# Mu Alpha Theta National Convention: Denver, 2001

## Trigonometry Topic Test – Mu Division

1. Convert 56° into radians.

   (A) $\frac{14\pi}{45}$ (B) $\frac{10080}{\pi}$ (C) $\frac{7\pi}{45}$ (D) $\frac{7}{50}$ (E) NOTA

2. Change $\frac{43\pi}{18}$ radians to degrees.

   (A) $\left(\frac{43\pi^2}{3240}\right)^\circ$ (B) 860° (C) $\left(\frac{43\pi^2}{6480}\right)^\circ$ (D) 430° (E) NOTA

3. Evaluate: $\sin 0 - \cos 30° + \tan\frac{\pi}{4} - \sec 60° + \csc\frac{\pi}{2} - \cot 120°$

   (A) $-\frac{1}{2}+\frac{\sqrt{3}}{3}$ (B) $\frac{9-\sqrt{3}}{6}$ (C) $2-\frac{3\sqrt{3}}{2}$ (D) $-\frac{\sqrt{3}}{6}$ (E) NOTA

4. If $\theta$ is an acute angle and $1+\tan^2 57° = \csc^2\theta$, then what is $\theta$?

   (A) 73.5° (B) 57° (C) 33° (D) 61.5° (E) NOTA

5. Evaluate: $\sin\left(\cos\left(\sqrt{\frac{\pi^2}{4}}\right)\right)$

   (A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) sin 1 (D) 0 (E) NOTA

6. Simplify: $2\sin^3 y\cos y + 2\cos^3 y\sin y$

   (A) tan *y* (B) sin (2*y*) (C) cos (2*y*) (D) 1 (E) NOTA

7. Evaluate: $\cos(73°)\cos(13°)+\sin(37°)\cos(8°)+\sin(73°)\sin(13°)+\cos(37°)\sin(8°)$

   (A) $\frac{1+\sqrt{2}}{2}$ (B) 1 (C) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (D) $\sqrt{2}$ (E) NOTA

8. If $\cot\omega + \tan\omega = \frac{143}{69}$ and $\frac{1}{\cot\omega} + \frac{1}{\tan\omega} = \frac{m}{n}$, where *m* and *n* are positive, relatively prime integers, what is *m* + *n*?

(A) 202 (B) 212 (C) 222 (D) 232 (E) NOTA

9. In $\Delta ABC$, $AB = 1$, $\angle ACB = 36°$, and $\angle CAB = 79°$. Which of the following is equal to *BC*?

(A) $\frac{\sin 36°}{\sin 79°}$ (B) $\frac{\sin 101°}{\cos 54°}$ (C) $\frac{\cos 79°}{\cos 36°}$ (D) $\frac{\cos 11°}{\sin 18°}$ (E) NOTA

10. Given that $\sec\theta = -\frac{25}{7}$, where $\frac{\pi}{2} \le \theta \le \pi$ and $f(x) = \csc x$, find $f'(\theta)$.

(A) $\frac{25}{7}$ (B) $-\frac{25}{24}$ (C) $\frac{175}{576}$ (D) $-\frac{25}{168}$ (E) NOTA

11. Two sides of a triangle are 3 and 8 while the sine of the acute angle between them is $\frac{48}{73}$. If this angle is doubled, what is the ratio of the area of the new triangle to the old?

(A) 2:1 (B) 73:55 (C) 110:73 (D) 73:48 (E) NOTA

12. What is the period of $f(x) = \sin(6x)$?

(A) $\frac{1}{6}$ (B) $\frac{\pi}{6}$ (C) $\frac{1}{3}$ (D) $\frac{\pi}{3}$ (E) NOTA

13. Given that $\cos u = \frac{5}{13}$ and $\sin v = \frac{4}{5}$, what is the value of $\cos(-u) + \sin(-v)\cos u$?

(A) $\frac{1}{169}$ (B) $\frac{1}{13}$ (C) $-\frac{1}{13}$ (D) $-\frac{1}{169}$ (E) NOTA

14. A plane intersects a sphere of radius 5 at a distance of $5\cos 37°$ units from its center. Which of the following is equal to the area, in square units, of the cross section formed?

(A) $\frac{25\pi - 25\pi\cos 74°}{2}$ (B) $25\pi + 25\pi\cos^2 37°$

(C) $25\pi + 25\pi\cos 74°$ (D) $10\pi\sin 37°$ (E) NOTA

15. A missile rises vertically from a point on the ground 75,000 feet from a radar station. If the missile is rising at a rate of 16,500 feet per minute at the instant when it is 37,500 feet high, what is the rate of change (in radians per minute) of the missile's angle of elevation from the station?

