Mu Alpha Theta National Convention: Denver, 2001

Matrices and Determinants Topic Test – Theta Division

1. Evaluate: $\begin{bmatrix} 1 & -2 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 5 & -2 \end{bmatrix}$

   (A) $\begin{bmatrix} 4 & 4 \\ 5 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 2 \\ \frac{5}{2} & 1 \end{bmatrix}$ (C) $\begin{bmatrix} -4 & 4 \\ 5 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & -4 \\ -5 & 2 \end{bmatrix}$ (E) NOTA

2. Evaluate: $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 7 & -8 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -2 & -1 \\ -1 & -3 & 2 \\ 1 & -1 & 2 \end{bmatrix}$

   (A) $\begin{bmatrix} -1 & -4 & 2 \\ -5 & 2 & -8 \\ 6 & -9 & 7 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 0 & -4 \\ -3 & 2 & -8 \\ 8 & -7 & 9 \end{bmatrix}$

   (C) $\begin{bmatrix} -1 & 0 & 4 \\ -3 & 8 & -8 \\ 6 & -7 & 7 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 4 & -4 \\ -5 & 2 & -8 \\ 6 & -9 & 9 \end{bmatrix}$ (E) NOTA

3. What is the sum of the elements in matrix A if $A = 2\begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix} - 3\begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$?

   (A) -6 (B) -7 (C) 8 (D) -9 (E) NOTA

4. If $4\begin{bmatrix} 1 & -3 \\ x & 2 \end{bmatrix} + 3\begin{bmatrix} y & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -18 \\ 4 & 17 \end{bmatrix}$, what is the value of $x + y$?

   (A) 0 (B) 1 (C) 2 (D) $\frac{11}{4}$ (E) NOTA

5. What is the sum of r and s if: $r\begin{bmatrix} 1 & 4 \end{bmatrix} + s\begin{bmatrix} -3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & -1 \end{bmatrix}$?

   (A) 1 (B) 0 (C) -1 (D) $-\frac{5}{4}$ (E) NOTA

6. Evaluate: $\begin{bmatrix} -1 & 2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 1 & -2 \end{bmatrix}$

(A) $\begin{bmatrix} -3 & -7 \\ 18 & 16 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & 7 \\ 22 & 8 \end{bmatrix}$ (C) $\begin{bmatrix} -3 & 7 \\ 18 & -8 \end{bmatrix}$ (D) $\begin{bmatrix} -3 & -7 \\ 22 & 8 \end{bmatrix}$ (E) NOTA

7. Evaluate: $\begin{bmatrix} 1 & -4 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 2 & -3 \\ 0 & 3 \end{bmatrix}$

(A) $\begin{bmatrix} -9 & 13 \\ 14 & -20 \end{bmatrix}$ (B) $\begin{bmatrix} -9 & 13 \\ 16 & -35 \\ -4 & 24 \end{bmatrix}$

(C) $\begin{bmatrix} -9 & 13 & -4 \\ 14 & -20 & 24 \end{bmatrix}$ (D) $\begin{bmatrix} -9 & 13 & 16 \\ 14 & -20 & -35 \\ -4 & 24 & 0 \end{bmatrix}$ (E) NOTA

8. Evaluate: $\begin{bmatrix} 2 & -2 & 3 \\ 0 & 4 & -1 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 7 & 2 \\ 1 & -1 \end{bmatrix}$

(A) $\begin{bmatrix} 7 & 17 \\ 9 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & -17 \\ 9 & 9 \end{bmatrix}$ (C) $\begin{bmatrix} -7 & 10 \\ 27 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} -17 & -7 \\ 27 & 9 \end{bmatrix}$ (E) NOTA

9. Evaluate: $\begin{bmatrix} 3 & -2 & 4 \\ 1 & b & -3 \\ -1 & 3 & 0 \end{bmatrix}\begin{bmatrix} -2 & 3 & 1 \\ 0 & 2 & -4 \\ -1 & 1 & a \end{bmatrix}$

(A) $\begin{bmatrix} -10 & 9 & 11+4a \\ 1+3b & 2b & 1-12ab \\ 2 & 3 & -13+a \end{bmatrix}$ (B) $\begin{bmatrix} -10 & 9 & 11+4a \\ 1 & 2b & 1-3a-4b \\ 2 & 3 & -13 \end{bmatrix}$

(C) $\begin{bmatrix} -10 & 9 & 11+4a \\ 1+3b & 2b & 1-7ab \\ 2 & 3 & -13+a \end{bmatrix}$ (D) $\begin{bmatrix} -10 & 9 & 11+4a \\ 1+3b & 2b & 1-3a-4b \\ 2 & 3 & -13+a \end{bmatrix}$ (E) NOTA

10. There exist matrices *A*, *B*, and *C* such that $A = BC$. If $A = \begin{bmatrix} -13 & -3 \\ -10 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$, what is the product of the elements in *C*?

