# Mu Alpha Theta National Convention: Denver, 2001
## Trigonometry Topic Test – Alpha Division

1. Which of the following always equals 1?

(A) $\sin^2 M + \cos^2 M$ (B) $2\cos^2 A - 1$

(C) $\dfrac{2\tan\Theta}{1-\tan^2\Theta}$ (D) $\sec^2 E - \cot^2 E$ (E) NOTA

2. Convert 78° into radians.

(A) $\dfrac{14040}{\pi}$ (B) $\dfrac{39}{200}$ (C) $\dfrac{13}{60}$ (D) $\dfrac{13\pi}{30}$ (E) NOTA

3. Evaluate: $\sin 0 - \cos 30° + \tan\dfrac{\pi}{4} - \sec 60° + \csc\dfrac{\pi}{2} - \cot 120°$

(A) $-\dfrac{1}{2}+\dfrac{\sqrt{3}}{3}$ (B) $\dfrac{9-\sqrt{3}}{6}$ (C) $2-\dfrac{3\sqrt{3}}{2}$ (D) $-\dfrac{\sqrt{3}}{6}$ (E) NOTA

4. If $\theta$ is an acute angle and $1+\cot^2 42° = \sec^2\theta$, then what is $\theta$?

(A) 42° (B) 69° (C) 48° (D) 66° (E) NOTA

5. When $\cot\kappa > 0$ and $\sec\kappa > 0$, the terminal side of angle $\kappa$ is in which quadrant?

(A) I (B) II (C) III (D) IV (E) NOTA

6. Change $\dfrac{57\pi}{36}$ radians to degrees.

(A) $\left(\dfrac{190}{3}\right)^{\circ}$ (B) 285° (C) $\left(\dfrac{57\pi^2}{6480}\right)^{\circ}$ (D) 570° (E) NOTA

7. Evaluate $\arctan\left(\sin\left(\sqrt{\dfrac{\pi^2}{4}}\right)\right)$, where $-\dfrac{\pi}{2} < \arctan x < \dfrac{\pi}{2}$ for all $x$.

(A) $\dfrac{\pi}{4}$ (B) 0 (C) $\dfrac{\pi}{2}$ (D) 1 (E) NOTA

8. Which of the following angles is complementary to 25°43'37"?

(A) 32°08'13.5" (B) 64°16'23" (C) 154°16'23" (D) 56°12'34" (E) NOTA

9. *ABC* is a triangle where *AC* = 7, *CB* = 24, and *AB* = 25. Let *D* be the foot of the altitude to *AB*. What is the value of $\sin\angle BCD + \csc\angle ACD$?

(A) $\frac{4}{3}$ (B) $\frac{793}{175}$ (C) $\frac{649}{168}$ (D) $\frac{775}{168}$ (E) NOTA

10. A pendulum in Richard's grandfather clock is 4 meters long in radius and swings back and forth along a half-meter arc. Find the measure of the angle, in radians, through which the pendulum passes during one swing.

(A) 2 (B) 8 (C) $\frac{1}{8}$ (D) $\frac{1}{2}$ (E) NOTA

11. Solve for *x*: $2x\cos^2\theta + 2x\sin^2\theta + 3x - 6 = 9\sec^2\theta - 9\tan^2\theta$

(A) 7 (B) $-\frac{3}{5}$ (C) 3 (D) $\frac{14}{3}$ (E) NOTA

12. In $\Delta ABC$, $AB = 1$, $\angle ACB = 25°$, and $\angle CAB = 68°$. Which of the following is equal to the length of *BC*?

(A) $\frac{\sin 112°}{\cos 25°}$ (B) $\frac{\cos 112°}{\sin 25°}$ (C) $\frac{\sin 68°}{\sin 25°}$ (D) $\frac{\sin 34°}{\cos 115°}$ (E) NOTA

13. What is the period of the graph of $y = \cos\frac{x}{4} + \sin\frac{x}{2}$?

(A) $16\pi$ (B) $8\pi$ (C) $4\pi$ (D) $2\pi$ (E) NOTA

14. Simplify: $\sin\left(\theta - \frac{3\pi}{2}\right) + \cos(\pi + \theta)$

(A) 0 (B) $2\cos\theta$ (C) $\sin\theta - \cos\theta$ (D) $-2\sin\theta$ (E) NOTA