(A) $\frac{11}{75}$ (B) $\frac{11}{250}$ (C) $\frac{22}{125}$ (D) $\frac{8}{5}$ (E) NOTA

16. Solve for $B$ on the interval $(0, 2\pi]$: $2\sin^2 B - \cos B = 1$.

(A) $\frac{\pi}{3}, \frac{2\pi}{3}, \pi$ (B) $\frac{\pi}{6}, \pi, \frac{11\pi}{6}$ (C) $\frac{\pi}{3}, \frac{5\pi}{3}$ (D) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$ (E) NOTA

17. In $\Delta ABC$, $a$ = 16947, $b$ = 49463, and $c$ = 50000, where $a$, $b$, and $c$ are the lengths of the sides opposite angles $A$, $B$, and $C$, respectively. Compute $b\cos A + a\cos B$.

(A) 16947 (B) 49463 (C) 50000 (D) 66410 (E) NOTA

18. If $y = \sin^3 x \cot^3 x \cos^2 x \sec^5 x$, find $y'$.

(A) $2\cos 2x$ (B) 0 (C) $-\csc x \cot x$ (D) $3\sin^2 x \cos x$ (E) NOTA

19. Solve for $\frac{\pi}{3} < \theta < \frac{11\pi}{6}$: $\tan\theta \sin\theta + \sin\theta = 1 + \tan\theta$.

(A) $\frac{3\pi}{4}, \frac{5\pi}{4}$ (B) $\frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}$ (C) $\frac{\pi}{2}, \frac{5\pi}{4}, \frac{7\pi}{4}$ (D) $\frac{3\pi}{4}, \frac{7\pi}{4}$ (E) NOTA

20. $ABC$ is a triangle where $AC$ = 5, $CB$ = 12, and $AB$ = 13. Let $D$ be the foot of the altitude to $AB$. What is the value of $\cos\angle BCD + \sec\angle ACD$?

(A) $\frac{229}{156}$ (B) $\frac{10}{13}$ (C) $\frac{34}{13}$ (D) $\frac{229}{65}$ (E) NOTA

21. Describe the behavior of the graph of $y = \sin x$ as $x$ increases on the interval $\frac{1473\pi}{23} < x \le \frac{773\pi}{12}$.

(A) decreases, then increases (B) strictly decreasing

(C) strictly increasing (D) increases, then decreases (E) NOTA

22. Find the maximum value of the derivative of $y = 2\sin\frac{5x}{3} + 5\cos\frac{3x}{4}$ with respect to $x$.

(A) 3 (B) 7 (C) $\frac{85}{12}$ (D) $\frac{5}{12}$ (E) NOTA

23. Solve for $x$: $\arctan\frac{22}{7} = \arctan\frac{3}{x} + \arctan\frac{17}{x}$, where $-\frac{\pi}{2} < \arctan t < \frac{\pi}{2}$ for all $t$.

(A) $\frac{17}{34}$ (B) 11 (C) 4 (D) $-\frac{51}{11}$ (E) NOTA

24. A drawbridge with two 10 foot spans is being raised at a rate of 2 radians per minute. How fast, in feet per minute, is the distance increasing between the ends of the spans when they are at an angle of $\frac{\pi}{4}$ radians above their horizontal rest position?

(A) $10\sqrt{2}$ ft/min (B) $20\sqrt{2}$ ft/min (C) $5\sqrt{2}$ ft/min (D) $30\sqrt{2}$ ft/min (E) NOTA

25. Evaluate: $\int \sin^3 t \cos^4 t \, dt$

(A) $\frac{1}{5}\sin^5 t - \frac{1}{7}\sin^7 t + C$ (B) $-\frac{1}{9}\cos^9 t + \frac{2}{7}\cos^7 t - \frac{1}{5}\cos^5 t + C$

(C) $\frac{1}{7}\cos^7 t - \frac{1}{5}\cos^5 t + C$ (D) $-\frac{1}{7}\sin^7 t + \frac{1}{3}\sin^6 t - \frac{1}{5}\sin^5 t + C$

(E) NOTA

26. If $\sin A = -\frac{28}{53}$, where $\pi < A \le \frac{3\pi}{2}$ and $B$ is a fourth-quadrant angle such that $\cos B = \frac{21}{29}$, what is $\cos(A+B)$?