(A) 0 (B) -120 (C) 160 (D) 180 (E) NOTA

11. Evaluate: $\begin{vmatrix} 2 & -2 \\ 3 & 5 \end{vmatrix}$

(A) 4 (B) 8 (C) 16 (D) 32 (E) NOTA

12. Evaluate: $\begin{vmatrix} -2 & -3 & 1 \\ 7 & 4 & 0 \\ -3 & 2 & 3 \end{vmatrix}$

(A) 55 (B) 60 (C) 65 (D) 70 (E) NOTA

13. Evaluate: $\begin{vmatrix} b & 4 & 1 \\ c & -2 & a \\ -1 & 2 & 0 \end{vmatrix}$

(A) –2 – 4a – 2b + 2c (B) –2 – 4a + 2c
(C) –2 – 4a – 2ab + 2c (D) –2 – 2ab + 2c (E) NOTA

14. What is the order of this matrix: $\begin{bmatrix} 1 & -4 & -2 & 0 \\ 5 & 0 & -3 & -1 \\ -3 & 1 & -2 & 7 \end{bmatrix}$?

(A) 4 x 3 (B) 3 x 4 (C) 12 (D) 10 (E) NOTA

15. Given: $\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$, determine $x + y$.

(A) -7 (B) -5 (C) 5 (D) 7 (E) NOTA

16. If $\begin{vmatrix} 4 & 3 \\ a & -3 \end{vmatrix} = \begin{vmatrix} -1 & 2 \\ 7 & 5 \end{vmatrix}$, what is the value of a?

(A) -4 (B) $\frac{5}{3}$ (C) 2 (D) $\frac{7}{3}$ (E) NOTA

17. Determine the sum of the elements in: $\begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix}^{-1}$

(A) -3 (B) -2 (C) 1 (D) 4 (E) NOTA

18. Determine the sum of the elements in: $\begin{bmatrix} 1 & -2 & 5 & 0 \\ 3 & 0 & 4 & -1 \\ -3 & 7 & -2 & 4 \end{bmatrix}^{-1}$

(A) -3 (B) -2 (C) 4 (D) 11 (E) NOTA

19. Which of the following matrices is singular?

(A) $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & -4 \\ 3 & -6 \end{bmatrix}$ (E) NOTA

20. What is the transpose of $\begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}$?

(A) $\begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$ (E) NOTA

21. Determine the cofactor of the element in row 2, column 2 of the matrix $\begin{bmatrix} 0 & -3 \\ 4 & 2 \end{bmatrix}$.

(A) -12 (B) 0 (C) 2 (D) 4 (E) NOTA

22. The system of equations

$$-x+4y+2z=517$$
$$3x-z=-112$$
$$2x-Ay+6z=121$$

is satisfied by exactly one ordered triple (x, y, z). Which value(s) of A is/are not possible?

(A) A = 10 (B) A = -16 (C) $A\in\{10,-16\}$ (D) $-16<A<10$ (E) NOTA

23. The system of equations

$$2Ax+4y=517+A$$
$$2x+Ay=121$$

is satisfied by exactly one ordered pair (x, y). Which value(s) of A is/are not possible?

(A) $A=\pm 2$ (B) $A=2$ (C) $A\le 2$ (D) $|A|>2$ (E) NOTA

24. Evaluate: $\begin{vmatrix} 4 & 3 & 2 & 1 \\ -1 & 2 & 3 & 4 \\ -4 & -3 & 2 & 1 \\ -1 & -2 & -3 & 4 \end{vmatrix}$

(A) 292 (B) 300 (C) 308 (D) 316 (E) NOTA

25. Which of the following matrices has a determinant different from all the others?

(A) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (B) $\begin{bmatrix} c & d \\ -a & -b \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & a & b \\ -2 & c & d \end{bmatrix}$ (D) $\begin{bmatrix} c & d \\ a & b \end{bmatrix}$ (E) NOTA

26. If A and B are 2 by 2 matrices whose (i,j)th elements are $2i+j$ and $i-j$ respectively, what is the sum of the largest and smallest entries in AB?

(A) 0 (B) 1 (C) 4 (D) 9 (E) NOTA

27. What is the inverse of $\begin{bmatrix} 3 & a \\ -1 & 2 \end{bmatrix}$?

(A) $\begin{bmatrix} \frac{1}{3} & \frac{1}{a} \\ -1 & \frac{1}{2} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{2}{a+6} & \frac{-a}{a+6} \\ \frac{1}{a+6} & \frac{3}{a+6} \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 \\ -a & 3 \end{bmatrix}$ (D) $\begin{bmatrix} \frac{2a-2}{a+6} & \frac{-3a}{a+6} \\ \frac{-2a+1}{a+6} & \frac{3}{2a+12} \end{bmatrix}$ (E) NOTA

28. What values of x will satisfy the inequality $\begin{vmatrix} 2 & x \\ 3x-1 & -2 \end{vmatrix} > -8$?

(A) $-1 < x < \frac{4}{3}$ (B) $-\frac{4}{3} < x < 1$

(C) $x < -1$ or $x > \frac{4}{3}$ (D) $x < -\frac{4}{3}$ or $x > 1$ (E) NOTA

29. Which of the following statements is/are true about matrix addition?

I. Matrices which are added together must have the same order.
II. The closure property is true for matrix addition.
III. The commutative property is true for matrix addition.
IV. Every matrix has an additive inverse.

