Mu Alpha Theta National Convention: Denver, 2001

Matrices and Vectors Topic Test – Alpha Division

1. Evaluate: $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 1 & -3 \\ 1 & -2 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -2 & -1 \\ -4 & -3 & 6 \\ 1 & -2 & 2 \end{bmatrix}$

(A) $\begin{bmatrix} -1 & 0 & 4 \\ -8 & 4 & -9 \\ 0 & 4 & -2 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 0 & 4 \\ 0 & 4 & -9 \\ 0 & 0 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & -4 & 4 \\ -8 & 4 & 3 \\ 0 & 4 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 4 & 2 \\ 0 & -2 & 3 \\ 2 & -4 & 2 \end{bmatrix}$ (E) NOTA

2. If $2\begin{bmatrix} 1 & -3 \\ x & 2 \end{bmatrix} - 4\begin{bmatrix} y & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 4 & -8 \end{bmatrix}$, what is the value of $x + y$?

(A) 1 (B) 0 (C) -1 (D) $-\frac{5}{4}$ (E) NOTA

3. Evaluate: $\begin{bmatrix} 5 & 0 \\ 3 & 2 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} -1 & -5 & 4 \\ 3 & 0 & -2 \end{bmatrix}$

(A) $\begin{bmatrix} -5 & -25 \\ 3 & -15 \end{bmatrix}$ (B) $\begin{bmatrix} -5 & -25 & 20 \\ 3 & -15 & 8 \\ -13 & -5 & 12 \end{bmatrix}$

(C) $\begin{bmatrix} -5 & -25 & -3 \\ 3 & -15 & 12 \\ 11 & 30 & -5 \end{bmatrix}$ (D) Cannot Be Done (E) NOTA

4. If A and B are 3 by 3 matrices whose (i,j)th elements are $i+j$ and $3i-j$ respectively, what is the sum of the largest and smallest entries in AB?

(A) 92 (B) 99 (C) 106 (D) 114 (E) NOTA

5. Evaluate: $\begin{vmatrix} 3 & -1 & -2 \\ a & 4 & 0 \\ -5 & 0 & 5 \end{vmatrix}$

(A) $5a + 20$ (B) $5a + 100$ (C) $-5a + 20$ (D) $-5a + 100$ (E) NOTA

6. What values of x will satisfy the inequality $\begin{vmatrix} 6 & -x \\ 3x-1 & -2 \end{vmatrix} > 2$?

(A) $x < -\frac{4}{3}$ or $x > -\frac{1}{3}$ (B) $-\frac{4}{3} < x < -\frac{1}{3}$
(C) $x < -2$ or $x > \frac{7}{3}$ (D) $-\frac{4}{3} < x < \frac{7}{3}$ (E) NOTA

7. Which of these matrices has a determinant different from the others?

(A) $\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$ (B) $\begin{bmatrix} c & f & i \\ -b & -e & -h \\ a & d & g \end{bmatrix}$
(C) $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ (D) $\begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix}$ (E) NOTA

8. The system of equations

$$-2x + y - 5z = 111$$
$$x - 2y + z = 67$$
$$3x - Ay = 37$$

is satisfied by exactly one ordered triple (x, y, z). Which value(s) of A is/are not possible?

(A) $A \le 9$ (B) $A = 9$ (C) $A = 12$ (D) $A = \pm 9$ (E) NOTA

9. Evaluate r minus s if: $r[1 \quad 2 \quad 3] + s[1 \quad -2 \quad 4] + t[-1 \quad 0 \quad -2] = [-2 \quad 5 \quad 3]$.

(A) 2 (B) $\frac{5}{2}$ (C) 3 (D) $\frac{7}{2}$ (E) NOTA

10. Harry was shown the solution of x and y (shown below) for a system of equations using Cramer's Rule. From this information, he was able to set up the solution for z. What value did he get for z?

$$x=\frac{\begin{vmatrix}-3&-1&-2\\1&4&0\\2&0&5\end{vmatrix}}{\begin{vmatrix}3&-1&-2\\2&4&0\\-5&0&5\end{vmatrix}} \quad y=\frac{\begin{vmatrix}3&-3&-2\\2&1&0\\-5&2&5\end{vmatrix}}{\begin{vmatrix}3&-1&-2\\2&4&0\\-5&0&5\end{vmatrix}}$$

(A) $-\frac{9}{10}$ (B) $-\frac{9}{22}$ (C) -1 (D) $-\frac{6}{5}$ (E) NOTA

11. What is the cofactor of the element in the first row, second column of the matrix $\begin{bmatrix}1&4&-5\\-2&3&-4\\-1&2&-3\end{bmatrix}$?

(A) -5 (B) -2 (C) 2 (D) 5 (E) NOTA

12. Evaluate: $\begin{bmatrix}2&-2\\-1&-1\end{bmatrix}^{-1}$

(A) $\begin{bmatrix}-1&1\\2&2\end{bmatrix}$ (B) $\begin{bmatrix}\frac{1}{4}&-\frac{1}{2}\\-\frac{1}{4}&-\frac{1}{2}\end{bmatrix}$ (C) $\begin{bmatrix}-2&1\\3&-1\end{bmatrix}$ (D) $\begin{bmatrix}-\frac{1}{2}&-\frac{1}{4}\\-\frac{1}{2}&\frac{1}{4}\end{bmatrix}$ (E) NOTA

13. Let A, B, and C be 2 by 2 matrices whose *i,j* th elements are $a_{i,j}$, $b_{i,j}$, and $c_{i,j}$, respectively. If $c_{i,j}=\sum_{n=1}^{2}\left(a_{n,i}b_{n,j}\right)$, which of the following is true?

(A) $C=AB$ (B) $C=A^TB$ (C) $C=AB^T$ (D) $C=\left(AB\right)^T$ (E) NOTA

14. Determine the sum of the elements in the multiplicative inverse of $\begin{bmatrix}z&3\\-1&-3\end{bmatrix}$.

(A) 2*z* - 4 (B) $\frac{4z+4}{6-z}$ (C) $\frac{4z-1}{3-z}$ (D) $\frac{z-5}{3-3z}$ (E) NOTA

15. What is the product of the eigenvalues of the matrix $\begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix}$?

(A) 14 (B) 15 (C) 16 (D) 17 (E) NOTA

16. Which of the following are eigenvectors of the matrix $\begin{bmatrix} -4 & 1 \\ 3 & -2 \end{bmatrix}$?

I. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ II. $\begin{bmatrix} -6 \\ -6 \end{bmatrix}$ III. $\begin{bmatrix} -2 \\ -6 \end{bmatrix}$ IV. $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(A) I only (B) III only (C) I & II only (D) I, II, & III only (E) NOTA

17. A matrix M is reduced by a series of elementary row operations to the matrix $2I$, where I is the identity matrix. What matrix will be the result of the same sequence of row operations applied to $12I$?

(A) $24M$ (B) $144M$ (C) $24M^{-1}$ (D) $144M^{-1}$ (E) NOTA

18. Suppose the 4 by 4 matrix A has a determinant of 4. What is the determinant of $4A$?

(A) 16 (B) 64 (C) 256 (D) 1024 (E) NOTA

19. If transformation matrix M is such that $M \times \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ z \\ x \\ w \\ v \end{bmatrix}$ for all values of v, w, x, y, and z, then what is $M^{-1} \times \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix}$?

(A) $\begin{bmatrix} w \\ y \\ x \\ z \\ v \end{bmatrix}$ (B) $\begin{bmatrix} x \\ w \\ y \\ v \\ z \end{bmatrix}$ (C) $\begin{bmatrix} y \\ w \\ v \\ x \\ z \end{bmatrix}$ (D) $\begin{bmatrix} z \\ y \\ x \\ v \\ w \end{bmatrix}$ (E) NOTA

20. What is the rank of the matrix $\begin{bmatrix} 1 & -3 & 5 & -1 \\ 2 & 0 & -4 & 3 \\ 3 & -3 & 1 & 2 \end{bmatrix}$?

(A) 12 (B) 11 (C) 3 (D) 2 (E) NOTA

21. Given non-coincident points *A* and *B* in two dimensions, what shape is the locus of points *P* where $\overrightarrow{AP} = t\overrightarrow{AB}$?

(A) a triangle (B) a hyperbola (C) a circle (D) a line (E) NOTA

22. A particle travels from the point (4, 7) to the point (8, 10) in 5 seconds. What is the particle's average velocity (in units per second) during this time interval?

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) $\left[\frac{4}{5}, \frac{3}{5}\right]$ (E) NOTA

23. If $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - \vec{k}$, and $\vec{c} = -3\vec{i} - \vec{j} + 2\vec{k}$, what is $2\vec{a} + \vec{b} - 4\vec{c}$?

(A) $5\vec{i} + 2\vec{j} - \vec{k}$ (B) $5\vec{i} + \vec{j} - 3\vec{k}$ (C) $2\vec{i} - 12\vec{j} + 5\vec{k}$ (D) $17\vec{i} - 7\vec{k}$ (E) NOTA

24. What is the magnitude of the vector [-1, 3, 4]?

(A) $2\sqrt{6}$ (B) 5 (C) $\sqrt{26}$ (D) $3\sqrt{3}$ (E) NOTA

25. What is the dot product of the vectors [1, 4] and [-3, -4]?

(A) -19 (B) -12 (C) -6 (D) 4 (E) NOTA

26. What is the angle between the vectors [3, -5] and [-2, 1] (to the nearest tenth of a degree)?

(A) 2.6° (B) 126.3° (C) 137.6° (D) 147.5° (E) NOTA

27. Given points *A* and *B* in two dimensions, what is the shape of the locus of points *P* where $\overrightarrow{AP} \cdot \overrightarrow{BP} = 0$?