(A) $\frac{993}{1537}$ (B) $-\frac{1505}{1537}$ (C) $-\frac{385}{1537}$ (D) $\frac{1080}{1537}$ (E) NOTA

27. What is the volume of the solid formed when the region bounded by the graphs of $y = \sqrt{\sin x}$ and $y = \sqrt{\cos x}$ on the interval $0 \le x \le \frac{\pi}{4}$ is revolved about the $x$-axis?

(A) $\pi\sqrt{2} + 2\pi$ (B) $\frac{\pi\sqrt{2}}{2}$ (C) $2\pi\sqrt{2} + \pi$ (D) $\pi\sqrt{2} - \pi$ (E) NOTA

28. Each of the six basic trigonometric functions (sin $x$, cos $x$, tan $x$, sec $x$, csc $x$, and cot $x$) are each written on its own slip of paper. These six slips of paper are placed in a hat, from which a mathematician randomly draws two slips. What's the probability that when the functions written on the chosen slips are squared and added together, the result is equal to 1 for all $x$?

(A) $\frac{1}{18}$ (B) $\frac{1}{15}$ (C) $\frac{1}{30}$ (D) $\frac{2}{5}$ (E) NOTA

29. Find the area bounded by the graphs of $y = 3\sin x - 4\sin^3 x$, $x = 0$, $x = \frac{\pi}{3}$, and $y = 0$.

(A) 2 (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) 6 (E) NOTA

30. What is the sum of all $\varphi$ such that $\sin^2 5\varphi - 1 = 0$ on the interval $\frac{47\pi}{50} < \varphi < \frac{8\pi}{5}$?

(A) $\frac{28\pi}{5}$ (B) $\frac{39\pi}{10}$ (C) $\frac{13\pi}{2}$ (D) $10\pi$ (E) NOTA

31. The course of a skateboard race consists of a 200-meter downhill run and a 150-meter level portion. When the starting point of the race is spotted from the finishing line, the angle of elevation is 25°. To the nearest degree, what angle does the hill make with the horizontal?

(A) 97° (B) 18° (C) 59° (D) 43° (E) NOTA

32. Evaluate: $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$

(A) $\frac{\sqrt{1-x^2}}{x^2}+C$ (B) $\frac{x}{\sqrt{1-x^2}}-\frac{2}{x^3}+C$

(C) $-\frac{\sqrt{1-x^2}}{x}+C$ (D) $\frac{x}{(1-x^2)^{3/2}}+C$ (E) NOTA

33. How many times does the graph of $y=\cos x-\sin 2x-\cos 3x$ intersect the $x$-axis on the interval $0\le x\le 2\pi$?

(A) 8 (B) 7 (C) 6 (D) 5 (E) NOTA

34. What is the period of $f(x)=2\sin(3x)-4\tan\left(\frac{5x}{4}\right)$?

(A) $8\pi$ (B) $4\pi$ (C) $\frac{16\pi}{15}$ (D) $\frac{8\pi}{15}$ (E) NOTA

35. Given that $\cos\beta=\frac{\sqrt{21}}{5}$ and $\frac{\pi}{18}<\beta<\frac{\pi}{4}$, what is $\sin 3\beta$?

(A) $-\frac{64}{125}$ (B) $\frac{118}{125}$ (C) $\frac{103}{125}$ (D) $-\frac{76}{125}$ (E) NOTA

36. Find $\frac{dy}{dx}$ if $y=(\sin x)^x$.

(A) $(\sin x)^x(\ln\sin x+x\cot x)$ (B) $(\sin x)^x(\ln\sin x)(\cos x)$

(C) $(\sin x)^x\cot x$ (D) $(\sin x)^x\left(\frac{\sin x}{x}+\ln x\cos x\right)$ (E) NOTA

37. A triangle has two sides of length 14 and 12, and the angle between them is $18°$. The area of this triangle is $a\sqrt{b}-a$, where $a$ and $b$ are positive integers and $b$ is not divisible by the square of any prime. What is the value of $a+b$?

(A) 26 (B) 61 (C) 47 (D) 86 (E) NOTA

38. How many of the following functions satisfy the differential equation $y + \sum_{n=1}^{3} \frac{d^n y}{dx^n} = 0$ ?

$y = \sin x$ $y = \cos x$ $y = \tan x$
$y = \sec x$ $y = \csc x$ $y = \cot x$

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

39. Determine the sum of the two smallest positive values of $\alpha$ for which $\cos(8\alpha) = \cos(9\alpha)$

(A) $\frac{\pi}{3}$ (B) $\frac{6\pi}{17}$ (C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{8}$ (E) NOTA

40. For how many values of x ( $0 \le x < 2\pi$ ) is $\frac{1+\sin x}{1+\cos x} = 2$ ?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA