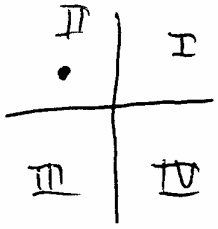
Mu Alpha Theta National Convention: Denver, 2001

Analytic Geometry Topic Test Solutions – Open Division

1. 

II B

2. $\frac{-5+1}{2} = \frac{-4}{2} = -2$

$\frac{6+2}{2} = \frac{8}{2} = 4$ B

3. $\sqrt{(3-8)^2+(4-7)^2} = \sqrt{121+9}$

$= \sqrt{130}$ D

4. $m = \frac{13-8}{9-11} = \frac{5}{-2} = -\frac{5}{2}$ A

5. $y = \frac{-b}{2a} = \frac{12}{4} = 3$ C

6. center: $(1,-1)$

slopes $= \pm\frac{4}{2} = \pm 2$

$y = 2x-3$, $y = -2x+1$ B

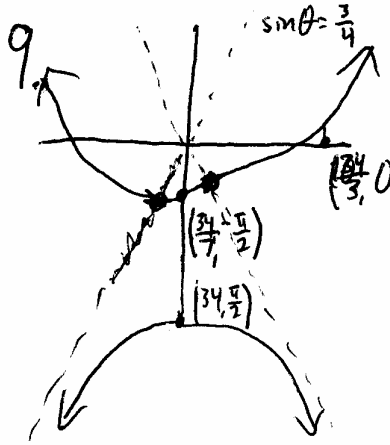
7. $(x-5)^2+(y+3)^2+(z+4)^2 = -1+25+9+16$

$= 49$

$r = \sqrt{49} = 7$ $V = \frac{4}{3}\pi r^3 = \frac{1372\pi}{3}$ A

8. $(x-3)^2+(y+7)^2 = 400+9+49$

center: $(3,-7)$ B

9. 

maybe hyperbola

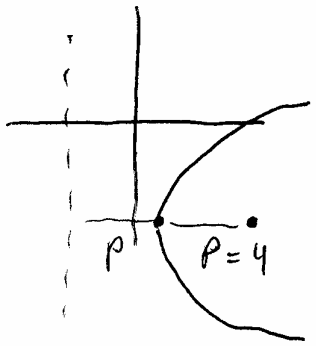
$r(3-4\frac{y}{r}) = 34$

$3r - 4y = 34$

$3r = 34+4y$

$9(x^2+y^2) = 16y^2+272y+1156$

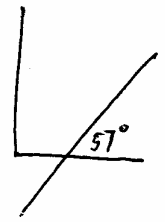
$9x^2-7y^2\ldots$ $\Rightarrow$ hyperbola D

10. 

$x = \frac{1}{4p}(y+4)^2+1$

$x = \frac{1}{16}(y+4)^2+1$

C

11. 

$m = \tan 57^\circ = 1.54$

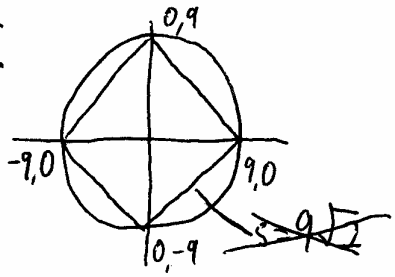
≈ 2 A

12. $\frac{3}{k} = \frac{5}{-1} \Rightarrow k = -\frac{3}{5}$

$2 = \frac{3}{5}h \Rightarrow h = \frac{10}{3} \Rightarrow -\frac{3}{8}\cdot\frac{10^2}{3} = -2$ C

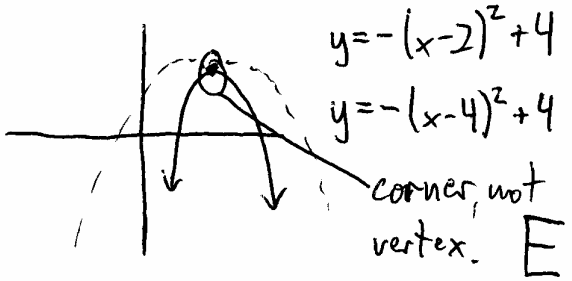
13. center $(3,12)$ $r=12$
center $(-4,12)$ $r=13$ zero area **A**
distance $= \sqrt{7^2+24^2} = 25$

14. $y = -\frac{1}{2}(x+1)^2 + 1$
vertex $= (-1,1)$
$\frac{1}{4p} = -\frac{1}{2} \Rightarrow p = -\frac{1}{2}$
focus $= (-1, \frac{1}{2})$ **B**

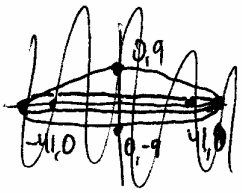
15. 