(A) a line (B) a circle (C) a hyperbola (D) a triangle (E) NOTA

28. Vector $\vec{a}$ in three dimensions makes equal acute angles with $\vec{i}$, $\vec{j}$, and $\vec{k}$. What is the measure of these angles, to the nearest tenth of a degree?

(A) 54.7 (B) 51.2 (C) 49.4 (D) 45.0 (E) NOTA

29. What is the cross product of the vectors [2, 0, -4] and [-2, 3, 2]?

(A) [-14, 7, 6] (B) [20, 32, -9] (C) [5, -12, -18] (D) [12, 4, 6] (E) NOTA

30. For what value of a will the vector $[a, -3, 4a]$ be parallel to the vector [2, -6, -8]?

(A) -2 (B) -1 (C) 1 (D) 2 (E) NOTA

31. Which of the following is an equation of the line through the points (1, 1, 2) and (3, 5, 8)?

(A) $[x, y, z] = [1, 1, 2] + t[2, 4, 6]$ (B) $[x, y, z] = [1, 1, 2] + t[5, 3, -1]$
(C) $[x, y, z] = [3, 5, 8] + t[-3, -1, 0]$ (D) $[x, y, z] = [3, 5, 8] + t[4, -3, 8]$ (E) NOTA

32. A particle starts at the origin and travels with velocity [1, 2] (measured in units/second) for a while. Then, it changes velocity, travelling at [-3, 1] (measured similarly) for another interval of time. How long (in seconds) has it traveled when it reaches the point (3, 12)?

(A) 6 seconds (B) $\frac{45}{7}$ seconds
(C) $\frac{49}{8}$ seconds (D) Cannot Be Determined (E) NOTA

33. Which of the following is an equation of the plane perpendicular to the vector [1, -2, 3] and passing though the point (2, -3, -4)?

(A) $x - 2y + 3z = -4$ (B) $x - 2y + 3z = 14$
(C) $2x - 3y - 4z = 29$ (D) $2x - 3y - 4z = 6$ (E) NOTA

34. Which of the following is an equation of the plane through the points (1, -4, 7), (1, -4, 2), and (-8, 3, 5)?

(A) $-2x + 3y - 8z = -70$ (B) $3x + 6y + 12z = 63$
(C) $-8x + 5y - z = -35$ (D) $7x + 9y = -29$ (E) NOTA

35. What is the distance from the point (-1, 3, 2) to the plane $2x + y - 5z = 12$?

(A) $\frac{7\sqrt{30}}{10}$ (B) $\frac{11\sqrt{10}}{15}$ (C) $\frac{5\sqrt{5}}{4}$ (D) $\frac{7\sqrt{5}}{2}$ (E) NOTA

36. Which of the following is an equation of the line which is the intersection of the plane $x - 2y - z = 4$ and the plane $-3x + y - z = 7$?

(A) $[x, y, z] = [3, -1, 5] + t[3, 4, -5]$ (B) $[x, y, z] = [0, 1, -6] + t[3, 4, -5]$
(C) $[x, y, z] = [0, 1, -6] + t[3, -4, -5]$ (D) $[x, y, z] = [2, -4, 1] + t[3, -4, -5]$ (E) NOTA

37. Which of these lines is parallel to the plane $6x + 2y - 3z = 4$?

I. $[x, y, z] = [1, 3, 4] + t[3, 1, 7]$
II. $[x, y, z] = [1, -3, 5] + t[1, -3, 0]$
III. $[x, y, z] = [2, 5, -7] + t[2, 3, 6]$

(A) I only (B) III only (C) I & II only (D) II & III only (E) NOTA

38. Given the points A (3, 4, 1) and B (2, -3, -5), what is the equation of the locus of points (x, y, z) that are equidistant from A and B?

(A) $3xy + 2z = 12$ (B) $x + 7y + 6z = -6$
(C) $x - 7y - 2z = 20$ (D) $xy - 7yz - 2xz = -22$ (E) NOTA

39. What is the area of the triangle formed by the points (-2, 7, 1), (4, 1, -3), and (5, -3, 0)?

(A) $\frac{45}{2}$ (B) $\frac{89}{4}$ (C) $\sqrt{491}$ (D) $\frac{269}{12}$ (E) NOTA

40. A particle travels at a speed of 2 units/second along the line $[x, y, z] = [4, 1, 7] + t[2, -2, -1]$, starting at the point (4, 1, 7), in a direction such that its x-coordinate is increasing. What is its position after 6 seconds?

(A) (12, -7, 3) (B) (4, -3, 11) (C) $\left(7, -2, \frac{11}{2}\right)$ (D) (16, -11, 1) (E) NOTA