Alpha Probability Solutions

1. $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16} \rightarrow C$

2. $1 - (\text{probability of neither}) = 1 - (\frac{1}{4} \times \frac{3}{4}) = 1 - \frac{3}{16} = \frac{13}{16} \rightarrow A$

3. 1 and 4 → 2 ways, 2 and 3 → 2 ways; so 4 total ways. $5 \times 5 = 25$ possible outcomes.
$\frac{4}{25} \rightarrow D$

4. Sixth game must be Mariners' win (W), other 5 in any order, so $\binom{5}{2} = 10$ ways.
$10 \times (\frac{2}{3})^4 (\frac{1}{3})^2 = \frac{160}{729} \rightarrow D$

5. $1 - (\text{probability none defective}) = 1 - (\frac{66}{72} \times \frac{65}{71} \times \frac{64}{70}) = \frac{347}{1491} \rightarrow B$

6. This is self-explanatory; after first card drawn, 39 of 51 remaining cards are different suits. $\frac{39}{51} \rightarrow \frac{13}{17} \rightarrow B$

7. Only identical outfit possible is red shirt and shorts.
$P(\text{Jim}) \times P(\text{Fred}) = (\frac{4}{9} \times \frac{1}{6}) \times (\frac{7}{17} \times \frac{3}{5}) = \frac{14}{765} \rightarrow D$

8. Easy way to think of this is first seat Mary randomly, then Jane.
$P(\text{Mary on either end}) \times P(\text{Jane adjacent}) + P(\text{Mary not on end}) \times P(\text{Jane adjacent}) = ?$
$(\frac{1}{3}) \times (\frac{1}{5}) + (\frac{2}{3} \times \frac{2}{5}) \rightarrow \frac{1}{3} \rightarrow D$

9. By either rotating or inverting the ring, all arrangements are identical → 1 → C

10. $(\frac{2}{3})^{11} \rightarrow \frac{2048}{177147} \rightarrow C$

11. $\frac{(\#\text{ of ways to choose two numbers} < 5) \times (\#\text{ of ways to choose a number} > 5)}{\text{total \# of possible choices}} = \frac{\binom{4}{2} \times \binom{4}{1}}{\binom{9}{4}} = \frac{4}{21}$
↓
A

Alpha Probability Solutions

12. $\frac{1}{2} \times \frac{1}{11} + \frac{1}{2} \times \frac{8}{9} = \frac{97}{198} \rightarrow C$

13. This interval is 11 periods of the cosine function; since cosine is positive for half its period, answer is $\frac{1}{2} \rightarrow A$

14. # of ways to get at least 1 four = 6^3 − (# ways to get no fours) = ?

$216 - 5^3 = 91$

# of ways to sum to 12 with at least one four:

2 and 4 and 6 → 3! → 6 ways ; 3 and 4 and 5 → 6 ways ; three 4s → one way

$\frac{13}{91} \rightarrow \frac{1}{7} \rightarrow E$

15. $\frac{14}{15} \times \frac{20}{21} = \frac{8}{9} \rightarrow C$

16. (# of ways to arrange 4 heads, 6 tails) $\times \left(\frac{1}{2}\right)^{10} = \binom{10}{4} / 2^{10} = \frac{105}{512} \rightarrow A$

17. Can flip either heads or tails first, then only one way to do all the rest. $2 \times \left(\frac{1}{2}\right)^7 = \frac{1}{64} \rightarrow B$

18. $\frac{\text{area of inner circle}}{\text{area of board}} = \frac{2^2}{8^2} = \frac{1}{16} \rightarrow C$

19. $\frac{7}{9} \times \frac{6}{8} = \frac{7}{12} \rightarrow C$

20. $1 - \left(\begin{matrix}\text{probability at least} \\ \text{2 empty adjacent}\end{matrix}\right) = 1 - \left[\begin{matrix}(\text{prob. only 2 adj.}) + \\ (\text{prob. 3 adj.})\end{matrix}\right] = 1 - \left[\frac{\begin{matrix}(\#\text{ ways 2 adj. on end}) + (\#\text{ ways 2 adj. not} \\ \text{on end}) + (\#\text{ ways 3 adj.})\end{matrix}}{\binom{11}{3}}\right]$

$1 - \left[\frac{(2 \times 8) + (8 \times 7) + (9)}{165}\right] = \frac{28}{55} \rightarrow C$

Alpha Probability Solutions

21. Each of the 24 chips has a $^7/_8$ probability of <u>not</u> being placed in the chosen cookie. $(^7/_8)^{24} = 0.0406 \Rightarrow C$

22. Search area is 4π mi^2. In 36 hours 3 square miles can be searched. $3/4\pi \rightarrow D$

23. $\dfrac{\sum_{n=10}^{20}\binom{20}{n}}{\sum_{n=0}^{20}\binom{20}{n}} = 0.5881 \rightarrow C$

24. blue from A: $\frac{6}{10} \times \left[\frac{\binom{4}{2}}{\binom{11}{2}} + \frac{\binom{7}{2}}{\binom{11}{2}}\right] = \frac{81}{275}$

red from A: $\frac{4}{10} \times \left[\frac{\binom{3}{2}}{\binom{11}{2}} + \frac{\binom{8}{2}}{\binom{11}{2}}\right] = \frac{62}{275}$

$\frac{81}{275} + \frac{62}{275} = \frac{13}{25} \rightarrow D$

25. period of $\frac{4\sqrt{3}}{3}\sin(3x+7)$ is $\frac{2\pi}{3}$; given interval is exactly 16 periods, so we need only consider one period.

