Mu Alpha Theta National Convention: Denver, 2001

Integration Topic Test – Mu Division

1. Evaluate: $\int_0^4 \left(x^3 + x + 1\right)dx$

   (A) 48 (B) 24 (C) 76 (D) 69 (E) NOTA

2. The acceleration of a particle moving along the *x*-axis after *t* seconds is given by $a(t) = 3t^2 - 6t + 2$, where $t \geq 0$. Given the particle was at rest initially, determine its displacement from its initial position after four seconds.

   (A) 16 (B) $\frac{33}{2}$ (C) 24 (D) $\frac{128}{15}$ (E) NOTA

3. Evaluate: $\int (2a+1)^8 \, da$

   (A) $\frac{1}{9}(2a+1)^9 + C$ (B) $\frac{1}{18}(2a+1)^9 + C$

   (C) $\frac{2}{9}(2a+1)^9 + C$ (D) $\frac{1}{16}(2a+1)^9 + C$ (E) NOTA

4. When the substitution $\tan u = x$ is made, $\int x^2(x^2+1)^2 \, dx$ becomes

   (A) $\int \tan^2 u \sec^4 u \, du$ (B) $\int \left(\tan^5 u + \tan^2 u\right)du$

   (C) $\int \frac{\tan^2 u}{\sin^4 u} \, du$ (D) $\int \left(\tan^2 u \sec^6 u\right)du$ (E) NOTA

5. Find the area of the closed region bounded by the graph of $y - 25 + x^2 = 0$ and the *x*-axis.

   (A) $\frac{100}{3}$ (B) $\frac{500}{3}$ (C) 250 (D) $\frac{250}{3}$ (E) NOTA

6. Using three rectangles, find the upper-sum approximation for $\int_0^7 x^2 \, dx$ where the width of the *n*th partition is given by 2^{n-1}.

   (A) 351 (B) 147 (C) 38 (D) 215 (E) NOTA

For questions 7 through 9, use the following table of values for differentiable functions *a* and *b*.

t	0	1	2	3
$a(t)$	4	5	6	7
$b(t)$	1	3	7	15

7. Evaluate: $\int_1^3 \left(a'(t)b(t)+b'(t)a(t)\right)dt$

(A) 90 (B) 42 (C) 105 (D) 19 (E) NOTA

8. Evaluate: $\int_0^1 a'(b(t))b'(t)\,dt$.

(A) 2 (B) 3 (C) 1 (D) 7 (E) NOTA

9. Evaluate: $\int_0^3 \frac{a'\left(\frac{t}{3}\right)b\left(\frac{t}{3}\right)-b'\left(\frac{t}{3}\right)a\left(\frac{t}{3}\right)}{\left[b\left(\frac{t}{3}\right)\right]^2}\,dt$.

(A) $-\frac{7}{3}$ (B) $-\frac{53}{5}$ (C) -7 (D) $-\frac{53}{15}$ (E) NOTA

10. Upon hearing news of Tom's eligibility, girls begin flocking to his home at a rate of $20+4\sqrt{t}$ girls per minute, where *t* is in minutes. How many girls are in Tom's house after 81 minutes?

(A) 1692 (B) 1812 (C) 2106 (D) 3564 (E) NOTA

11. Consider a continuous, odd function $f(x)$ on the interval $-5 \le x \le 5$, where $f(x)>0$ for $x>0$. Two distinct integers from this interval are randomly chosen and used as limits of integration for $\int f(x)\,dx$. What's the probability the integral yields a value of zero?

(A) $\frac{1}{11}$ (B) $\frac{6}{55}$ (C) $\frac{2}{11}$ (D) $\frac{1}{22}$ (E) NOTA

12. Evaluate: $\int_{-2}^1 |x+1|\,dx$

(A) 1 (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) 2 (E) NOTA

13. Find the average value of $E(n) = n^2$ over the interval $0 \le n \le 2$.

(A) $\frac{4}{3}$ (B) 2 (C) $\frac{8}{3}$ (D) $\frac{10}{3}$ (E) NOTA

14. Suppose $n(z)$ is a continuous, even function where $\int_{-2}^{2} n(z)\,dz = 4$ and $\int_{-1}^{1} n(z)\,dz = -6$. What is the value of $\int_{1}^{2} n(z)\,dz$?