A

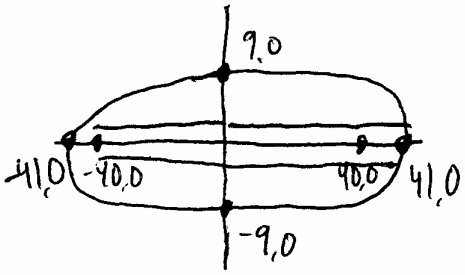
16. $0,0$ is on the line $\frac{x}{7} + \frac{y}{24} = 0$
The distance b/t these lines is $\frac{1-0}{\sqrt{\left(\frac{1}{7}\right)^2 + \left(\frac{1}{24}\right)^2}} = \frac{7 \cdot 24}{25} = \frac{168}{25}$ **B**

17. 
$y = -(x-2)^2 + 4$
$y = -(x-4)^2 + 4$
corner, not vertex. **E**

18. $9(x+2)^2 + 4(y-3)^2 = -36+36+36$
$\Rightarrow x = -2$ **A**

19. 

He can walk the major axis twice
$\Rightarrow 2 \cdot 82 = 164$

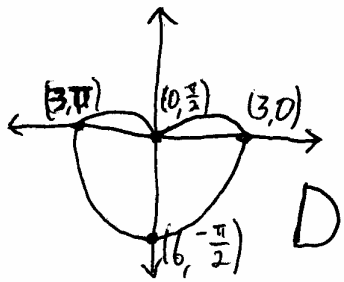


$40+1+81+40 = 162$ **C**

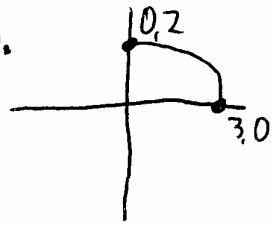
20. x-distance from zero $\Rightarrow$ **C**

21. B' = midpoint of $AC = (6,2,2)$
$BB' = \sqrt{4+16+25} = \sqrt{45} = 3\sqrt{5}$ **B**

22. $r = 3 - 3\frac{y}{r}$
$r^2 = 3r - 3y$



D

23. 

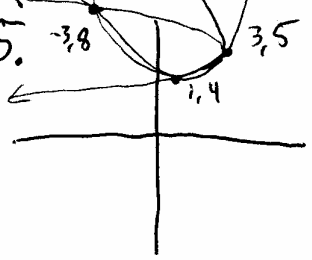
$V = \frac{1}{2} \cdot \frac{4}{3}\pi \cdot 2 \cdot 2 \cdot 3 = 8\pi$
(like $V = \frac{4}{3}\pi r^3$) **B**

24. $a=4, b=3$, ~~$b=\sqrt{7}$~~

$c=\sqrt{a^2+b^2}=5$

$e=\frac{c}{a}=\frac{5}{4}$ C

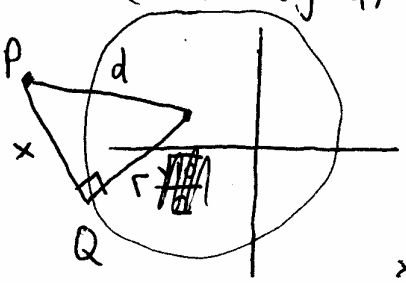
25.



parabolae don't have to be aligned with the axes, so infinitely many. D

26. $2(x+\frac{3}{4})^2+2(y-\frac{1}{4})^2=5+\frac{9}{8}+\frac{1}{8}=\frac{50}{8}=\frac{25}{4}$

$(x+\frac{3}{4})^2+(y-\frac{1}{4})^2=\frac{25}{8} \Rightarrow r=\frac{5\sqrt{2}}{4}$



$d=\sqrt{(\frac{9}{4})^2+(\frac{7}{4})^2}$

$=\frac{3\sqrt{10}}{4}$

$x=\sqrt{\frac{90}{16}-\frac{50}{16}}=\sqrt{\frac{40}{16}}=\sqrt{\frac{5}{2}}=\frac{\sqrt{10}}{2}$

~~$\sqrt{\frac{90}{16}}=\sqrt{\frac{25}{4}}=\frac{5}{2}$~~ B

27. in the x-z plane, it's an ellipse.
in other two (yz, xy) is a hyperbola
$\Rightarrow$ C (contrast w/ $16x^2-y^2-25z^2=144$ which has two sheets.)

