

Mu Alpha Theta National Convention: Denver, 2001
Algebra Topic Test – Theta Division

1. What is the name of the property demonstrated by the following equality?

$$b + c = c + b$$

- (A) distributive law of multiplication (B) associative law of addition
(C) commutative law of addition (D) closure for addition of real numbers (E) NOTA

2. What is the slope of the line $\frac{5x}{2} + \frac{7y}{2} = \frac{49}{10}$?

- (A) $\frac{5}{7}$ (B) $\frac{7}{5}$ (C) $-\frac{5}{7}$ (D) $-\frac{7}{5}$ (E) NOTA

3. How many times do the graphs of the equations $y = x^2 + 3x - 2$ and $y = 5x^3$ intersect in the Cartesian plane?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

4. If $f(x) = x^2 - 3x$ and $g(x) = 5x + 2$, which of the following is equal to $f(g(x)) - g(f(x))$?

- (A) 0 (B) $20x^2 + 20x - 4$
(C) $30x^2 - 20x$ (D) $15x^2 - 12x + 4$ (E) NOTA

5. What is the distance between the points $(-2, 5)$ and $(3, 8)$?

- (A) $\sqrt{10}$ (B) $\sqrt{34}$ (C) 6 (D) $\sqrt{35}$ (E) NOTA

6. At what point do the lines $y = 5x + 1$ and $y = 3x + 2$ intersect?

- (A) $(1, 7)$ (B) $\left(\frac{3}{8}, \frac{113}{40}\right)$ (C) $\left(\frac{3}{2}, \frac{13}{2}\right)$ (D) $\left(\frac{1}{2}, \frac{7}{2}\right)$ (E) NOTA

7. Which quadrant does the graph of $f(x) = (x - 5)^2 - 1$ not pass through?

- (A) I (B) II (C) III (D) IV (E) NOTA

8. Find the roots of $x^4 - 17x^2 + 16 = 0$.

- (A) -1, 4 (B) 1, 4 (C) $\pm 1, \pm 4$ (D) -1, -4 (E) NOTA

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9. A teacher has a system of awarding cards to his students to reward good scores on tests. For each score between 80 and 90, a student receives a white card. For each score between 91 and 100, a student receives a yellow card. Two white cards can be traded in for a yellow card. Five yellow cards can be traded in for a blue card. Three blue cards can be traded in for five pieces of candy. What is the price of each piece of candy in white cards?
- (A) 4 (B) 5 (C) 6 (D) 7 (E) NOTA
10. The sum of the digits in a two-digit number is 10. If the tens digit is decreased by 2 and the units digit is increased by 2 and then the digits are reversed, the resulting number is 36 less than the original number. What was the original number?
- (A) 64 (B) 73 (C) 82 (D) 91 (E) NOTA
11. In their roller hockey game, Deb, George, and Brian all scored goals. Twice the number of goals Deb scored was three times the sum of the number of goals scored by George and Brian. Twice the number of goals George scored, plus half the number of goals Brian scored, is equal to the number of goals Deb scored. Deb also scored one more goal than the sum of the number of goals George scored and twice the number of goals Brian scored. What was the total number of goals scored by Deb, George, and Brian?
- (A) 13 (B) 14 (C) 15 (D) 16 (E) NOTA
12. The Ideal Gas Law states that $PV = nRT$, where P is the pressure, V is the volume, n is the amount of gas present, R is a constant, and T is the absolute temperature. If the pressure is increased, and only one other variable is affected, which of the following describes a possible effect?
- I. The absolute temperature increases
II. The volume decreases
III. The amount of gas present decreases
- (A) I only (B) II only (C) I & II only (D) I & III only (E) NOTA
13. What is the slope of a line that is perpendicular to the line $5x + 3y = 13$?
- (A) $\frac{5}{3}$ (B) $\frac{3}{5}$ (C) $-\frac{3}{5}$ (D) $-\frac{5}{3}$ (E) NOTA
14. What is the product of the roots of $2x^2 + 5x - 3 = 0$?
- (A) 2 (B) -2 (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$ (E) NOTA

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15. Simplify the expression: $\frac{x^3 - x^2 - 10x - 8}{x^3 + 2x^2 - 11x - 12}$

- (A) $\frac{x^2 + 2x - 8}{x^2 + x - 12}$ (B) $\frac{x^2 - 2x - 8}{x^2 + x - 12}$ (C) $\frac{x^2 - 2x + 7}{x^2 + x - 12}$ (D) $\frac{x^2 - 2x - 8}{x^2 + 3x - 12}$ (E) NOTA

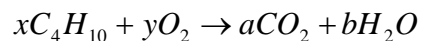
16. One long-distance provider charges a \$1.00 connection fee, and \$.04/minute during each call. Another charges a flat \$.05/minute during each call (no connection fee). A third provider charges a \$.03/minute for the first ten minutes of each call, and \$.06/minute for every minute after that (no connection fee). What is the difference in price between the cheapest and most expensive providers for a twenty-minute call?