$|f(x_0)| > \frac{2\sqrt{3}}{3} \rightarrow \left|\frac{4\sqrt{3}}{3}\sin(3x+7)\right| > \frac{2\sqrt{3}}{3} \rightarrow |\sin(3x+7)| > \frac{1}{2} \rightarrow$ (considering period $0 < 3x+7 \le 2\pi$)

$\frac{\pi}{6} < 3x+7 < \frac{5\pi}{6}$ or $\frac{7\pi}{6} < 3x+7 < \frac{11\pi}{6} \rightarrow \frac{\frac{8\pi}{6}}{2\pi} \rightarrow \frac{2}{3} \rightarrow D$

26. $1 - (\text{probability he doesn't realize}) = 1 - (^1/_3)^5 = \frac{242}{243} \rightarrow A$

27. $f(x) < 2 \rightarrow 2x^2 < 2 \rightarrow x^2 < 1 \rightarrow x < 1 \rightarrow {}^1/_3 \rightarrow D$

Alpha Probability Solutions

28. # of ways with:

no 2s: 12×44 ← choose a non-2 rank, choose one other card

one 2: $\binom{4}{3} \times 12 \times \binom{4}{1} \times 44$ ← choose 3 suits within non-2 rank, choose non-2 rank, choose one 2, choose one other card

two 2s: $\binom{4}{2} \times 12 \times \binom{4}{2} \times 44$ ← choose 2 suits within non-2 rank, choose non-2 rank, choose two 2s, choose one other card

three 2s: $\left(\frac{48 \times 44}{2}\right) \times \binom{4}{3}$ ← choose 2 different non-2 cards, choose 3 twos

total of above is 32208

$$\frac{32208}{\binom{52}{5}} = .01239 \rightarrow E$$

---

29. There are $\binom{6}{4} = 15$ sets of 4 distinct numbers; each set has only one way to be in descending order.

$$\frac{15}{6^4} = \frac{5}{432} \rightarrow C$$

---

30. P(Bob arrives between 1:20 and 2:50) × P(Jane's time overlaps) +
P(Bob between 1:00 and 1:20) × P(overlap) + P(Bob 2:50 to 3:00) × P(overlap) =

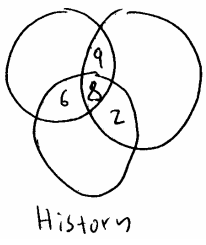
$$\left(\frac{3}{4}\right) \times \left(\frac{1}{4}\right) + \left(\frac{1}{6}\right) \times \left(\frac{1}{6}\right) + \left(\frac{1}{12}\right) \times \left(\frac{5}{24}\right) = \frac{67}{288} \rightarrow C$$

---

31. Must ignore the extraneous info! Each person equally likely to get "X": $\frac{1}{5} \rightarrow C$

---

32. English, Science, History



$17 + 14 + 10 - 2x = 199 - 174 \rightarrow x = 8$

$$\frac{8}{199} \rightarrow A$$

Alpha Probability Solutions

33. (Arrange brands in circle) × (order within brands) = 2

$(3!/3) \times (4! \times 5! \times 3!) = 34560 \rightarrow C$

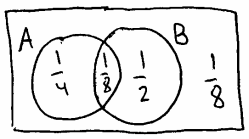
34. $1 - P(6) = 1 - \frac{6^2}{6^2+1} = \frac{1}{37} \rightarrow A$

35. Numbers which qualify are squares of prime numbers:

$2^2 = 4$, $3^2 = 9$, $5^2 = 25$, $7^2 = 49$, ~~$11^2 = 121$~~ ← too big!

So $\frac{4}{50} \rightarrow \frac{2}{25} \rightarrow B$

36.



$\frac{1}{8} \rightarrow B$

37. $\frac{1}{4} + \frac{1}{8} = \frac{3}{8} \rightarrow A$

38. $1 - P(\text{35¢ or more}) = 1 - P(Q \text{ and } D) - P(Q \text{ and } Q) = 1 - \frac{(\binom{8}{1} \times \binom{5}{1})}{\binom{16}{2}} - \frac{\binom{8}{2}}{\binom{16}{2}} = \frac{13}{30} \rightarrow A$

39. At least 3 correct → all correct → $(\frac{1}{4}) \times (\frac{1}{3}) \times (\frac{1}{2}) \times (1) = \frac{1}{24} \rightarrow B$

40. $\frac{\binom{6}{1} \times \binom{10}{3}}{\binom{16}{8}} = \frac{8}{143} \rightarrow B$