Mu Alpha Theta National Convention: Denver, 2001
Sequences and Series Topic Test – Mu Division

1. Find the sum of all prime numbers p such that $p \equiv 1 \pmod{3}$ and $p < 40$.

   (A) 110 (B) 87 (C) 107 (D) 132 (E) NOTA

2. For $n>3$, find the value of the sum $1 - 2 + 3 - 4 + 5 - 6 + \ldots + (2n - 1) - (2n)$.

   (A) –1 (B) $-n + 1$
   (C) cannot be determined (D) $-n$ (E) NOTA

3. The nth triangular number is defined as the sum of the first n positive integers. Find the 35th triangular number.

   (A) 1260 (B) 630 (C) 1225 (D) 105 (E) NOTA

4. Steve opens up a savings account with 6% annual interest, compounded monthly. He deposits \$32 in it each month, hoping to save up enough money for a new car after 5 years. Assuming he deposits his money at the beginning of each month, how much money (in dollars, rounded to the nearest cent) will be in Steve's savings account at the end of 5 years?

   (A) \$2,232.64 (B) \$2,243.80 (C) \$2,589.79 (D) \$1,929.60 (E) NOTA

5. The 10th term of an arithmetic sequence is 27. The 20th term is 12. Find the 40th term.

   (A) 18 (B) –18 (C) 15 (D) -3/2 (E) NOTA

6. Evaluate the sum of the arithmetic sequence: 6 + 10 + 14 + … + 502.

   (A) 31750 (B) 31496 (C) 127508 (D) 63246 (E) NOTA

7. Evaluate: $\lim_{n\to\infty}\left\{\left(\frac{\sin(n^2)}{n^2+1}\right)^n\right\}$.

   (A) e (B) e^2 (C) 1 (D) 0 (E) NOTA

8. The series $\sum_{n=0}^{\infty} k^n$ converges for what values of k?

   (A) $-1 < k < 1$ (B) $|k| > 1$
   (C) $k < 1$ (D) $-1 < k < 1$ (for nonzero k) (E) NOTA

9. Find the sum of the infinite geometric sequence with $r = \dfrac{1}{\sqrt{2}-1}$ and $a_1 = \sqrt{8}$.

(A) diverges (B) $4 + 2\sqrt{2}$ (C) $4 - 2\sqrt{2}$ (D) $4\sqrt{2}$ (E) NOTA

10. For which value of v does the equality $(1 + 2 + 3 + \ldots + n)^2 = 1^v + 2^v + 3^v + \ldots + n^v$ hold true? (Assume n is an integer > 3.)

(A) no value (B) 2 (C) 4 (D) 3 (E) NOTA

11. Consider the recurrence relation $a_{n+1} = 3a_{n-1} + \frac{1}{2}a_n$ for $a_1 = 1$ and $a_2 = 2$. Find the 1735th term.

(A) 2^{1735} (B) $\dfrac{1735(1735^2 - 3\cdot 1735 + 8)}{6}$ (C) 2^{1734}
(D) $\dfrac{1735^4 - 6\cdot 1735^3 + 23\cdot 1735^2 - 18\cdot 1735 + 24}{24}$ (E) NOTA

12. Evaluate: $1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cdots}}}}$

(A) $\dfrac{1+\sqrt{3}}{2}$ (B) $\dfrac{-1+\sqrt{13}}{2}$ (C) $\dfrac{1+\sqrt{13}}{2}$ (D) $1 + \dfrac{\sqrt{3}}{2}$ (E) NOTA

13. Find the value of $\displaystyle\sum_{n=1}^{10}\left(\sum_{m=1}^{10}\left(m^2 + n^2\right)\right)$.

(A) 7700 (B) 770 (C) 4235 (D) 6050 (E) NOTA

14. There exist two arithmetic sequences, *A* and *B*. *A* and *B* have the same first and the same last term, *p* and *q* respectively, where *p* and *q* are positive. The sum of *A* and the sum of *B* differ by $(p + q)$. How many more terms does the sequence with the larger sum have than the sequence with the smaller sum?

(A) 2 (B) cannot be determined
(C) 0 (D) 1 (E) NOTA

15. Consider the recurrence relation for which $a_n = \dfrac{a_{n-1} - 1}{a_{n-1} + 1}$ with $a_1 = 2$. Find a_{1023}.

(A) 1/3 (B) -1/2 (C) –3 (D) 2 (E) NOTA

16. Evaluate: $\displaystyle\sum_{k=2}^{\infty} \frac{3}{2^{k+1}}$

(A) diverges (B) 3/8 (C) 3/4 (D) 3/16 (E) NOTA

17. Find the 13th Fibonacci number (let $F_1 = F_2 = 1$).

(A) 144 (B) 377 (C) 169 (D) 233 (E) NOTA

18. Evaluate: $\displaystyle\sum_{k=0}^{90} \sin^2(k°)$

(A) 45 (B) 90 (C) 181/2 (D) 91/2 (E) NOTA

19. A right isosceles triangle is drawn with legs of 12 cm. A triangle is then inscribed by connecting the midpoints of all the sides of the first triangle. If triangles are continuously inscribed in the same fashion, what is the sum of the areas of every triangle drawn?

(A) 144 cm^2 (B) 48 cm^2 (C) 72 cm^2 (D) 96 cm^2 (E) NOTA

20. Evaluate $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$

(A) 2e (B) 4e (C) e^2 (D) $e^2/2$ (E) NOTA

21. Given the relation $a_n = \sqrt{a_{n-1} + 8}$ with $a_1 = 1$, evaluate $\lim_{n\to\infty} a_n$.

(A) divergent (B) 17/5 (C) $\dfrac{\sqrt{33}+1}{2}$ (D) $\dfrac{\sqrt{33}-1}{2}$ (E) NOTA

22. Find the sum of the first four terms of the harmonic series.

(A) 7/12 (B) 247/210 (C) 15/8 (D) 25/12 (E) NOTA

23. A cube is inscribed in a sphere of radius 3. Another sphere is inscribed inside the cube and a second cube is inscribed in this sphere. If this pattern continues, what is the surface area of the 21st sphere?

