Mu Alpha Theta National Convention: Denver, 2001

Limits & Derivatives Topic Test – Mu Division

1. Find $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ if $f(x)=x^2$.

   (A) $2x$ (B) $\frac{1}{3}x^3$ (C) 0 (D) 1 (E) NOTA

2. Which of the following is an equation of the line tangent to the graph of $y=-6x^3+x^5+3x^4+1$ at (2, 33)?

   (A) $y=104x-176$ (B) $\frac{y+175}{104}=x$

   (C) $\frac{104x}{173}-\frac{y}{173}=1$ (D) $104x-y=177$ (E) NOTA

3. A function's derivative is given by $r(z)=3z^2-30z+48$. Over what intervals is the function strictly increasing as z increases?

   (A) $2 < z < 8$ (B) $z > 5$

   (C) $z < 2$ or $z > 8$ (D) $z < 5$ (E) NOTA

4. Evaluate: $\lim_{w\to 3}(w^5-2w^2+6)$

   (A) 231 (B) 69 (C) 331 (D) 15 (E) NOTA

5. What value does the sequence $a_n=\frac{3^n+5^n}{6^n}$ converge to as n approaches positive infinity?

   (A) 0 (B) ∞ (C) 6 (D) $\frac{4}{3}$ (E) NOTA

6. Find $\frac{dr}{dt}$ for the implicit equation $2r^2+1=\frac{t^3+r^3}{t+r}$.

   (A) $\frac{t+2r}{2t-r}$ (B) $\frac{2t}{2r+1}$ (C) $\frac{2t-r}{t+2r}$ (D) $\frac{2r}{2t-1}$ (E) NOTA

For questions 7 through 9, use the following table of values for differentiable functions *f*, *g*, and *h*.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
1	0	–1	5	8	4	2
2	–9	4	3	–6	–5	4
3	11	2	–3	4	0	6
4	8	7	1	–1	12	8

7. If $H(x) = f(2x)g(x)$, determine $H'(2)$.

(A) –7 (B) –6 (C) –27 (D) 78 (E) NOTA

8. Find the derivative of $f(h(x))g(x)$ with respect to x at $x = 1$.

(A) 134 (B) 112 (C) 99 (D) 54 (E) NOTA

9. What is $\frac{d}{dx} f(g(h(x)))$ at $x = 1$?

(A) –16 (B) –4 (C) 0 (D) –24 (E) NOTA

10. Given that $\sin c = \frac{4}{5}$, where $\frac{\pi}{2} < c < \pi$, what is $\frac{d}{dp}\left[\sec p\right]_{p=c}$?

(A) $\frac{5}{4}$ (B) $\frac{20}{9}$ (C) $\frac{7}{25}$ (D) $\frac{15}{16}$ (E) NOTA

11. The legs of an isosceles right triangle are shrinking at a rate of 3 inches per minute. How fast (in square inches per second) is the area of this triangle decreasing the moment the hypotenuse has length $2\sqrt{6}$ inches?

(A) $\frac{\sqrt{6}}{2}$ (B) $\frac{\sqrt{3}}{4}$ (C) $\frac{\sqrt{3}}{10}$ (D) $3\sqrt{6}$ (E) NOTA

12. When the substitution $u = \frac{1}{x}$ is made, $\lim_{x \to 0^-} \left(x^2 + \frac{1}{x} \right)^x$ becomes

(A) $\lim_{u \to -\infty} \left(u + \frac{1}{u^2} \right)^{\frac{1}{u}}$ (B) $\lim_{u \to 0^-} \left(u^2 + \frac{1}{u} \right)^u$

(C) $\lim_{u \to 0^-} \left(\frac{1}{u^2} + u \right)^{\frac{1}{u}}$ (D) $\lim_{u \to \infty} \left(\frac{1}{u^2} + u \right)^{\frac{1}{u}}$ (E) NOTA

13. What is $\frac{d^{500} y}{dx^{500}}$ if $y = \sec^3 x \cos^3 x$?

(A) $\cos x$ (B) 0 (C) $-\sin x$ (D) 1 (E) NOTA

14. A particle moving along the *y*-axis has its position given by $y(t) = t \sin(2t)$ for $t \geq 0$. What is the acceleration of the particle at $t = \pi$?

