Mu Alpha Theta National Convention: Denver, 2001

Complex Numbers Topic Test Solutions – Mu Division

1.
$$\begin{array}{r} 5+2i\sqrt{3} \\ -4-3i \\ +6-2i\sqrt{3} \\ +3-i\sqrt{3} \\ \hline 10-(3+\sqrt{3})i \end{array}$$
D

2. $1-\frac{z}{x} = 1-\frac{7+4i}{4+i}\cdot\frac{4-i}{4-i}$

$= 1-\frac{32+9i}{17} = \frac{-15-9i}{17}$ A

3.
$$\begin{array}{r} +i \\ \hline -2-3i \\ +4+5i \\ \hline \downarrow \\ \hline +28+29i \\ \hline \end{array}$$
$= i + 7(2+2i)$

$= 14+15i$ A

4. $① - 2② \Rightarrow 7B = 21-7i$

$B = 3-i$

$A = 8$

$AB = 24-8i$ A

5. $-4i(2+i) - 3\cdot(1-2i)$

$-8i+4-3+6i$

$1-2i$ E

6. $(2+2i)^6 = (2\sqrt{2}\,e^{i\pi/4})^6 = 512\,e^{3i\pi/2}$

$= -512i$ B

7. $(\sqrt{2}\,e^{-i\pi/4})^4 (2e^{-i\pi/3})^5$

$-4(32e^{\pi/3})$

$-64-64i\sqrt{3}$ C

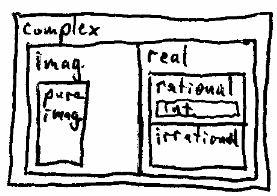
8. $4=4$

$\frac{-5\pi}{3} = 60^\circ$ coterminal B

9. $\frac{3\cdot 8}{6} = 4$ $\quad \frac{2\pi}{3}+\pi-\frac{\pi}{3} = \frac{4\pi}{3}$ A

10. $|15+20i| = \sqrt{15^2+20^2} = 25$ B

11. $\frac{2-i}{1-\frac{i}{2}} = \frac{2-i}{\frac{2-i}{2}} = 2$ A

12. 

II & IV C

13. $(x+3)(3x^2+2x+1)=0$

$x=-3 \quad x = \frac{-2\pm\sqrt{4-12}}{6}$

$= \frac{-1\pm i\sqrt{2}}{3}$ B

14. $x^2-8x+32$ has roots $4\pm4i$
so does any multiple $\Rightarrow$ I & III D

15. a cubic equation always has 3
(remember that reals are a subset
of the complex numbers) C

16. sum of roots $= -\frac{-7}{3} = \frac{7}{3}$ B

17. product $= \frac{(-1)^n c}{a} = -\frac{-9}{2} = \frac{9}{2}$ A

18. $(r^2+s^2) = (r+s)^2 - 2rs = (-8)^2 - 2\cdot 24$
$= 64 - 48 = 16$ B

19. $\frac{1}{r}+\frac{1}{s}+\frac{1}{t} = \frac{st+rt+rs}{rst} = \frac{\frac{-6}{2}}{\frac{-7}{2}} = \frac{6}{7}$ A

20. $z^6 = z^7 + (17-3i)$
$0 = z^7 - z^6 + (17-3i) \Rightarrow A = 7$
$B = 1$
$C = -(17-3i)$
$-9+3i$ D

21. $3+i \Rightarrow 3-i$
product $= 5 \Rightarrow \frac{1}{2}$ II A

22. It is not stated that
k & y are real, so we cannot
infer any of the other roots
(though we still know they sum
to zero) D

23. $2-i \Rightarrow 2+i \Rightarrow x^2-4x+5$
$(x-5)(x^2-4x+5) = x^3-9x^2+25x-25$
Ooops... we're not asked for
real coefficients...
$(x-5)(x-(2-i)) = x^2-(7-i)x+(10-5i) = C$
C

24. 2 is a 6^{th} root of 64.
others are symmetric about
origin in complex plane. III C

25. $x^2 = -i$ sum $= 0$
$x^3 = i$ sum $= 0$ $\Rightarrow 0$ B

26. $x = \frac{bi \pm \sqrt{-b^2+8}}{2} \Rightarrow$ D

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27. I. $\frac{e^1+e^{-1}}{2} + \frac{e^1-e^{-1}}{2} = \frac{2e}{2} = e$ ✓

II. $-1+1=0$ ✗

III. e^i ✗

IV. $4\cdot\frac{\pi}{4} = \pi$ ✗ $\Rightarrow B$

28. $\sinh(i) = \frac{e^i - e^{-i}}{2}$

$= \frac{(\cos 1 + i\sin 1) - (\cos 1 - i \sin 1)}{2} = i \sin 1$

B

29. $(x-3)(x^2+2x+4)=0$

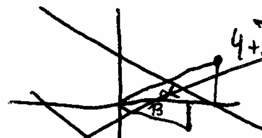
$x=3 \quad x = \frac{-2\pm\sqrt{4-16}}{2}$