15. Which of the following is equivalent to $\frac{\tan x \csc x - \cos x}{(1-\cos x)(1+\cos x)}$?

(A) $\cos x$ (B) $\sin x$ (C) $\sec x$ (D) $\csc x$ (E) NOTA

16. If $z = 2\cos\frac{\pi}{7} + 2i\sin\frac{\pi}{7}$, what is $z^{14}$?

(A) $-2^7$ (B) $2^7$ (C) $-2^{14}$ (D) $2^{14}$ (E) NOTA

17. Simplify: $\ln\left|\sec u - \tan u\right| + \ln\left|\sec u + \tan u\right| + e^{\ln\left(\sin^2 u + \cos^2 u\right)}$

(A) $e$ (B) 2 (C) 1 (D) 0 (E) NOTA

18. Whenever Brett sees girls, his breathing rate speeds up and the amount of air in his lungs $t$ seconds after seeing them is given by $A(t) = 100 - 100\cos\frac{\pi t}{2}$. Suppose Brett is locked up in a room with several girls for an hour. How many full breaths will he have taken in that span of time?

(A) 7200 (B) 14400 (C) 9000 (D) 4500 (E) NOTA

19. Given that $\cos u = \frac{1}{\sqrt{3}}$ and $\sin v = \frac{2}{3}$, what is $\cos(-u) + \sin(-v)\cos(-u) + \sin^2(-u)$?

(A) $\frac{6-\sqrt{3}}{9}$ (B) $\frac{6-5\sqrt{3}}{9}$ (C) $\frac{6+5\sqrt{3}}{9}$ (D) $\frac{6+\sqrt{3}}{9}$ (E) NOTA

20. Evaluate: $\cos 2925° - \tan 2400° + \csc 2970°$

(A) $\frac{2+\sqrt{2}-2\sqrt{3}}{2}$ (B) $\frac{\sqrt{2}-2\sqrt{3}+1}{2}$

(C) $\frac{3\sqrt{2}-2\sqrt{3}}{6}$ (D) $\frac{3\sqrt{2}-\sqrt{3}}{6}$ (E) NOTA

21. Describe the behavior of the graph of $y = \sin x$ as x increases on the interval $\frac{43\pi}{3} < x < \frac{31\pi}{2}$.

(A) decreases, then increases (B) strictly increasing

(C) strictly decreasing (D) increases, then decreases (E) NOTA

22. Which of the following sinusoids of the form $A\cos(\omega t - \phi)$ is equivalent to $4\cos 4t - 4\sqrt{3}\sin 4t$ ?

(A) $8\cos\left(4t + \frac{7\pi}{6}\right)$ (B) $8\cos\left(4t + \frac{2\pi}{3}\right)$

(C) $8\cos\left(4t - \frac{5\pi}{3}\right)$ (D) $8\cos\left(4t - \frac{11\pi}{6}\right)$ (E) NOTA

23. Compute the value of $\frac{(9\,\text{cis}\,41°)(3\,\text{cis}\,64°)}{(27\,\text{cis}\,27°)(18\,\text{cis}\,18°)}$, where $\text{cis}\,x = \cos x + i\sin x$ and $i = \sqrt{-1}$.

(A) $\frac{(1+i)\sqrt{2}}{36}$ (B) $\frac{1+i\sqrt{3}}{36}$ (C) $\frac{(3+i)\sqrt{3}}{54}$ (D) $\frac{\sqrt{3}+i}{36}$ (E) NOTA

24. The expression $\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x}$, when defined, is equivalent to which of the following?

(A) 2 csc $x$ (B) 2 tan $x$ (C) 2 cot $x$ (D) 2 sin $x$ (E) NOTA

25. Which of the following expressions are never negative for real values of $x$?

I. $a(x) = \cos(2x) + \sin^2 x$
II. $b(x) = \sec^2 x + 2\tan x$
III. $c(x) = \tan^3 x + 5$
IV. $d(x) = 4\cos^2 x - \sin^2(2x)$

(A) I, III, IV (B) II, III, IV (C) I, II, IV (D) I, II, III (E) NOTA

26. In calculus, a technique for computing the **indefinite integral** of the function $f(x) = \cos^3 x \sin^4 x$ with respect to $x$ is to manipulate $f$ into the form $P(u)d'(x)$, where $u = \sin x$ and $d'(x) = \cos x$. What is $P(u)$?

(A) $-u^8 + 2u^6 - u^4$ (B) $u^6 - u^4$

(C) $u^4 - u^6$ (D) $-u^6 + 2u^5 - u^4$ (E) NOTA

27. A function is defined as $f(n) = \sin^2 n + \cos(2n) + \tan(3n)$. Find $f\left(\frac{\pi}{12}\right)$.

(A) $\frac{7+3\sqrt{3}}{4}$ (B) $\frac{6+3\sqrt{3}}{4}$ (C) $\frac{5+2\sqrt{3}}{4}$ (D) $\frac{6+\sqrt{3}}{4}$ (E) NOTA

28. If $x$ and $y$ acute angles such that $\tan x + \tan y = \sqrt{3}(1 - \tan x \tan y)$ and $\sqrt{2}\sin y = 1$, what is the measure of $x$, in degrees?

