Mu Alpha Theta National Convention: Denver, 2001
Matrices & Vectors Topic Test Solutions – Mu Division

1. $-4x-3=4 \Rightarrow x=\frac{-7}{4}$

$-4+3y=7 \Rightarrow y=\frac{11}{3}$

$\frac{-21}{12}+\frac{44}{12}=\frac{23}{12}$ C

2. $\begin{bmatrix}1+12+3 & -4-12+0\\ -3+0+1 & 12+0+0\end{bmatrix}=\begin{bmatrix}16 & -16\\ -4 & 12\end{bmatrix}$ C

3. $\begin{bmatrix}2&3&4&5\\3&4&5&6\\4&5&6&7\\5&6&7&8\end{bmatrix}+\begin{bmatrix}3&&&\\&&&\\&&&\\&&&12\end{bmatrix}=\begin{bmatrix}5&&&\\&&&\\&&&\\&&&20\end{bmatrix}$

$5+20=25$ E

4. $\begin{vmatrix}-3&2\\-3&2\end{vmatrix}-\begin{vmatrix}-1&-3\\-3&2\end{vmatrix}=0--11=11$ D

5. $|A^{-1}|=\frac{1}{|A|}A^{adj}=\frac{1}{6}\begin{bmatrix}2&-4\\1&1\end{bmatrix}$ C

6. $-3+5+2+-8=-4$ D

7. $\begin{vmatrix}-1&-1&2\\0&2&-2\\4&A&3\end{vmatrix}=0=-\begin{vmatrix}2&-2\\A&3\end{vmatrix}+4\begin{vmatrix}-1&2\\2&-2\end{vmatrix}$

$=-6-2A-8=-2A-14 \Rightarrow A=-7$ D

8. $\begin{vmatrix}3-\lambda&8\\-2&6-\lambda\end{vmatrix}=\lambda^2-9\lambda+16=0$

Sum = 9 B

9. $\begin{vmatrix}3-\lambda&1\\6&-2-\lambda\end{vmatrix}=\lambda^2-\lambda-12=0$

$\lambda=4,-3$

$\begin{bmatrix}-6&1\\6&-6\end{bmatrix}\Rightarrow\begin{bmatrix}1\\1\end{bmatrix}$ I, III, IV

$\begin{bmatrix}6&1\\6&1\end{bmatrix}\Rightarrow\begin{bmatrix}1\\-6\end{bmatrix}$ D

10. $2+3: 0\ 0\ 6\ 0\ 0$ ←
$4-5: 0\ 0\ 0\ 8\ 0$ } dependent
$1-4: 0\ 0\ 6\ 0\ 0$ ←

$|A|=0$ E

11. swapping rows or columns changes sign, as does multiplying a row/column by -1.

$A \neq B = C \Rightarrow$ A

12. $z=\frac{\begin{vmatrix}1&-1&4\\1&3&7\\2&-2&4\end{vmatrix}}{\begin{vmatrix}1&-1&1\\1&3&-1\\2&-2&4\end{vmatrix}}=\frac{-16}{8}=-2A$

13. $(A^{adj})_{1,2}=\text{cofactor}(A_{2,1})$

$=-\begin{vmatrix}3&-3\\-1&0\end{vmatrix}=3$ A

14. $A^{-1}=\frac{1}{|A|}A^{adj}=\frac{1}{3e+2}\begin{bmatrix}2&-e\\3&1\end{bmatrix}$

$\text{Sum}=\frac{2+3+1\bar{4}e}{3e+2}=\frac{6\bar{4}e}{3e+2}$ A

15. $|A^{-1}| = \frac{1}{|A|}$ guess C

$|A| = -f\begin{vmatrix}-3 & -1\\ -2 & 4\end{vmatrix} + 2\begin{vmatrix}-3 & -1\\ 1 & 0\end{vmatrix}$

$= 14f + 2 \Rightarrow$ C

16.

