Mu Alpha Theta National Convention: Denver, 2001

Matrices and Vectors Topic Test – Mu Division

1. If $-4\begin{bmatrix}1 & -3\\ x & 2\end{bmatrix}+3\begin{bmatrix}y & -2\\ -1 & 3\end{bmatrix}=\begin{bmatrix}7 & 6\\ 4 & 1\end{bmatrix}$, what is the value of $x + y$?

   (A) $\frac{7}{4}$ (B) $\frac{11}{6}$ (C) $\frac{23}{12}$ (D) 3 (E) NOTA

2. Evaluate: $\begin{bmatrix}-1 & -6 & 3\\ 3 & 0 & -1\end{bmatrix}\begin{bmatrix}-1 & 4\\ -2 & 2\\ 1 & 0\end{bmatrix}$

   (A) $\begin{bmatrix}12 & 8 & -4\\ -16 & 16 & 16\\ 1 & 0 & 12\end{bmatrix}$ (B) $\begin{bmatrix}12 & -19\\ 1 & 0\end{bmatrix}$

   (C) $\begin{bmatrix}16 & -16\\ -4 & 12\end{bmatrix}$ (D) $\begin{bmatrix}-16 & 4\\ 12 & -4\end{bmatrix}$ (E) NOTA

3. If A and B are 4 by 4 matrices whose (i,j)th elements are $i+j$ and $2i+j$ respectively, what is the sum of the largest and smallest entries in $A+B$?

   (A) 338 (B) 352 (C) 387 (D) 392 (E) NOTA

4. Evaluate: $\begin{vmatrix}-1 & -3 & 1\\ -3 & 2 & 0\\ -3 & 2 & -1\end{vmatrix}$

   (A) -8 (B) -1 (C) 6 (D) 11 (E) NOTA

5. Evaluate: $\begin{bmatrix}1 & 4\\ -1 & 2\end{bmatrix}^{-1}$

   (A) $\begin{bmatrix}\frac{1}{4} & \frac{5}{6}\\ 0 & \frac{1}{2}\end{bmatrix}$ (B) $\begin{bmatrix}\frac{1}{4} & \frac{1}{3}\\ -\frac{1}{4} & \frac{1}{2}\end{bmatrix}$ (C) $\begin{bmatrix}\frac{1}{3} & -\frac{2}{3}\\ \frac{1}{6} & \frac{1}{6}\end{bmatrix}$ (D) $\begin{bmatrix}\frac{1}{3} & \frac{1}{2}\\ -\frac{1}{3} & \frac{1}{6}\end{bmatrix}$ (E) NOTA

6. What is the trace of $\begin{bmatrix} -3 & -3 & 6 & 9 \\ -4 & 5 & 8 & -12 \\ 12 & 0 & 2 & 0 \\ 24 & 3 & 9 & -8 \end{bmatrix}$?

(A) 18 (B) 16 (C) -1 (D) -4 (E) NOTA

7. The system of equations

$$-x - y + 2z = 14$$
$$2y - 2z = -1$$
$$4x + Ay + 3z = 117$$

is satisfied by exactly one ordered triple (x, y, z). Which value(s) of A is/are not possible?

(A) $A < 12$ (B) $A = 12$ (C) $A > -7$ (D) A = -7 (E) NOTA

8. What is the sum of the eigenvalues of the matrix $\begin{bmatrix} 3 & 8 \\ -2 & 6 \end{bmatrix}$?

(A) 8 (B) 9 (C) -12 (D) -16 (E) NOTA

9. Which of the following are eigenvectors of the matrix $\begin{bmatrix} 3 & 1 \\ 6 & -2 \end{bmatrix}$?

I. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ II. $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ III. $\begin{bmatrix} 1 \\ -6 \end{bmatrix}$ IV. $\begin{bmatrix} -6 \\ -6 \end{bmatrix}$

(A) I & II only (B) I & III only
(C) II & III only (D) I, III, & IV only (E) NOTA

10. Evaluate: $\begin{vmatrix} 1 & 2 & 3 & 4 & 5a \\ -1 & 2 & 3 & 4 & -5a \\ 1 & -2 & 3 & -4 & 5a \\ 1 & 2 & -3 & 4 & 5a \\ 1 & 2 & -3 & -4 & 5a \end{vmatrix}$

(A) 16 (B) a + 16 (C) 5a + 16 (D) 25a + 16 (E) NOTA

11. Which of the following matrices has a determinant different from all the others?

(A) $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ (B) $\begin{bmatrix} d & e & f \\ a & b & c \\ g & h & i \end{bmatrix}$ (C) $\begin{bmatrix} a & c & b \\ d & f & e \\ g & i & h \end{bmatrix}$ (D) $\begin{bmatrix} a & b & c \\ -d & -e & -f \\ g & h & i \end{bmatrix}$ (E) NOTA

12. Solve the system of equations for z.

$$x - y + z = 4$$
$$x + 3y - z = 7$$
$$2x - 2y + 4z = 4$$

(A) -2 (B) -1 (C) 0 (D) $\frac{3}{4}$ (E) NOTA

13. What is the element in the first row, second column of the adjoint of the matrix $\begin{bmatrix} 1 & 3 & -3 \\ 3 & -4 & 1 \\ 2 & -1 & 0 \end{bmatrix}$?