(A) 1/19683 (B) $1/(6561\sqrt{3})$ (C) $1/(19683\sqrt{3})$ (D) 1/6561 (E) NOTA

24. Evaluate $\sum_{n=0}^{\infty} \cot^n\left(\frac{x}{2}\right)$ for $x = \frac{2\pi}{3}$.

(A) $\frac{-1-\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}-1}{2}$ (C) $\frac{3-\sqrt{3}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$ (E) NOTA

25. Which of the following is an accurate description of $\ln\left(\sum_{k=1}^{\infty} e^{-k}\right)$?

(A) positive and less than 1
(B) finite and negative
(C) positively infinite
(D) negatively infinite
(E) NOTA

26. Evaluate: $2\prod_{k=1}^{\infty} 3^{(5^{-k})}$

(A) $2\sqrt[4]{3}$ (B) $\sqrt[4]{6}$ (C) 3/4 (D) 3/2 (E) NOTA

27. Evaluate: $\sum_{n=4}^{\infty} \frac{1}{n^2-9}$

(A) 49/20 (B) 29/20 (C) 29/120 (D) 49/120 (E) NOTA

28. Which of the following is a sufficient condition for $\sum_{n=b}^{\infty} a_n$ (b a positive integer) to converge?

I. $\int a_n \, dn$ over the elementary functions exists

II. $\int_c^{\infty} a_n \, dn$ ($c \geq b$) converges

III. $\lim_{n \to 0} a_n$ is finite

(A) II & III only (B) II only (C) I & II only (D) III only (E) NOTA

29. Which of the following is an accurate description of $\sum_{n=1}^{\infty}\frac{(-1)^n}{3n+1}$?

(A) divergent
(B) conditionally convergent
(C) absolutely convergent
(D) monotonically increasing
(E) NOTA

30. For sequences *a* and *b* with positive terms defined for all *n*, the following information is given: $b_n < a_n$ and $\lim_{n\to\infty} a_n = 0$. Which of the following statements must be true?

(A) $\sum_{n=1}^{\infty} b_n$ converges
(B) $\sum_{n=1}^{\infty} b_n$ diverges
(C) $\lim_{n\to\infty} b_n < \lim_{n\to\infty} a_n$
(D) b_n diverges
(E) NOTA

31. Given that $\lim_{n\to\infty}\frac{a_n}{a_{n+1}} = 0$ for sequence *a*, which of the following statements about *a* is true?

(A) *a* converges
(B) *a* is strictly decreasing
(C) $\lim_{n\to\infty} a_n = 0$
(D) *a* diverges
(E) NOTA

32. Evaluate: $\sum_{n=0}^{\infty}\frac{2^n(-1)^n}{(2n)!}$

(A) $\cos\sqrt{2}$
(B) $\sin\sqrt{2}$
(C) sin 2
(D) e^{-2}
(E) NOTA

33. Evaluate the sum of the base 7 geometric series $.5_7 - .05_7 + .005_7 - .0005_7 + \ldots$. Express your answer as a fraction in base 7.

(A) $\frac{5_7}{11_7}$
(B) $\frac{5_7}{9_7}$
(C) $\frac{5_7}{8_7}$
(D) $.625_7$
(E) NOTA

34. Evaluate: $\sum_{n=0}^{\infty}\frac{n^3}{n!}$

(A) $5e$
(B) e
(C) e^3
(D) $3e$
(E) NOTA

35. Evaluating the improper integral $\lim_{b\to\infty}\int_1^b \frac{1}{x^2+1}\,dx$ shows that:

(A) $\sum_{n=0}^{\infty}\frac{1}{n^2+1}$ diverges
(B) $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$ diverges
(C) $\sum_{n=0}^{\infty}\frac{1}{n^2+1}$ converges
(D) $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$ could either converge or diverge
(E) NOTA

36. Evaluate: $\sum_{n=3}^{\infty}\frac{1}{n^4-5n^2+4}$.

(A) 25/144 (B) 5/36 (C) 5/144 (D) 5/16 (E) NOTA

37. Approximate $\int_{4/3}^{5/3}(\ln x)^{-1}\,dx$ using the first two non-zero terms of the Taylor series expansion of $\ln(x)$ centered about $x=1$.

(A) $\ln\left(\frac{5}{2}\right)$ (B) $\ln\left(\frac{5}{4}\right)$ (C) $\ln\left(\frac{25}{22}\right)$ (D) $\ln\left(\frac{11}{8}\right)$ (E) NOTA

38. Evaluate: $\dfrac{\left(\frac{1+\sqrt{5}}{2}\right)^{16}-\left(\frac{1-\sqrt{5}}{2}\right)^{16}}{\sqrt{5}}$

(A) 390625 (B) 1344 (C) 987 (D) 377 (E) NOTA

39. Let a_n and b_n be two arithmetic progressions with $n>0$, the sum of the first n terms of which are $S_a(n)$ and $S_b(n)$, respectively. Given that $\frac{S_a(n)}{S_b(n)}=\frac{5n+9}{2n+8}$ and $a_2=4$, determine b_4.

(A) $\frac{8}{3}$ (B) $\frac{11}{4}$ (C) 8 (D) 12 (E) NOTA

40. Evaluate: $\sum_{x=1}^{\infty} \frac{x^2}{6^x}$

(A) 1/25 (B) 42/125 (C) 6/25 (D) 48/125 (E) NOTA