$M = \begin{bmatrix}0&1&0&0&0\\0&0&0&0&1\\0&0&1&0&0\\0&0&0&1&0\\1&0&0&0&0\end{bmatrix}$

$M^{-1}\begin{bmatrix}w\\z\\x\\y\\v\end{bmatrix} = \begin{bmatrix}v\\w\\x\\y\\z\end{bmatrix}$

$\Rightarrow M^{-1} = \begin{bmatrix}0&0&0&0&1\\1&0&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&1&0&0&0\end{bmatrix}$

$M^{-1}\begin{bmatrix}v\\w\\x\\y\\z\end{bmatrix} = \begin{bmatrix}z\\v\\x\\y\\w\end{bmatrix}$ C

17. $3 \cdot 3^3 = 3^4 = 81$ C

18. rank is order of largest square sub-matrix w/ $\neq 0$ determinant.

$3\times3 \Rightarrow \begin{vmatrix}-1&-1\\0&1\end{vmatrix} + \begin{vmatrix}2&3\\-1&-1\end{vmatrix} = -1+1 = 0$

$2\times2 \Rightarrow \begin{vmatrix}1&2\\0&-1\end{vmatrix} = -1$ ✓ 2 D

19. This is just sigma notation for the algorithm for calculating an element in a matrix product. D

20. $B = A^{-1} = \frac{1}{|A|}A^{adj} = \frac{1}{x}\begin{bmatrix}0&1\\-1&2\end{bmatrix}$

$\frac{2}{5} = \frac{2}{x} \Rightarrow x = 5$ A

21. all points two away from A $\Rightarrow$ circle B

22. $\sqrt{4+25+9} = \sqrt{38}$ B

23. $\vec{A}\cdot\vec{B} = 4\cdot3 + -2\cdot3 = \sqrt{16+4}\cdot\sqrt{9+9}\cos\theta$

$\cos\theta = \frac{6}{2\sqrt{5}\cdot3\sqrt{2}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{10}} = \sqrt{\frac{9}{10}}$

$= \frac{3\sqrt{10}}{10}$ B

24. $\vec{A}\cdot\vec{B} = 1\cdot5 + 4\cdot3 + 4\cdot-5 = 5+12-20 = -3$ D

25. no value : E

26. $\vec{A}\cdot\vec{B} = -3+2a-8 = 0$

$2a = 11 \Rightarrow a = \frac{11}{2}$ A

27. unit vector in direction: $\frac{3i+4j}{5} = \underline{u}$

projection $P = |A|\cos\theta\,\underline{u} = \frac{|A|}{|B|}\cos\theta\,\underline{B}$

$= \frac{(\vec{A}\cdot\vec{B})\vec{B}}{|B|^2} = \frac{(3\cdot 13 + 9\cdot 4)(3i+4j)}{25}$

$= \frac{75(3i+4j)}{25} = 9i+12j$ A

28. $\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 2 & -3 & 0 \\ 1 & 1 & -4 \end{vmatrix} = [12, 8, 5]$ D

29. $5t-3u = 14$ ①
$3t+7u = 12$ ②

$2\cdot$② + ① $\Rightarrow 11t+11u = 38 \Rightarrow \frac{38}{11}$ C

30. $\vec{A} = [2,-5,-1]$
$\vec{B} = [-7,-2,0]$
$\vec{A}\times\vec{B} = [-2,7,-39]$
$-2x+7y-39z = 96$ D

31. $\vec{A} = [2,2,-6] \sim [1,1,-3]$ D

32. 2 seconds $\Rightarrow$ 6 units
$|[2,1,-2]| = \sqrt{4+1+4} = 3 \Rightarrow \Delta t = 2$
$(-1,4,0) \Rightarrow t = -1 + 2 \Rightarrow t = 1$
$\Rightarrow (3,6,-4)$ D