(A) –4 (B) –2 (C) 2 (D) 4 (E) NOTA

15. What is $P'(n)$ if $P(2n+6) = 2n^2 + 18n + 400$?

(A) $n+3$ (B) $2n+9$ (C) $4n+18$ (D) $n-15$ (E) NOTA

16. Find $y''(v)$ when $y = \sin(v^2)$.

(A) $-2v \sin(v^2)$ (B) $-4v^2 \sin(v^2)$

(C) $2\cos^2 v - 2\sin^2 v$ (D) $2\cos(v^2) - 4v^2 \sin(v^2)$ (E) NOTA

17. A slice of pizza in the shape of a circular sector has a perimeter of four inches. What is the radius, in inches, of the pizza slice of maximum area?

(A) $\frac{3}{2}$ (B) 2 (C) 1 (D) $\frac{1}{2}$ (E) NOTA

18. If $\frac{d}{d\theta} \sin^2 \theta = \sin a\theta$, what is *a*?

(A) $\frac{3}{2}$ (B) 2 (C) $\frac{\pi}{2\theta} - 2$ (D) $\frac{1}{2}$ (E) NOTA

19. Suppose (*c*, *d*) is a critical as well as an inflection point of the graph of $y = x^4 - 12x^3 + 48x^2 - 64x + 1$ while (*a*, *b*) is its global minimum. What is $ac - bd$?

(A) 27 (B) –288 (C) –22 (D) 30 (E) NOTA

20. Evaluate: $\lim_{C \to A} \left(\dfrac{\cos A \cos C - \sin A \sin C}{\cos C \sin A + \sin C \cos A} \right)$

(A) $\cot(2A)$ (B) 1 (C) $\dfrac{2\tan 2A}{1 - \tan^2 2A}$ (D) $\tan(2A)$ (E) NOTA

21. The line tangent to the graph of $\sqrt{x} + \sqrt{y} = 4$ at the point (1, 9) has *x*-intercept (*a*, 0) and *y*-intercept (0, *b*). Evaluate *a* + *b*.

(A) $-\dfrac{80}{7}$ (B) 16 (C) $\dfrac{112}{3}$ (D) $\dfrac{80}{21}$ (E) NOTA

22. Determine $y'(4)$ in terms of *a* if $y(x) = \dfrac{\log_2 x}{x}$ and $a = \ln 2$.

(A) $-\dfrac{1}{16}$ (B) $\dfrac{2a+1}{16a}$ (C) $\dfrac{2a+1}{16}$ (D) $\dfrac{1-2a}{16a}$ (E) NOTA

23. Evaluate: $\lim_{x \to 3} \dfrac{x^2 - 4x + 3}{x - 3}$

(A) 2 (B) 3 (C) 4 (D) undefined (E) NOTA

24. If $x^2 y + y x^2 = 6$, then $\dfrac{dy}{dx}$ at (1, 3) is

(A) –18 (B) –6 (C) 6 (D) 18 (E) NOTA

25. Let $B(n) = \begin{cases} 2n - n^2 & n \le 1 \\ n^2 + kn + p & n > 1 \end{cases}$. For what ordered pair (*k*, *p*) will *B* be continuous and differentiable?

(A) (3, 1) (B) (0, 1) (C) (–2, 3) (D) (–2, 2) (E) NOTA

26. Evaluate: $\frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right)$

(A) $\frac{1}{2}\sec\frac{x}{2}\tan\frac{x}{2}$ (B) $\csc x\cot x-\csc^2 x$

(C) $\frac{1}{1+\cos x}$ (D) $-\cot x$ (E) NOTA

27. The tangent lines to the graphs of $y=x^2+6x+4$ at (1, 11) and $y=e^x+4$ at (0, 5) intersect at which point?

(A) $\left(\frac{2}{7},\frac{37}{7}\right)$ (B) $\left(\frac{2}{3},\frac{13}{5}\right)$ (C) $\left(-\frac{5}{3},\frac{10}{3}\right)$ (D) $\left(-\frac{5}{7},\frac{30}{7}\right)$ (E) NOTA

28. Evaluate: $\lim_{\alpha\to\frac{\pi}{2}}\left(\text{Arctan}\left(\tan\left(\alpha\right)\right)\right)$

(A) 1 (B) 0 (C) $\frac{\pi}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) NOTA

29. If $y(x)=3x^2-\frac{4}{x}$, evaluate $y'(2)$.

(A) $\frac{23}{2}$ (B) 12 (C) 13 (D) $\frac{27}{2}$ (E) NOTA

30. Given that $\lim_{t\to 0}\frac{7t^2+14\cos t-14}{t^4}=\frac{m}{n}$ where *m* and *n* are relatively prime positive integers, find m^2+n^3.