28. The points need to be collinear $\Rightarrow \frac{2k-4}{1-3k}=\frac{6k-4}{5-3k}$

$-6k^2+22k-20=-18k^2+18k-4$

$12k^2+4k-16=0$

$3k^2+k-4=0$

$(3k+4)(k-1)=0$

$k=\frac{-4}{3}, 1$ C

29. I can't think of a good proof, but I believe 4 is the maximum. A

30. I. Standard definition, similar to pencil/string definition of ellipse.

II. defines an ellipse by eccentricity

III. known as a "rectangular hyperbola"

C

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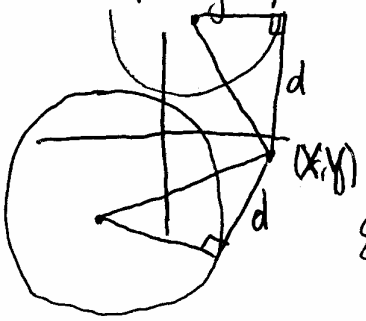
31. $(x+4)^2+(y+6)^2=28+16+36=80,\ r=4\sqrt{5}$

$(x-2)^2+(y-6)^2=-20+4+36=20,\ r=2\sqrt{5}$



$(X+4)^2+(Y+6)^2=80$

$=(X-2)^2+(Y-6)^2+20$

$8X+16+12Y+36=80$

$=-4X+4-12Y+36-20$

$12X+24Y=48$

$X+2Y=4 \Rightarrow$ E

32. $y=2(x-3)^2+7$

vertex $=(3,7)$

$\frac{1}{4p}=2 \Rightarrow p=\frac{1}{8}$

focus $=(3,\frac{57}{8})$ A

33. $\pi\frac{M}{2}\frac{m}{2}=\frac{\pi Mm}{4}$ E

(similar to πr^2 for a circle)

34. n=1 n=2

n=3 n=4

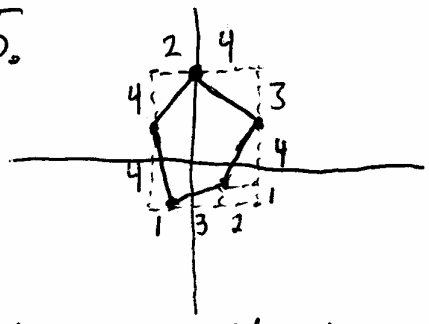
$1+3+5+7+9+11+13+15+17+19$

$=100$

$4\sum_{n=1}^{10} n$

$400 \Rightarrow 320$ A

35.

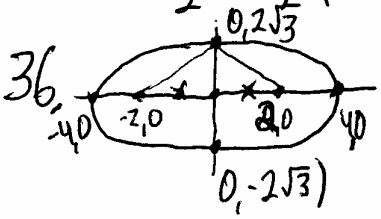


$A=6\cdot8-1\cdot2-\frac{1}{2}(2\cdot4+4\cdot3+4\cdot2+3\cdot1+4\cdot1)$

$=46-\frac{1}{2}(35)$

$=\frac{57}{2}$ A

36.



$x+y=8 \Rightarrow xy=x(8-x)$

$=-x^2+8x$

max value at $x=4 \Rightarrow 16$

min @ $x=0 \Rightarrow 0$

$x^2+y^2=(x+y)^2-2xy$

$=64-32=32$ B

37.

x	4	3	2	1	0
y	27	16	9	4	1
		-11	-7	-5	-3

$y=2(x-\frac{3}{4})^2+\frac{47}{8}$

$=2x^2-3x+\frac{56}{8} \Rightarrow 6$ D

38. $5cx+3y=1$

$2x-cy=1$

$x=\frac{(1+cy)}{2}$

$\frac{5c+5c^2y}{2}+3y=1$

$\frac{5c+(5c^2+6)y}{2}=1$

$5c+(5c^2+6)y=2$

$y=\frac{2-5c}{5c^2+6}>0$

$2-5c>0$

$c<\frac{2}{5}$

$y=\frac{2x-1}{c}$

$5cx+\frac{6x-3}{c}=1$

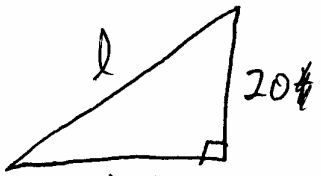
$(5c^2+6)x=c+3$

$x=\frac{c+3}{5c^2+6}\geq 0$

$c>-3$

B

39.



"unwind" it

$l=\sqrt{15^2+20^2}=25$ C

40.

$y=-\frac{2}{3}x^2+6$

$y(1.5)=-\frac{3}{2}+6=\frac{9}{2}$ C