Mu Alpha Theta National Convention: Denver, 2001

Probability Topic Test – Mu Division

1. Consider rain and wind as independent events. The probability of rain is $\frac{2}{3}$, and the probability of wind is $\frac{1}{3}$. What is the probability of both rain and wind?

   (A) 1 (B) $\frac{1}{3}$ (C) $\frac{2}{9}$ (D) $\frac{2}{3}$ (E) NOTA

2. In conditions of problem 1 above, what is the probability of either rain or wind (or both)?

   (A) 1 (B) $\frac{7}{9}$ (C) $\frac{8}{9}$ (D) $\frac{2}{9}$ (E) NOTA

3. On planet Morg a fair 5-sided die can be constructed. If two such dice are labeled with the numbers 1-5 and tossed, what is the probability that the sum of the numbers displayed is 4?

   (A) $\frac{1}{12}$ (B) $\frac{1}{5}$ (C) $\frac{3}{10}$ (D) $\frac{3}{25}$ (E) NOTA

4. The probability the Mariners win any baseball game is $\frac{2}{3}$. What is the probability that they win a seven-game series in exactly five games? A team wins a seven-game series by winning four of the games. The series is over once this occurs, and no further games are played.

   (A) $\frac{1}{8}$ (B) $\frac{10}{21}$ (C) $\frac{64}{243}$ (D) $\frac{80}{243}$ (E) NOTA

5. Wally's Widget Works sends out a shipment of 72 individually packaged widgets, six of which are defective. If Ike the Inspector opens two packages, what is the probability that at least one defective widget is found?

   (A) $\frac{137}{852}$ (B) $\frac{1}{6}$ (C) $\frac{1}{144}$ (D) $\frac{11}{71}$ (E) NOTA

6. Two distinct cards are drawn randomly from a standard 52-card deck. What is the probability that the cards are of the same suit?

   (A) $\frac{1}{17}$ (B) $\frac{2}{17}$ (C) $\frac{4}{17}$ (D) $\frac{8}{17}$ (E) NOTA

7. Jim has two blue shirts, three red shirts, and four green shirts, as well as one pair of jeans, three pairs of khakis, and two pairs of shorts. Fred has six red shirts, seven brown shirts, and four white shirts, as well as six pairs of shorts and four pairs of sweats. Jim and Fred each choose an outfit at random. What is the probability that their choices are identical in terms of shirt color and legwear style?

(A) $\frac{1}{8}$ (B) $\frac{1}{17}$ (C) $\frac{1}{85}$ (D) $\frac{2}{85}$ (E) NOTA

8. Seven students are seated in a row; what is the probability that Mary and Jane sit next to each other?

(A) $\frac{1}{360}$ (B) $\frac{3}{320}$ (C) $\frac{1}{6}$ (D) $\frac{2}{7}$ (E) NOTA

9. Steve borrows Sean's circular keyring, which holds 4 keys. Steve promptly drops the keyring, and the keys fall off. If Steve replaces the keys on the keyring, what is the probability that they are placed in an order identical to the original order? Assume that the direction the keys are facing does not affect order.

(A) $\frac{1}{24}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) NOTA

10. The probability of an American winning a medal in any one event at a track and field meet is $\frac{2}{3}$. If a meet has 12 events, what is the probability that an American wins a medal in every event?

(A) $\frac{3721}{436{,}520}$ (B) $\frac{3819}{459{,}349}$ (C) $\frac{4096}{531{,}441}$ (D) $\frac{8191}{1{,}062{,}882}$ (E) NOTA

11. Four distinct numbers are chosen from the first ten natural numbers. What is the probability that 6 is the second largest of those chosen?

(A) $\frac{3}{21}$ (B) $\frac{4}{21}$ (C) $\frac{4}{25}$ (D) $\frac{3}{50}$ (E) NOTA

12. The probability of Josh having an asthma attack during a tennis match is $\frac{1}{2}$. If he has such an attack his probability of winning is $\frac{1}{9}$, otherwise it is $\frac{7}{8}$. What is the probability of Josh winning any tennis match?

(A) $\frac{8}{17}$ (B) $\frac{7}{72}$ (C) $\frac{71}{144}$ (D) $\frac{1}{2}$ (E) NOTA

13. Given that $f(x)=\cos(x)+1$, and that a real value of *x* is chosen at random on the interval $-6\pi \le x < 14\pi$, what is the probability that $f(x)>0$?

(A) 0 (B) 1 (C) $\frac{143{,}999}{144{,}000}$ (D) $\frac{\sqrt{3}}{3}$ (E) NOTA

14. On Tom's drive to work, each traffic light he must wait at costs him 2 minutes. His drive passes 10 traffic lights. Without any traffic lights the drive would take him 2 minutes. His probability of having to wait at any given light is $\frac{1}{3}$. What is his probability of reaching work exactly ten minutes after departure?