(A) –1 (B) 10 (C) 5 (D) –2 (E) NOTA

15. What is the total area of the two regions between the graphs of $y = 3x^3 - x^2 - 10x$ and $y = 2x - x^2$?

(A) 6 (B) 12 (C) 18 (D) 24 (E) NOTA

16. Evaluate: $\int \frac{dx}{x^2 + 12x + 45}$

(A) $\ln\left|x^2 + 12x + 45\right| + C$ (B) $\arcsin\left(\frac{x+6}{3}\right) + C$

(C) $\frac{3}{x^2 + 18x + 135} + C$ (D) $\frac{1}{3}\arctan\left(\frac{x+6}{3}\right) + C$ (E) NOTA

17. The lamina in the xy-plane bounded by the graphs of $y = \sec^2 x$, $x = \pm\frac{\pi}{6}$, and $y = 0$ has a uniform density of 18 kilograms per square unit. Find the mass of the lamina.

(A) $18\sqrt{3}$ kg (B) $36\sqrt{3}$ kg (C) $6\sqrt{3}$ kg (D) $12\sqrt{3}$ kg (E) NOTA

18. Find the value that satisfies the conclusion of the Mean Value Theorem for Integrals for $I(u) = u^2 + 1$ on the interval $2 \le u \le 5$.

(A) $\sqrt{42}$ (B) $\sqrt{13}$ (C) $\sqrt{14}$ (D) $\sqrt{41}$ (E) NOTA

19. Find the ordered pair (*a*, *b*) such that $\int_{a}^{b} \left(10y - 16 - y^2\right) dy$ is a maximum.

(A) (4, 8) (B) (2, 8) (C) (0, 4) (D) (4, 10) (E) NOTA

20. Evaluate: $\int_e^{e^2}\left(\frac{1}{x}+\frac{1}{x^2}\right)dx$

(A) $\frac{e^2+1}{e^4}$ (B) $\frac{e^2+e-1}{e^2}$ (C) $\frac{e^2+e-1}{e^3}$ (D) $\frac{e^3+e^2-1}{e^4}$ (E) NOTA

21. Given that $F(t)=\int_0^{t^3}\sin(\phi^2)d\phi$, what is $F'(t)$?

(A) $3t^2\sin(t^6)$ (B) $2t^3\cos(t^6)$ (C) $\sin(t^6)$ (D) $t^3\sin(t^5)$ (E) NOTA

22. Evaluate: $\int \frac{r^2+3r^3-r^5}{r\sqrt{r}}\,dr$

(A) $\frac{5}{6}r^{\frac{1}{2}}+\frac{15}{8}r^{\frac{3}{2}}-\frac{1}{2}r^{\frac{5}{2}}+C$ (B) $-\frac{2}{9}r^{\frac{9}{2}}+\frac{6}{5}r^{\frac{5}{2}}+\frac{2}{3}r^{\frac{3}{2}}+C$

(C) $\frac{15}{2}r^{\frac{5}{2}}-\frac{9}{2}r^{\frac{9}{2}}+\frac{3}{2}r^{\frac{3}{2}}+C$ (D) $\frac{1}{2}r^{-\frac{1}{2}}+\frac{9}{2}r^{\frac{1}{2}}-\frac{7}{2}r^{\frac{5}{2}}+C$ (E) NOTA

23. The line $x=\pi$ divides the region in the first quadrant bounded by the graphs of $y=x+\sin x$, the x-axis, and $x=2\pi$ into two pieces. Find the ratio of the larger area to the smaller.

(A) $\frac{3\pi^2+4}{\pi^2-4}$ (B) $\frac{\pi+2}{\pi-2}$ (C) $\frac{\pi^2+2}{\pi^2-2}$ (D) $\frac{3\pi^2-4}{\pi^2+4}$ (E) NOTA

24. What is the volume of the solid formed by revolving the region bounded by the graph of $y=x^3-x^2+x+1$ and the x-axis on the interval $0\le x\le 1$ about the y-axis?

(A) $\frac{47\pi}{30}$ (B) $\frac{17\pi}{12}$ (C) $\frac{218\pi}{105}$ (D) $\frac{13\pi}{60}$ (E) NOTA

25. Evaluate $\int_0^{10}\lfloor x\rfloor\,dx$, where $\lfloor x\rfloor$ stands for the greatest integer less than or equal to x.

(A) 90 (B) 100 (C) 45 (D) 55 (E) NOTA

26. Evaluate: $\int (\sin x \cos x) dx$

(A) $\sin^2 x + C$ (B) $\cos^2 x + C$ (C) $\frac{\sin^2 2x}{8} + C$ (D) $-\frac{\cos 2x}{4} + C$ (E) NOTA

27. What is the coefficient of the x^5 th term of $\int (2x-1)^{12}\ dx$?

(A) –4224 (B) –440 (C) 1584 (D) 8448 (E) NOTA

28. Which of the following is an equation of a curve that passes through (–2, 1) where the slope of the *normal* line at the point (*x*, *y*) is equal to $x^2 y$?