- (A) \$0.90 (B) \$0.80 (C) \$0.20 (D) \$0.10 (E) NOTA

17. Billy and Tommy wish to build a paper airplane air force. If it takes two minutes for Billy to fold an airplane, and ninety seconds for Tommy to do the same, how long (to the nearest minute) will it take for them to build 500 planes working together?

- (A) 7 hrs, 9 min. (B) 7 hrs, 7 min. (C) 7 hrs, 6 min. (D) 7 hrs, 10 min. (E) NOTA

18. Chemists know that C_4H_{10} (isobutane, a molecule composed of four carbon atoms and ten hydrogen atoms) reacts with O_2 (oxygen, a molecule composed of two oxygen atoms) to form CO_2 (carbon dioxide, a molecule composed of one carbon atom and two oxygen atoms) and H_2O (water, a molecule composed of two hydrogen atoms and one oxygen atom). In this reaction, each kind of atom remains that kind of atom, for example carbon cannot become hydrogen, it must remain carbon. Chemists represent this reaction with an equation of the form:



where x , y , a , and b have integral values which cause the number of atoms of each type on each side of the equation to be equal (the number of carbon atoms on the left of the equation is the same as that on the right). In addition, there is no natural number greater than one which is a factor of all of x , y , a , and b . Determine the values of x , y , a , and b and express them as an ordered quadruplet (x, y, a, b) .

- (A) (2, 13, 8, 10) (B) (2, 15, 8, 10) (C) (1, 6, 3, 5) (D) (1, 7, 4, 5) (E) NOTA

19. The product of two numbers is 378 and their sum is 39. What is the positive difference between the two numbers?

- (A) 3 (B) 17 (C) 5 (D) 15 (E) NOTA

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20. There exist two complex numbers, x and y , such that $x^3 - y^3 = 5x^2 - 5y^2$, and $x + y = 6$. Determine the value of xy .
- (A) 5 (B) 7 (C) 9 (D) 11 (E) NOTA
21. CDs and DVDs have the same size cases, but a DVD can hold 7 times as much information. A CD holds 70 minutes of music and a DVD holds 490 minutes of music. If David has 186 cases and 31080 minutes of music, how many DVDs does he have?
- (A) 43 (B) 75 (C) 118 (D) 143 (E) NOTA
22. A teacher has a problem with students turning their work in late. To help alleviate it, he uses the scale $G = g\left(\frac{20-n}{20}\right)^n$, where g is the actual grade the assignment deserved (having a value between 0 and 100), n is the number of days late the assignment was (always an integer), and G is the final grade. If a student knows he has a score of 70 at the time the assignment is due, how many days late could he turn in his assignment and still receive a higher score (assuming he puts in some time raising the value of g)?
- (A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA
23. What is the product when r^x is multiplied by r^p ?
- (A) r^{x+p} (B) r^{xp} (C) r^{x-p} (D) r^{p-x} (E) NOTA
24. Let $A = 2 + 3i$ and $B = 4 - 2i$. Evaluate: $AB - (A + B)$
- (A) $8 + 7i$ (B) $8 - 7i$ (C) $4 - 15i$ (D) $-4 + 15i$ (E) NOTA
25. If $\log A = \frac{27}{10}$, $\log B = \frac{41}{10}$, and $\log C = \frac{16}{5}$, evaluate $\log\left(\frac{AC}{B}\right)$?
- (A) $\frac{9}{5}$ (B) $\frac{21}{10}$ (C) $\frac{18}{5}$ (D) $\frac{23}{5}$ (E) NOTA
26. Jerry invested \$8,000 in two types of bonds. The interest rate of one of the investments was 4% and on the other 5%. If the total annual income from both investments was \$340, how much was invested in the account with the lower interest rate?
- (A) \$6000 (B) \$5000 (C) \$4000 (D) \$3000 (E) NOTA

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27. Solve for x :

$$\frac{x+3}{4} + 2 = \frac{3}{2} - \frac{2x-1}{3}$$

- (A) -1 (B) 0 (C) 1 (D) 2 (E) NOTA

28. A semi-truck driver's eyes are about nine feet above the ground, but the hood of his truck extends eight feet in front of him and is six feet above the ground, obscuring part of his view. What is the minimum distance in front of the driver an object at ground level can still be visible to the driver?