(A) 75° (B) 15° (C) 45° (D) 30° (E) NOTA

29. Find the sum of all $\varphi$ such that $\sin^2(5\varphi) - 1 = 0$ on the interval $\frac{3\pi}{5} < \varphi < \frac{9\pi}{5}$.

(A) $\frac{48\pi}{5}$ (B) $\frac{36\pi}{5}$ (C) $\frac{12\pi}{5}$ (D) $10\pi$ (E) NOTA

30. A triangle is inscribed in a circle of radius 2. If the measures of two of its angles are 45° and 30°, what is the perimeter of the triangle?

(A) $2 + \sqrt{2} + \sqrt{6}$ (B) $2 + 3\sqrt{2} + \sqrt{6}$

(C) $2\sqrt{3} + 3\sqrt{2} + 2\sqrt{6}$ (D) $2\sqrt{3} + \sqrt{2} + \sqrt{6}$ (E) NOTA

31. How many values of $\theta$ on the interval $\left[\frac{\pi}{12}, \frac{25\pi}{12}\right)$ are solutions to the equation

$$16\sin^2\theta\cos^2\theta - 12\sin\theta\cos\theta + 11 = 7 + 5\sin(2\theta) - 2\sin^2(2\theta)$$

(A) 6 (B) 3 (C) 5 (D) 4 (E) NOTA

32. The course of a skateboard race consists of a 300-meter downhill run and a 200-meter level portion. When the starting point of the race is spotted from the finishing line, the angle of elevation is 36°. To the nearest degree, what angle does the hill make with the horizontal?

(A) 23° (B) 98° (C) 59° (D) 62° (E) NOTA

33. How many times does the graph of $y = \cos x - \sin(2x) - \cos(3x)$ intersect the *x*-axis on the interval $-2\pi \le x \le 2\pi$?

(A) 14 (B) 13 (C) 12 (D) 11 (E) NOTA

34. A triangle has a side of length 8 opposite an angle with a measure of $\frac{\pi}{6}$. What is the area of the triangle's circumscribed circle?

(A) $\frac{255\pi}{4}$ (B) $63\pi$ (C) $\frac{127\pi}{2}$ (D) $64\pi$ (E) NOTA

35. If $\sin A = -\frac{28}{53}$, where $\pi < A \le \frac{3\pi}{2}$ and *B* is a fourth-quadrant angle such that $\cos B = \frac{21}{29}$, what is $\tan(A+B)$?

(A) $\frac{1488}{385}$ (B) $-\frac{312}{1505}$ (C) $\frac{312}{1505}$ (D) $-\frac{1488}{385}$ (E) NOTA

36. Each of the basic six trigonometric functions (sin *x*, cos *x*, tan *x*, sec *x*, csc *x*, and cot *x*) is written on its own slip of paper. These six slips of paper are placed in a hat, from which a mathematician randomly draws two slips. What's the probability that when the functions written on the chosen slips are squared and added together, the result is equal to 1 for all *x*?

(A) $\frac{1}{15}$ (B) $\frac{2}{5}$ (C) $\frac{1}{18}$ (D) $\frac{1}{30}$ (E) NOTA

37. Let $P = (r, \theta)$ be a point in polar coordinates. Which of the following transformations does not change the position of *P*?

(A) $(-r, \theta + 5\pi)$ (B) $\left(2r, \frac{\theta}{2}\right)$ (C) $\left(-r, \theta + \frac{\pi}{2}\right)$ (D) $(-2r, \theta - 2\pi)$ (E) NOTA

38. Two sides of a triangle are 3 and 8 while the sine of the acute angle between them is $\frac{48}{73}$. If this angle is doubled, what is the ratio of the area of the new triangle to the old?

(A) 2:1 (B) 73:55 (C) 110:73 (D) 73:48 (E) NOTA

39. What is the period of $y = \sin(3x)$?

(A) $2\pi$ (B) $6\pi$ (C) 6 (D) $\frac{2\pi}{3}$ (E) NOTA

40. What is the maximum possible area of a triangle that has a side of length $\sqrt{6}$ and another side of length $2\sqrt{3}$?

(A) $6\sqrt{2}$ (B) $3\sqrt{2}$ (C) $\frac{3\sqrt{2}}{2}$ (D) $2\sqrt{6}$ (E) NOTA