33. $\vec{A} = [5,1,-2]$
$\vec{B} = [5,2,2]$
$\vec{C} = [3,-1,-1]$
$\vec{D} = [-2,4,2]$
$\vec{A}\cdot\vec{B} = 25+2-4 \neq 0$
$\vec{A}\cdot\vec{C} = 15-1+2 \neq 0$
$\vec{A}\cdot\vec{D} = -10+4-4 \neq 0$
none are $\perp$ E

34. $\vec{A} = [1,-5,2]$
$\vec{B} = [-1,1,4]$
$\vec{A}\times\vec{B} = [-22,-6,-4]$
$[11,3,2]$ direction of line
$\Rightarrow$ A, C, or (D) — 1, -1, 1 is on both planes

35. $A = \frac{1}{2}ab\sin\theta = \frac{1}{2}|\vec{a}\times\vec{b}|$
$\vec{a} = [3,-6,0]$
$\vec{b} = [-4,0,1]$
$\vec{a}\times\vec{b} = [-6,-3,-24]$
$A = \frac{1}{2}\cdot 3\sqrt{4+1+64} = \frac{3\sqrt{69}}{2}$ B

36. $l_1 = [39,22,5] + t[-54,-27,-15]$
$= [39,22,5] + u[18,9,5]$
$l_2 = [27,-2,-7] + s[24,6,-2]$
$= [27,-2,-7] + r[12,3,-1]$

$39+18u = 27+12r \Rightarrow 6u-4r=-4$
$3u-2r=-2$ ①

$22+9u = -2+3r \Rightarrow 9u+3r=-24$
$3u+r=-8$ ②

② − ① $\Rightarrow 3r=-6$

$r=-2 \Rightarrow u=-2$

$z = 5+5\cdot-2 = -5 = -7--2 = -5$ ✓
$y = -2-3\cdot-2 = 4 \quad x = 27-2\cdot12 = 3$
$4+3-5=2$ C

37. $A\to B = [-4,-2,1]$
$B\to A = [4,2,-1]$

$[3,-1,-2]\cdot[-4,-2,1] = -12+2-2 \overset{=12}{<} 0$
A is heading away from B

$[1,2,-3]\cdot[4,2,-1] = 4+4+3 \overset{=11}{>} 0$
B is heading towards A, but slower (just barely) than A is heading away.

Initial distance is the minimum
$\sqrt{4^2+2^2+1^2} = \sqrt{21}$ C

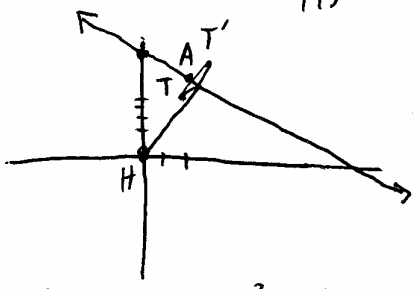
38. $\frac{dy}{dx} = f'(x) = -6x$

$\left.\frac{dy}{dx}\right|_{2,8} = -12$

$\vec{v} = \frac{i-12j}{\sqrt{145}} \quad a+b = \frac{1+-12}{\sqrt{145}}$

$= \frac{-11\sqrt{145}}{145}$ C

39.



$A = (x,y) \quad y = -\frac{3}{5}x+12$
$y = \frac{5}{3}x+\frac{2}{3}$

$0 = \left(\frac{5}{3}+\frac{3}{5}\right)x - \frac{34}{3}$

$\frac{34}{3} = \frac{34}{15}x \Rightarrow x = 5 \Rightarrow T' = (8,14)$
$y = 9$

$H\to T' = \sqrt{64+196} = \sqrt{260} = 2\sqrt{65}$ C

40. $\nabla\cdot\vec{F} = \frac{\partial}{\partial x}F_x + \frac{\partial}{\partial y}F_y + \frac{\partial}{\partial z}F_z$

$\nabla\cdot\vec{F} = 3y+0-1 = 3y-1$ C