$= -1 \pm i\sqrt{3} \Rightarrow A$

30. $\vec{A}\cdot\vec{B} = 12-2 = 2\sqrt{5}\cdot\sqrt{10}\cos\theta$

$\frac{10}{10\sqrt{2}} = \frac{\sqrt{2}}{2} = \cos\theta$

$\Rightarrow \theta = \frac{\pi}{4}$ B



~~$\alpha = \text{Arctan}\,\frac{1}{2}$~~

~~$= \frac{\pi}{2} - \text{Arctan}\,2$~~

~~$\beta = \text{Arctan}\,\frac{1}{3}$~~

~~$= \frac{\pi}{2} - \text{Arctan}\,3$~~

31. $4 \pm i\sqrt{3} \Rightarrow b = -8 \qquad x^2 - 8x + 4$

$\pm 2i \Rightarrow c = 4 \qquad x = \frac{8\pm\sqrt{64-16}}{2}$

$= 4 \pm 2\sqrt{3}$ B

32. $r^4 = 16$

$\Rightarrow r = 2, -2, 2i$, or $-2i$

4th term $= 8, -8, -8i, 8i$ D

33. $(5-i)\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$

$\frac{5\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i - \frac{5\sqrt{2}}{2}i - \frac{\sqrt{2}}{2}$

$\frac{4\sqrt{2}}{2} - \frac{6\sqrt{2}}{2}i = 2\sqrt{2} - 3\sqrt{2}i$ A

34. $z^2 - z\sqrt{3} - i = 0$

$z = \frac{\sqrt{3} \pm \sqrt{3+4i}}{2}$

$3+4i = (a+bi)^2 = (a^2-b^2) + (2ab)i$

$a=2,\ b=1$

$\frac{\sqrt{3} \pm (2+i)}{2}$ D

35. $3i(z^3-z) + (z^4 - (4+i)z^2 + (3+i)) = 0$

$3iz(z^2-1) + (z^2-1)(z^2-(3+i)) = 0$

$(z^2-1)(z^2+3iz-(3+i)) = 0$

$(z+1)(z-1)(\qquad) = 0$

$z = \pm 1 \qquad z = \frac{-3i \pm \sqrt{-9+12+4i}}{2}$

$= \frac{-3i \pm (2+i)}{2}$ C

36. $r_1 = 1+i$

$r_2 = c+di$

$r_1 + r_2 = (c+1)+(d+1)i \Rightarrow d=-3$

$r_1 r_2 = (c-d)+(c+d)i \Rightarrow c+3=5$

$c=2$

$a = -(c+1) = -3$

$b = c+d = -1$

sum $=-4$ A

37. $z^i = -1 = e^{i(2n+1)\pi}$

$z = e^{(2n+1)\pi} \Rightarrow$ II B

38. ~~imag~~ imaginary means it includes an "i" component, i.e. non-real.

(pure imaginary means no real component)

III. $\sqrt{-e} = i\sqrt{e} \Rightarrow$ I, II, III E

39. $z = \ln(-x) = \ln(-1 \cdot x)$

$\cancel{e^z = -x}$ $= \ln(-1) + \ln x$

$= y + \ln x$ where $y = \ln(-1)$

$e^y = -1 = e^{i(2n+1)\pi} \Rightarrow y = i(2n+1)\pi$

$z = \ln x + i(2n+1)\pi$ A

40. $y = e^{rt}$

$y' = re^{rt}$

$y'' = r^2 e^{rt}$

$r^2 + 5r + 7 = 0$

$r = \frac{-5 \pm \sqrt{25-28}}{2} = \frac{-5 \pm i\sqrt{3}}{2}$

$y = Ae^{\left(\frac{-5+i\sqrt{3}}{2}\right)t} + Be^{\left(\frac{-5-i\sqrt{3}}{2}\right)t}$

$\cancel{+(A+B)e^{\frac{5}{2}}}$

$= Ae^{-5/2\,t}\left(\cos\frac{\sqrt{3}}{2}t + i\sin\frac{\sqrt{3}}{2}t\right)$

$+ Be^{-5/2\,t}\left(\cos\frac{\sqrt{3}}{2}t - i\sin\frac{\sqrt{3}}{2}t\right)$

$= e^{-\frac{5}{2}t}\left((A+B)\cos\left(\frac{\sqrt{3}}{2}t\right) + (A-B)\sin\frac{\sqrt{3}}{2}t\right)$

B