention: Denver, 2001
Logarithms and Exponents Topic Test – Alpha Division

1. Given that $\frac{\log_{10} A}{\log_{10} B} = \frac{A}{B} = \frac{2}{3}$, what are A and B , in that order?
 (A) 2 and 3 (B) 1 and $\frac{3}{2}$ (C) $\left(\frac{2}{3}\right)^4$ and $\left(\frac{2}{3}\right)^3$ (D) $\left(\frac{2}{3}\right)^3$ and $\left(\frac{2}{3}\right)^2$ (E) NOTA

2. Solve for x : $\log_5(\log_3(\log_6 x)) = 0$
 (A) 216 (B) 125 (C) 36 (D) 243 (E) NOTA

3. Evaluate: $\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}}}$
 (A) 3 and -2 (B) 3 (C) -2 (D) 6 (E) NOTA

4. Which of the following is equal to i^i ?
 (A) $e^{i\pi}$ (B) e^π (C) $e^{-\frac{\pi}{2}}$ (D) $e^{-i\pi}$ (E) NOTA

5. Solve for x : $a^x b^x = c$
 (A) $\frac{\ln c}{\ln a - \ln b}$ (B) $\frac{\ln c}{\ln(a + b)}$ (C) $\log_{ab} c$ (D) $\frac{\ln c}{\ln a \ln b}$ (E) NOTA

6. Solve for x : $\frac{2}{3} \ln x^3 + \frac{1}{2} \ln x^4 - \frac{1}{2} \ln x^2 = 6$
 (A) e (B) e^3 (C) 3^6 (D) e^2 (E) NOTA

7. Solve for x : $18^{x^2+2x+4} = (54\sqrt{2})^{x^2+4}$
 (A) 2 (B) 4 (C) 3 (D) $\frac{3}{2}$ (E) NOTA

8. What is the product of the two solutions for x : $-\frac{2}{3} \log_x a + \frac{4}{3} \log_{ax} a + \frac{7}{3} \log_{a^2x} a = 0$
 (A) a (B) $\sqrt[3]{a^2}$ (C) a^{-1} (D) $a^{\frac{4}{3}}$ (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001
Logarithms and Exponents Topic Test – Alpha Division

9. If \$52,000 is invested at a 4% annual rate compounded continuously, how many years (to the nearest year) will it take for it to triple?

- (A) 31 (B) 32 (C) 19 (D) 27 (E) NOTA

10. Evaluate: $\frac{1}{\log_{\left(\frac{1}{3}\right)} 144} + \frac{1}{\log_3 144} + \frac{1}{\log_{12} 144}$

- (A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) NOTA

11. Which of the following is equivalent to: $\frac{1}{2} \ln(9) + \ln(2) + \frac{1}{3} \ln(8^2)$

- (A) $\ln\left(\frac{167}{6}\right)$ (B) $\ln(167) + \ln(6)$ (C) $\frac{11}{6} \ln(1152)$ (D) $\ln(3) + \ln(8)$ (E) NOTA

12. How many solutions are there to the following problem: $x * x^{\frac{1}{x}} = x^x$

- (A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

13. If you multiply together the solutions from problem 12, what do you get?

- (A) -2 (B) -1 (C) $\frac{1+\sqrt{5}}{2}$ (D) $\frac{1-\sqrt{5}}{2}$ (E) NOTA

14. $A \left(2^{\frac{x}{y}}\right) = B$. Solve for x with respect to y .

- (A) $y \log_2 A$ (B) $y \log_2 \left(\frac{B}{A}\right)$ (C) $Ay^2 \log_2 B$ (D) $\log_2 \left(\frac{Ay}{B}\right)$ (E) NOTA

15. Determine the sum of all solutions to $3^{x \log_3(x+2)} = x + 2$.

- (A) 2 (B) 1 (C) 0 (D) 3 (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001
Logarithms and Exponents Topic Test – Alpha Division

16. Given that $0 \leq x < 2\pi$, solve for x : $3^{\sin x} = \frac{1}{3}$
- (A) $\frac{3\pi}{2}$ (B) $\frac{\pi}{2}$ (C) $-\pi$ (D) $\frac{-\pi}{3}$ (E) NOTA
17. Given that $0 \leq x < 2\pi$, determine the sum of all values of x for which $\ln(\sin x) - \ln(\cos x) = 0$.
- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{2}$ (E) NOTA
18. What is the product of the solutions for x ? $(2 \log_4 x^{\log_4 x}) - (4 \log_4 x) + 3 = 9$
- (A) 64 (B) 32 (C) -6 (D) 16 (E) NOTA
19. Which of the following is equal to $9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9$?
- (A) 3^{22} (B) 3^{19} (C) 3^{24} (D) 3^{20} (E) NOTA
20. Solve for x : $4^x - 16(2^x)^2 + 64 \cdot 4^x = 98$
- (A) 2 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) NOTA
21. Solve for x : $\sqrt{x^2 - 5x + 40} = 6$
- (A) 1 (B) 4 and 1 (C) $\frac{5 \pm \sqrt{76}}{2}$ (D) 3 and 6 (E) NOTA
22. Solve for x : $\log_3 x = \log_x 27$
- (A) $\sqrt{3}$ (B) 9 (C) 3 (D) $3\sqrt{3}$ (E) NOTA
23. If a , b , and c are rational and $54^a \cdot 50^b \cdot 126^c = 2160$, evaluate $a + b + c$.
- (A) 3 (B) 4 (C) 1 (D) 5 (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001
Logarithms and Exponents Topic Test – Alpha Division