(A) $\frac{x^3 y^2 - 4}{x^3} = \frac{3}{2}$ (B) $y = e^{x^2/2-2}$

(C) $y = e^{x^3/3-8/3}$ (D) $y^2 = \frac{2x+2}{x}$ (E) NOTA

29. Evaluate: $\int \theta^2 \cos\theta\ d\theta$

(A) $\theta^2 \sin\theta + 2\theta\cos\theta - 2\sin\theta + C$ (B) $\theta^2 \sin\theta - 2\theta\cos\theta + 2\sin\theta + C$

(C) $\theta^2 \sin\theta - 2\theta\cos\theta - 2\sin\theta + C$ (D) $-\theta^2 \sin\theta + 2\theta\cos\theta + 2\sin\theta + C$ (E) NOTA

30. A sequence of nonnegative terms is given by $a_{n+1} = \int_{\frac{2}{3}a_{n-1}}^{a_n} 3\ dx$, where $a_0 = 0$ and $a_1 = 1$. Find $a_1 + a_2 + a_3 + \ldots + a_{2000} + a_{2001}$.

(A) $2^{2001} - 1$ (B) $2^{2002} - 2003$ (C) $2^{2002} + 1999$ (D) $2^{2002} - 2$ (E) NOTA

31. The graphs of $y = \sin(x)$ and $y = \cos(x)$ intersect infinitely many times, forming regions of equal area. What is the area of one of these regions?

(A) $\sqrt{2}$ (B) $\frac{\sqrt{2}}{2} + 1$ (C) $\sqrt{2} - 1$ (D) $2\sqrt{2}$ (E) NOTA

32. Evaluate: $\int 10^x \, dx$

(A) $\frac{10^x}{\ln(10)} + C$ (B) $10^x + C$ (C) $10^x \ln(10) + C$ (D) $e^x \ln(10) + C$ (E) NOTA

33. What is the volume generated when the region satisfying $x < y < 2x$ and $4 < x < 6$ is rotated about the *x*-axis?

(A) 152π (B) 160π (C) 180π (D) 196π (E) NOTA

34. What is the area of the region bounded by the graphs of $y = \frac{x^3 + x + 1}{1 + x^2}$, $x = \frac{\sqrt{3}}{3}$, $x = \sqrt{3}$, and the *x*-axis?

(A) $\frac{\pi}{6} - \frac{1}{2}$ (B) $\frac{2\sqrt{3}}{3} + \frac{\pi}{6}$ (C) $\frac{4}{3} + \ln 3$ (D) $\frac{4}{3} + \frac{\pi}{6}$ (E) NOTA

35. Find the sum of all natural numbers $x \le 67$ such that $\int_0^x 5n^4 - 1 \, dn$ is divisible by 5.

(A) 1190 (B) 910 (C) 2278 (D) 568 (E) NOTA

36. Let *R* be the region bounded by the graph of $y = kx^2$ (where *k* is positive) and the *x*-axis on the interval $0 \le x \le 1$. If the volume generated when *R* is revolved about the line y = –1 is $\frac{56\pi}{5}$, what is *k*?

(A) $\frac{336}{5}$ (B) 6 (C) 2 (D) 9 (E) NOTA

37. Find $\int f(x) \, dx$, where $f(x)$ satisfies the identity $3f(x + 2y) + 2f(x - 3y) = 5x^2 + 30y^2$.

(A) $x^2 + C$ (B) $2x^3 + C$ (C) $\frac{x^3}{3} + C$ (D) $5x^2 + C$ (E) NOTA

38. *R* is a solid whose base is the region bounded by the graphs of the *x*-axis and $y = \sqrt{x}$ on the interval $0 \le x \le 4$. If the cross sections of *R* perpendicular to the *x*-axis are regular octagons, find the volume of *R*.

(A) $24 + 16\sqrt{2}$ (B) $\frac{32\sqrt{2} + 32}{3}$ (C) $16\sqrt{2} + 16$ (D) $\frac{512\sqrt{2} + 512}{15}$ (E) NOTA

39. Evaluate: $\int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x}\, dx$

(A) $\frac{1}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{1}{2}$ (D) $\frac{\pi}{2}$ (E) NOTA

40. What is the volume generated when the region satisfying $2x < y < 3x$ and $3 < x < 5$ is rotated about the *y*-axis?

(A) $\frac{157\pi}{5}$ (B) $\frac{169\pi}{3}$ (C) $\frac{182\pi}{5}$ (D) $\frac{196\pi}{3}$ (E) NOTA