(A) $\frac{6{,}430}{19{,}683}$ (B) $\frac{4{,}480}{19{,}683}$ (C) $\frac{6{,}230}{59{,}049}$ (D) $\frac{6{,}530}{59{,}049}$ (E) NOTA

15. In the conditions of problem 14 above, what is Tom's probability of reaching work ten minutes or less after his departure (to the nearest ten-thousandth)?

(A) 0.2530 (B) 0.4281 (C) 0.6636 (D) 0.7869 (E) NOTA

16. Ten fair coins are tossed. What is the probability that exactly 5 faces display heads?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{63}{256}$ (D) $\frac{81}{512}$ (E) NOTA

17. One fair coin is tossed eight consecutive times. What is the probability that no two heads or tails appear consecutively?

(A) $\frac{1}{128}$ (B) $\frac{3}{128}$ (C) $\frac{1}{256}$ (D) $\frac{3}{256}$ (E) NOTA

18. A dartboard is made up of 3 concentric circles with radii of 2, 4, and 6 inches. A dart thrown will strike the board in a random location. What is the probability that it will strike within the innermost circle?

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{9}$ (D) $\frac{\pi}{18}$ (E) NOTA

19. Mike and Jim are pirates on a ship with a crew of nine. Three crewmembers must be sent out in a rowboat to an island, and lots are drawn to see who must go. What is the probability that neither Mike nor Jim will be forced to go?

(A) $\frac{4}{21}$ (B) $\frac{1}{18}$ (C) $\frac{5}{12}$ (D) $\frac{7}{18}$ (E) NOTA

20. Eight students will be placed randomly in 12 desks in a row. What is the probability that no two empty desks will be adjacent?

(A) $\frac{1}{3}$ (B) $\frac{2}{11}$ (C) $\frac{31}{99}$ (D) $\frac{3}{14}$ (E) NOTA

21. 32 chocolate chips considered to have negligible volume are mixed randomly into 32 ounces of cookie dough. The dough is then randomly divided into eight 4-ounce cookies. If one cookie is selected at random what is the probability (to the nearest ten-thousandth) that it will have zero chips?

(A) 0.0102 (B) 0.0139 (C) 0.0845 (D) 0.0125 (E) NOTA

22. A plane crash has one survivor, considered to have negligible size. He can survive 36 hours in cold ocean water before he must eat his disgusting survival rations. He is somewhere within a search radius of 3 miles. The coast guard can search 2 square miles per 24-hour day. What is the probability that the survivor is found before he must eat his survival rations?

(A) $\frac{1}{3}$ (B) 1 (C) $\frac{\pi}{6}$ (D) $\frac{1}{3\pi}$ (E) NOTA

23. When one of Paul's "friends" receives an invitation to a party that he is throwing, their probability of attending is $\frac{1}{2}$. If Paul invites 30 people, what is the probability (to the nearest ten-thousandth) that at least 10 people attend?

(A) 0.9544 (B) 0.9613 (C) 0.9627 (D) 0.9786 (E) NOTA

24. Seven blue socks and three red socks are in drawer A. Three blue socks and seven red socks are in drawer B. One sock is drawn at random from drawer A and placed in drawer B. If two socks are then drawn at random from drawer B, what is the probability that the two form a matching pair?

(A) $\frac{4}{121}$ (B) $\frac{3}{11}$ (C) $\frac{28}{55}$ (D) $\frac{141}{275}$ (E) NOTA

25. Given that $f(x)=\frac{4\sqrt{3}}{3}\sin(3x+7)$ and that x_0 is chosen at random on the interval $-22\pi \le x_0 < -10\pi$, what is the probability that $|f(x_0)|>2$?

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) NOTA

26. The Seahawks are equally likely to begin a drive at any field position *Y* between their own goal line ($Y=0$) and midfield ($Y=50$), and the probability that $0<Y<50$ is 1. The probability that they will score a touchdown on a drive starting at position *Y* is given by $T(Y)=\frac{1}{2e^{50}}e^{Y}$. On a drive, what is the probability that a touchdown is scored?

(A) $\frac{1}{25}$ (B) $\frac{1}{50}$ (C) $\frac{1}{100}$ (D) $\frac{1}{200}$ (E) NOTA

27. Given that $f(x)=x^2$ and $0<x<3$, if a value of x is chosen at random, what is the probability that $f(x)<2$?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{2}{9}$ (E) NOTA

28. Two players are playing a game in which they take turns rolling two six-sided dice. On a given turn, if a 2, 7, or 12 is rolled, the player whose turn it is wins. If a 3 or 11 is rolled, the other player wins. If neither of the above happens, it is the other player's turn. If each player takes two turns and no one has won yet, the game is declared a tie. What is the probability that a game ends in a tie?

(A) $\frac{2}{9}$ (B) $\frac{3}{16}$ (C) $\frac{16}{81}$ (D) $\frac{255}{1296}$ (E) NOTA

29. One fair six-sided die is tossed 5 times, what is the probability that the numbers are in descending order? Equal numbers cannot be placed in descending order.

(A) $\frac{1}{7{,}776}$ (B) $\frac{1}{3{,}888}$ (C) $\frac{1}{1{,}944}$ (D) $\frac{1}{1{,}296}$ (E) NOTA

30. Bob and Jane will each arrive at the train station at random times between one and two o'clock. Bob will wait 5 minutes; Jane will wait 10 minutes. What is the probability that they will be at the station at the same time?

(A) $\frac{67}{288}$ (B) $\frac{1}{10}$ (C) $\frac{1}{12}$ (D) $\frac{63}{181}$ (E) NOTA

31. The probability that a person is happy is directly proportional to their net worth, unless their net worth is greater than 1.5 million dollars, in which case they are guaranteed to be happy, or if their net worth is below zero dollars, in which case they cannot possibly be happy. If a person's net worth is one million dollars, their probability of happiness is $\frac{2}{3}$. A person's net worth (w) in dollars is related to his high school GPA (g) by the equation $w(g) = 10{,}000g^3 + 20{,}000g^2 + 10{,}000$. A person is chosen to have a random high school GPA in the continuous range of 0.0 - 4.0. What is the probability that the person is happy?

(A) $\frac{83}{435}$ (B) $\frac{83}{440}$ (C) $\frac{83}{445}$ (D) $\frac{83}{450}$ (E) NOTA

32. Sally's extension school offers three subjects: English, Science, and History. A student may be enrolled in one, two, or three subjects. 18 are enrolled in Science and English, 12 in History and English, and 10 in History and Science. 198 total students are enrolled, with 174 taking only one subject. If a student is chosen at random, what is the probability that he is enrolled in all three subjects?

(A) $\frac{7}{198}$ (B) $\frac{4}{99}$ (C) $\frac{9}{198}$ (D) $\frac{5}{99}$ (E) NOTA

33. A particle is at some position on the x-axis, $-2 < x < 2$. The probability it is at any location x is proportional to $\left|x^3\right|$. What is the probability that the particle is at $x = 1$?

(A) $\frac{1}{8}$ (B) $\frac{1}{16}$ (C) 1 (D) 0 (E) NOTA

34. In the conditions of problem 33 above, what is the probability that the particle is between $x = \frac{1}{2}$ and $x = \frac{3}{2}$?

(A) $\frac{5}{256}$ (B) $\frac{5}{128}$ (C) $\frac{5}{64}$ (D) $\frac{5}{32}$ (E) NOTA

35. One of the smallest 100 natural numbers is chosen at random. What is the probability that it has exactly 3 positive integral factors?

(A) $\frac{1}{25}$ (B) $\frac{9}{100}$ (C) $\frac{1}{20}$ (D) 0 (E) NOTA

36. The probability of event A occurring if event B occurs is $\frac{2}{5}$. The probability event B occurs if event A occurs is $\frac{1}{2}$. The probability of both events occurring is $\frac{1}{6}$. What is the probability of neither event occurring?

(A) $\frac{7}{12}$ (B) $\frac{1}{4}$ (C) $\frac{5}{12}$ (D) $\frac{1}{3}$ (E) NOTA

37. In the conditions of problem 36 above, what is the probability that event A occurs?

(A) $\frac{7}{12}$ (B) $\frac{1}{4}$ (C) $\frac{5}{12}$ (D) $\frac{1}{3}$ (E) NOTA

38. A game of poker is played in which a player is dealt five cards from a standard 52-card deck. How many ways are there for the resulting hand to be a flush, if 2's are considered wild cards? Note: a flush is a hand in which all the cards are the same suit (clubs, diamonds, hearts, or spades), but in which the rank of the cards (K, 3, 7, etc.) does not matter. Some examples of hands that would meet the criteria of this problem are A-2-7-9-Q of hearts, 3-7-6-10 of diamonds and the 2 of clubs, or 5-8 of spades and the 2's of clubs, diamonds, and hearts.

(A) 6,340 (B) 8,815 (C) 17,472 (D) 27,372 (E) NOTA

39. An apartment building has 5 units with separate mailboxes. One letter is sent to each unit, but the mailman is in a hurry and thus places one letter in each box at random. What is the probability that at least 3 letters are placed in the correct boxes?

(A) $\frac{1}{120}$ (B) $\frac{11}{120}$ (C) $\frac{1}{10}$ (D) $\frac{1}{12}$ (E) NOTA

40. Three points are on an 8 by 8 grid of unit squares: point A in the upper left corner, point B two units below and five units to the right of A, and point C in the lower right corner (eight units below and eight units to the right of A). If a path of length 16 is drawn from A to C along the gridlines, what is the probability that it passes through point B?

(A) $\frac{96}{715}$ (B) $\frac{97}{715}$ (C) $\frac{98}{715}$ (D) $\frac{1176}{2431}$ (E) NOTA