24. In Problem 23, what is $3a+2c$?

- (A) 4 (B) 3 (C) 2 (D) 1 (E) NOTA

25. Which of the following is equivalent to $\log_x 25 + \log_5 x - \frac{2}{\log_5 x}$?

- (A) $\frac{1}{\log_{25} x}$ (B) $\log_{25} x - \log_5 x$ (C) $\log_x 5$ (D) $\frac{1}{\log_x 5}$ (E) NOTA

26. Which of the following is equivalent to $(2^a 3^{a+2})^{a-2}$?

- (A) $\left(\frac{6^a}{(2^a 3^2)}\right)^2$ (B) $\frac{(2^a 3^a)^2}{2^2 3^2}$ (C) $\frac{3^{a^2-2a-4} 2^{a^2}}{2^a}$ (D) $3^{a^2-4} 2^{a^2-2a}$ (E) NOTA

27. Which of the following is equivalent to $\frac{c^{3n+2} b^{2n-1}}{b^{2n+3} (c^2)^{n+1}}$?

- (A) $\frac{c^{n-3}}{b^{4-n}}$ (B) $\frac{c^n}{b^{2-n}}$ (C) $\frac{c^n}{b^4}$ (D) $\frac{c^{4+n}}{b^{n-3}}$ (E) NOTA

28. Evaluate: $7^{\ln(25)}$

- (A) $5^{\ln 49}$ (B) $7^{\ln 2 \cdot \ln 5}$ (C) $5^{\ln 7}$ (D) $\ln(e^{\ln 7 \cdot \ln(25)})$ (E) NOTA

29. If $2^x = 3$, evaluate $\frac{2 \cdot 4^x - 3 \cdot 2^x}{4} + \frac{8^x}{4}$.

- (A) $\frac{13}{2}$ (B) $\frac{35}{4}$ (C) 9 (D) 7 (E) NOTA

30. Solve for x : $a^{\log_{10} a^{x^2}} = m^{\log_{10} m}$

- (A) $\sqrt{\log_m a}$ (B) am (C) $\log_m a$ (D) $\log_a m$ (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001
Logarithms and Exponents Topic Test – Alpha Division

31. Solve for x : $\log_7(2x + 3) + \log_7(3x - 1) = 1$
- (A) $\frac{2}{3}$ and -1 (B) $\frac{5}{6}$ and -2 (C) 2 and $-\frac{2}{3}$ (D) 3 and 2 (E) NOTA
32. Simplify: $\frac{3^x 3^{1-x} 9^x}{27^{\frac{2}{3}x}}$
- (A) 9 (B) 3 (C) 3^{x-1} (D) 9^{1-x} (E) NOTA
33. Consider the equation $\log_{10}(3x + 2) + \frac{1}{\log_{2^{x-1}} 10} = 1$, which is satisfied for two rational values of x . These two rational values can be expressed as $\frac{a}{b}$ and $\frac{c}{d}$, where a and b are relatively prime, as are c and d . If either $\frac{a}{b}$ or $\frac{c}{d}$ is negative, treat a or c as the negative quantity, not b or d . Evaluate $a + b + c + d$.
- (A) 6 (B) 7 (C) 8 (D) 9 (E) NOTA
34. If $\log_2(4x + 8) - \log_2(x) \leq 3$, what values of x are possible.
- (A) $x \geq 2$ (B) $x \leq 3$ (C) $x \leq 2$ (D) $x \geq 3$ (E) NOTA
35. Which of the following is equivalent to $\log_4 x^3$?
- (A) $\frac{2\log_2 x}{3}$ (B) $6\log_2 x$ (C) $\frac{3\log_2 x}{2}$ (D) $3\log_{\sqrt{4}} x$ (E) NOTA
36. What is the sum of all the positive integral factors of 630?
- (A) 900 (B) 1530 (C) 1242 (D) 1872 (E) NOTA
37. If $\log_{10} 5 = a$, $\log_{10} 7 = b$, and $\log_{10} 2 = c$, what is $\log_2 10 + \log_{35} 2$ in terms of a , b , and c .
- (A) $\frac{a+b+c^2}{ac+bc}$ (B) $\frac{ac+b}{ab^2}$ (C) $\frac{ac+b}{ab^2c}$ (D) $\frac{ab-c}{abc}$ (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001
Logarithms and Exponents Topic Test – Alpha Division

38. Which is equivalent to: $\frac{r^{-4} - s^{-4}}{r^{-2} - s^{-2}}$?

- (A) $r^2 - s^2$ (B) $s^{-2} + r^{-2}$ (C) $\frac{r^3 - s^3}{r - s}$ (D) $\frac{r^3 - s^3}{r + s}$ (E) NOTA

39. Evaluate: $\frac{2i - 3}{3i + 2}$

- (A) $-i$ (B) $\frac{i}{13}$ (C) i (D) $\frac{-i}{13}$ (E) NOTA

40. Which of the following is equal to: $\log_a\left(\frac{b}{c}\right) + \frac{1}{\log_b(a^3)} + \frac{1}{\log_c(a)}$

- (A) $\log_c(\sqrt[3]{a^2b})$ (B) $\log_a(\sqrt[3]{b^4})$ (C) $\log_b(\sqrt[3]{ac})$ (D) $\log_c\left(\frac{\sqrt{ab}}{b}\right)$ (E) NOTA