(A) 3 (B) 1 (C) -1 (D) -3 (E) NOTA

14. Determine the sum of the elements in: $\begin{bmatrix} 1 & e \\ -3 & 2 \end{bmatrix}^{-1}$

(A) $\frac{6-e}{3e+2}$ (B) $\frac{6e+2}{e-2}$ (C) $\frac{8e-3}{14-4e}$ (D) $\frac{1}{e-1}$ (E) NOTA

15. What is the determinant of A^{-1} if $A = \begin{bmatrix} 0 & -3 & -1 \\ f & 1 & 0 \\ 2 & -2 & 4 \end{bmatrix}$?

(A) $\frac{3}{5f+6}$ (B) $\frac{4f}{6-5f}$ (C) $\frac{1}{14f+2}$ (D) $\frac{12f-3}{2f+8}$ (E) NOTA

16. If transformation matrix M exists such that $M \times \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} w \\ z \\ x \\ y \\ v \end{bmatrix}$ for all values of v, w, x, y, and z, what is $M^{-1} \times \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix}$?

(A) $\begin{bmatrix} w \\ v \\ x \\ y \\ z \end{bmatrix}$ (B) $\begin{bmatrix} w \\ x \\ v \\ z \\ y \end{bmatrix}$ (C) $\begin{bmatrix} z \\ v \\ x \\ y \\ w \end{bmatrix}$ (D) $\begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix}$ (E) NOTA

17. Suppose the 3x3 matrix A has a determinant of 3. What is the determinant of $3A$?

(A) 9 (B) 27 (C) 81 (D) 243 (E) NOTA

18. What is the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$?

(A) 9 (B) 7 (C) 3 (D) 2 (E) NOTA

19. If A and B are n by n matrices, which of the following matrices is equal to matrix C, given that $c_{ij} = \sum_{q=1}^{n} \left(a_{iq} b_{qj} \right)$?

(A) AB^{-1} (B) BA^{-1} (C) BA^{t} (D) AB (E) NOTA

20. Let $A = \begin{bmatrix} 2 & -1 \\ x & 0 \end{bmatrix}$ and B the inverse of A. If the entry in the second row, second column of B is $\frac{2}{5}$, find x.

(A) 5 (B) $\frac{3}{2}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) NOTA

21. Given point A in a plane, what is the shape of the locus of all points P in the plane such that $\left|\overrightarrow{AP}\right| = 2$?

(A) a line (B) a circle (C) a square (D) a hyperbola (E) NOTA

22. What is the magnitude of the vector [2, 5, -3]?

(A) 6 (B) $\sqrt{38}$ (C) $2\sqrt{10}$ (D) $2\sqrt{11}$ (E) NOTA

23. What is the sine of the smaller angle between the vectors [4, -2] and [3, 3]?

(A) $\frac{3\sqrt{10}}{20}$ (B) $\frac{3\sqrt{10}}{10}$ (C) $\frac{3\sqrt{5}}{20}$ (D) $\frac{3\sqrt{5}}{10}$ (E) NOTA

24. What is the scalar product of the vectors [1, 4, 4] and [5, 3, -5]?

(A) 12 (B) 10 (C) 4 (D) -3 (E) NOTA

25. For what value of a will the vector [a, -3, 4a] be parallel to the vector [2, -6, -8]?

(A) 2 (B) 1 (C) -1 (D) -2 (E) NOTA

26. For what value of a will the vector [1, 2a, -4] be perpendicular to the vector [-3, 1, 2]?

(A) $\frac{11}{2}$ (B) 4 (C) -1 (D) -3 (E) NOTA

27. What is the projection of [13, 9] on [3, 4] ?

(A) $[9, 12]$ (B) $\left[\frac{15}{2}, 10\right]$ (C) $\left[7, \frac{28}{3}\right]$ (D) $[7, 9]$ (E) NOTA

28. Vector *a* is [2, -3, 0], while vector *b* is [1, 1, -4]. Evaluate $\vec{a} \times \vec{b}$.

(A) [2, -3, 5] (B) [2, 14, 1] (C) [-6, -3, 10] (D) [12, 8, 5] (E) NOTA

29. A particle starts at the origin and travels with velocity [5, 3] (measured in units/second) for a while. Then, it changes velocity, travelling at [-3, 7] (measured similarly) for another interval of time. What is the total duration of its journey (in seconds) when it reaches the point (14, 12)?