(A) I only (B) I and IV only
(C) I, III, and IV only (D) All of the Above (E) NOTA

30. Which of the following statements is/are true about matrix multiplication?

I. Matrices which are multiplied together must have the same order.
II. The closure property is true for multiplication of square matrices of the same order.
III. The commutative property is true for matrix multiplication.
IV. Every matrix has a multiplicative inverse.

(A) I only (B) II only (C) II & III only (D) None of them (E) NOTA

31. Jim desperately want to know how much money his older brother Joe has in his piggybank, but instead of telling Jim, Joe gives him a puzzle. Joe says he only has nickels and dimes, and that if you add twice the number of nickels to five times the number of dimes, you'll get 15. Also, he says that if Jim adds six times the number of nickels to the number of dimes multiplied by Joe's present age, Jim will get 120. As he leaves the room with Jim thinking furiously, Joe laughs and says he lied… the puzzle he gave Jim can't even be solved! How old is Joe?

(A) 14 (B) 15 (C) 16 (D) 17 (E) NOTA

32. Harry was shown the solution of x and y (shown below) for a system of equations using Cramer's Rule. From this information, he was able to set up the solution for z. What value did he get for z?

$$x = \frac{\begin{vmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \\ -1 & 1 & 1 \end{vmatrix}}$$

(A) 1 (B) -1 (C) 3 (D) -4 (E) NOTA

33. A matrix M is reduced by a series of elementary row operations to the matrix $3I$, where I is the identity matrix. Applying the same sequence of row operations to $5I$ will yield what matrix?

(A) $125M^{-1}$ (B) $125M^{5}$ (C) $15M^{-1}$ (D) $15M^{T}$ (E) NOTA

34. Suppose the 3x3 matrix A has a determinant of 4. What is the determinant of $3A$?

(A) 324 (B) 108 (C) 36 (D) 12 (E) NOTA

35. What is the element in the third row, second column of the adjoint of the matrix $\begin{bmatrix} 3 & 0 & -2 \\ 5 & 4 & 1 \\ -1 & 1 & -3 \end{bmatrix}$?

(A) 5 (B) 4 (C) -3 (D) -2 (E) NOTA

36. Which of the following are eigenvectors of the matrix $\begin{bmatrix} -1 & 2 \\ 3 & -2 \end{bmatrix}$?

I. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ II. $\begin{bmatrix} -2 \\ 2 \end{bmatrix}$ III. $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ IV. $\begin{bmatrix} 4 \\ -4 \end{bmatrix}$

(A) I & III only (B) I & IV only (C) II & IV only (D) I, II, & IV only (E) NOTA

37. What is the sum of the eigenvalues of the matrix $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$?

(A) 4 (B) 3 (C) 2 (D) 1 (E) NOTA

38. What is the determinant of A^{-1} if $A = \begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 2 & 1 & 4 \\ -3 & -2 & 0 & 2 \\ -1 & 0 & -2 & x \end{bmatrix}$?

(A) $\frac{1}{11x+160}$ (B) $\frac{11x+160}{4x-7}$ (C) $\frac{4x-7}{11x+160}$ (D) $4x-7$ (E) NOTA

39. If transformation matrix M is such that $M \times \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ z \\ x \\ w \\ v \end{bmatrix}$ for all values of v, w, x, y, and z, then what is $M^T \times \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix}$?

(A) $\begin{bmatrix} z \\ y \\ x \\ v \\ w \end{bmatrix}$ (B) $\begin{bmatrix} x \\ y \\ z \\ w \\ v \end{bmatrix}$ (C) $\begin{bmatrix} y \\ z \\ x \\ w \\ v \end{bmatrix}$ (D) $\begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix}$ (E) NOTA

40. At Carol's restaurant you have the best gravy you've ever tasted. You beg her for the recipe, but she refuses to give it away. She does give you the following information, however:

I make one gallon of gravy at a time.
I measure all my ingredients in gallons.
My gravy contains only drippings, wine, and milk.
The amount of wine is three times the amount of drippings minus the amount of milk.
The amount of drippings is equal to the amount of wine minus the amount of milk.

How many gallons of wine are in a gallon of gravy?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) Not enough information (E) NOTA