- (A) 12 feet (B) 18 feet (C) 24 feet (D) 36 feet (E) NOTA

29. Simplify $\sqrt{208x y^{34} z^5}$ where x , y , and z are positive real numbers.

- (A) $4y^{17} z^2 \sqrt{13xz}$ (B) $16y^{17} z^2 \sqrt{xz}$
(C) $16y^{17} z^2 \sqrt{13xz}$ (D) $4y^{15} z \sqrt{13xz^3}$ (E) NOTA

30. In a given function, y is directly proportional to k and inversely proportional to h . Which of the following is an equation that satisfies these conditions?

- (A) $y = \frac{x}{hk}$ (B) $y = \frac{hx}{k}$ (C) $y = h k x$ (D) $y = \frac{kx}{h}$ (E) NOTA

31. The length of a rectangle is 3 inches more than the width. If the length is increased by 2 inches and the width by 1 inch, the area is increased by 26 square inches. What is the width of the original rectangle, in inches?

- (A) 9 inches (B) 8 inches (C) 7 inches (D) 6 inches (E) NOTA

32. What is the quotient when $x^5 - 5x^4 + 11x^3 - 23x^2 + 25x - 3$ is divided by $x - 3$?

- (A) $x^4 + 2x^3 + 5x^2 - 8x + 1$ (B) $x^4 - 2x^3 + 5x^2 - 8x + 1$
(C) $x^4 + 2x^3 + 5x^2 - 8x - 1$ (D) $x^4 + 2x^3 - 5x^2 - 8x + 1$ (E) NOTA

33. What is the coefficient of x^3 in the expansion of $(2x - 5)^6$?

- (A) 20,000 (B) 1,000 (C) -15,633 (D) -1,250 (E) NOTA

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34. Which of the following equations will give the following data points

x	1	3	5	7
y	3	11	27	51

- (A) $y = 2x + 1$ (B) $y = 8x - 5$ (C) $y = x^2 + 2$ (D) $y = 4x^2 - 3x + 2$ (E) NOTA

35. You have 12 gallons of a 15% (by volume) sugar-water solution. How much more water, to the nearest tenth of a gallon, must you add to get it to be a 7.4% (by volume) sugar-water solution?

- (A) 12.3 gallons (B) 14.5 gallons (C) 18.7 gallons (D) 24.3 gallons (E) NOTA

36. A hotel currently has a swimming pool that takes three hours to drain and four hours to fill. However, they are in the process of building a pool that is capable of holding twice the current volume of water. To accommodate the new filtration/purification system required to bring the system up to code, the radius of the drainage pipe of the new pool will be double that of the old pool, meaning it will drain water four times as fast. The new system is so expensive that the hotel has decided to use the current filling hoses to fill the new pool. Once it is completed, how long will it take to drain and then refill the new pool (something which must be done every month to meet health standards)?

- (A) 8 hours (B) $\frac{33}{4}$ hours (C) $\frac{19}{2}$ hours (D) $\frac{35}{4}$ hours (E) NOTA

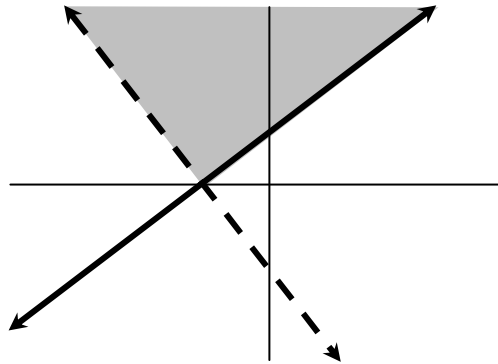
37. Each digit in a base-ten addition problem was replaced with a letter, with the following result. Each instance of a given letter represents the same digit, and every instance of a given digit was replaced with the same letter. What is the greatest possible value for B ?

$$\begin{array}{r}
 A \ B \ C \ D \\
 + \ B \ C \ A \ B \\
 \hline
 D \ E \ B \ F
 \end{array}$$

- (A) 6 (B) 7 (C) 8 (D) 9 (E) NOTA

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38. What set of equations could describe the graph below?



- (A) $y > x + 4$
 $y \geq -x - 4$ (B) $y \leq x + 4$
 $y > -x - 4$ (C) $y \geq x + 4$
 $y > -x - 4$ (D) $y \geq x + 4$
 $y > x - 4$ (E) NOTA

39. A baseball player currently has 50 hits in 317 at-bats. What is the minimum number of hits he needs in the rest of the season in order to average one hit in every four at-bats?

- (A) 37 (B) 39 (C) 41 (D) 43 (E) NOTA

40. What is the sum of the abscissas of the following set of points?

x	1.2	2.7	3.0	5.8	6.7	8.0	12.7	15.9	17.2
y	10.0	10.0	9.8	8.2	7.6	7.4	5.1	4.3	2.8

- (A) 65.2 (B) 67.4 (C) 70.3 (D) 73.2 (E) NOTA