(A) $\frac{38}{7}$ seconds (B) $\frac{38}{9}$ seconds (C) $\frac{38}{11}$ seconds (D) $\frac{38}{13}$ seconds (E) NOTA

30. What is the equation of the plane through the points (5, 4, -2), (7, -1, -3), and (-2, 2, -2)?

(A) $19x - 23y + 6z = -9$ (B) $3x + 5y - 2z = 39$
(C) $-8x + 12y + 11z = -14$ (D) $-2x + 7y - 39z = 96$ (E) NOTA

31. Which of the following is an equation of the line through the points (2, -3, 4) and (4, -1, -2)?

(A) $[x, y, z] = [4, -1, -2] + t[2, -3, 4]$ (B) $[x, y, z] = [2, -3, 4] + t[2, -3, 4]$
(C) $[x, y, z] = [4, -1, -2] + t[2, -2, 6]$ (D) $[x, y, z] = [2, -3, 4] + t[2, 2, -6]$ (E) NOTA

32. A particle travels at a speed of 3 units/second along the line $[x, y, z] = [1, 5, -2] + t[2, 1, -2]$, starting at the point (-1, 4, 0) in such a way that its *x*-position is increasing. What are the particle's coordinates after 2 seconds?

(A) (-6, 1, 4) (B) (13, 11, -14) (C) (5, 7, -6) (D) (3, 6, -4) (E) NOTA

33. Which of the lines listed intersects the line $[x, y, z] = [1, -1, -3] + t[5, 1, -2]$ at a right angle?

I. $[x, y, z] = [1, 0, -3] + t[5, 2, 2]$
II. $[x, y, z] = [2, 1, 2] + t[3, -1, -1]$
III. $[x, y, z] = [4, 2, -3] + t[-2, 4, 2]$

(A) II only (B) III only (C) I & II only (D) I, II, and III (E) NOTA

34. Which of the following is an equation of the line which is the intersection of the plane $x - 5y + 2z = 8$ and the plane $-x + y + 4z = 2$?

(A) $[x, y, z] = [2, 4, 13] + t[22, 6, 4]$ (B) $[x, y, z] = [5, -1, -1] + t[-3, 1, 0]$
(C) $[x, y, z] = [-3, 3, -1] + t[22, 6, 4]$ (D) $[x, y, z] = [1, -1, 1] + t[11, 3, 2]$ (E) NOTA

35. What is the area of the triangle defined by the points (1, -3, -2), (4, 3, -2), and (-3, -3, -1)?

(A) $6\sqrt{5}$ (B) $\frac{3\sqrt{69}}{2}$ (C) $6\sqrt{6}$ (D) $\frac{15}{2}$ (E) NOTA

36. Line A passes through the points $(39, 22, 5)$ and $(-15, -5, -10)$. Line *B* passes through the points $(27, -2, -7)$ and $(51, -8, -9)$. What is the sum of the *x*, *y*, and *z* coordinates of the point at which lines *A* and *B* intersect?

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

37. Consider two particles moving in straight lines in three dimensions. At 6AM, particle *A* is at the point (5, 3, -2), while particle *B* is at the point (1, 1, -1). Particle *A*'s velocity vector is [3, -1, -2], measured in units per minute, while particle *B*'s velocity vector is [1, 2, -3], measured similarly. What is the closest these points will come to one another, if they were stationary until 6AM?

(A) 5 (B) $2\sqrt{7}$ (C) $\sqrt{21}$ (D) $4\sqrt{2}$ (E) NOTA

38. If $\vec{v} = a\vec{i} + b\vec{j}$ is the unit vector tangent to $f(x) = 4 - 3x^2$ at (2, -8), where $a > 0$ and $b < 0$, and both *a* and *b* are fractions in lowest terms with rationalized denominators, what is the value of $a + b$?

(A) $\sqrt{2}$ (B) 2 (C) $-\frac{11\sqrt{145}}{145}$ (D) $-\frac{\sqrt{1145}}{14}$ (E) NOTA

39. The Lone Mathematician likes to use a coordinate system in which his home is (0, 0), the town is (2, 4), and the river is the line $y = -\frac{3}{5}x + 12$, with all coordinates expressed in miles. If the Lone Mathematician is in town and must get water at the river on his way home, what is the smallest number of miles he can travel?

(A) $\sqrt{258}$ (B) 16 (C) $2\sqrt{65}$ (D) $4\sqrt{17}$ (E) NOTA

40. What is the divergence of the vector field defined by $\vec{F}(x, y, z) = 3xy\vec{i} + 2x\vec{j} - (z + y)\vec{k}$?

(A) $2x + z$ (B) $-x + y - 2yz$ (C) $3y - 1$ (D) 5 (E) NOTA