(A) 193 (B) 487 (C) 1777 (D) 14020 (E) NOTA

31. If $z(r)=\sin\left(r^2\right)$, find $z^{(3)}(0)$.

(A) $\frac{1}{24}$ (B) 0 (C) $-\frac{1}{6}$ (D) 2 (E) NOTA

32. Use first-order differentials for $f(x) = x^{1/5}$ at $x = 32$ to approximate $f(33)$.

(A) $2\frac{1}{80}$ (B) $2\frac{1}{40}$ (C) $2\frac{5}{96}$ (D) $2\frac{2}{5}$ (E) NOTA

33. One of the most important graphs in statistics is the **normal curve**, which has the equation $y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, where μ and σ are constants. For which values does this curve have points of inflection?

(A) $x = \mu \pm 1$ (B) $x = \mu \pm \sqrt{\sigma}$ (C) $x = \mu \pm \sigma$ (D) $x = \mu \pm \sigma^2$ (E) NOTA

34. Which of the following is a function whose critical values are two more than those of $f(x) = 3x^5 - 25x^3 + 60x - 2001$?

(A) $A(x) = 3x^5 - 30x^4 + 95x^3 - 90x^2 + 1$
(B) $B(x) = 3x^5 - 30x^4 + 95x^3 - 90x^2 + 45x - 2$
(C) $C(x) = 3x^5 - 30x^4 + 35x^3 + 270x^2 - 540x + 3$
(D) $D(x) = 3x^5 + 30x^4 + 95x^3 + 90x^2 - 4$
(E) NOTA

35. Evaluate: $\lim_{x\to\infty} \frac{5x^2 - 3x + 1}{2x^2 + 17}$

(A) $\frac{1}{17}$ (B) $\frac{19}{9}$ (C) 4 (D) $\frac{5}{2}$ (E) NOTA

36. The radius of a sphere is increasing at a constant rate of 2 inches per minute. How fast is the rate of change of the volume changing at the instant the radius is 6 inches?

(A) $288\pi \frac{\text{in}^3}{\text{min}^2}$ (B) $144\pi \frac{\text{in}^3}{\text{min}^2}$ (C) $192\pi \frac{\text{in}^3}{\text{min}^2}$ (D) $96\pi \frac{\text{in}^3}{\text{min}^2}$ (E) NOTA

37. A sequence is defined recursively as: $a_0 = -\frac{47}{17}$, $a_n = \frac{2}{a_{n-1} + 2}$ for natural numbers *n*. Evaluate $\lim_{n\to\infty}(a_n)$.

(A) $1 - \sqrt{3}$ (B) $\sqrt{3} - 1$ (C) $\frac{3}{4}$ (D) $\frac{\sqrt{2}}{2}$ (E) NOTA

38. Which of the following is equivalent to $\frac{dy}{dx}$ if $y = x^y$?

(A) $(\ln x)x^y$ (B) $\frac{y^2}{x - xy\ln x}$ (C) yx^{y-1} (D) $\frac{xy\ln x + y^2}{x - xy}$ (E) NOTA

39. Which of the following is an expression that is equivalent to
$\frac{(x^2+4x+3)(3x^2+6x-1)+(2x+4)(3+x-3x^2-x^3)}{x^2+4x+3}$ for all x other than –1 and –3?

(A) $2x+4$ (B) $1-6x-3x^2$
(C) $\frac{-x^2-4x-3}{(x-1)^2}$ (D) x^2+4x+3 (E) NOTA

40. For $h(x) = \arcsin x$, determine $\lim_{c\to 0}\frac{h(x+2c)-2h(x+c)+h(x)}{c^2}$.

(A) $\frac{x}{\sqrt{(1-x^2)^3}}$ (B) $\frac{2x}{\sqrt{1-4x^2}}$ (C) $\frac{1}{2\sqrt{1-x^2}}$ (D) $\frac{-1}{\sqrt{(1-x^2)^3}}$ (E) NOTA