Mu Alpha Theta National Convention: Denver, 2001
Number Theory Topic Test – Theta Division

1. For how many ordered pairs of integers (m,n) does m multiplied by n equal 30?

   (A) 3 (B) 4 (C) 8 (D) 16 (E) NOTA

2. Find the sum of the positive proper integral factors of 48.

   (A) 76 (B) 80 (C) 112 (D) 240 (E) NOTA

3. In how many consecutive zeros does the number 134! end?

   (A) 26 (B) 31 (C) 32 (D) 37 (E) NOTA

4. What is the smallest counting number with exactly 12 positive integral factors?

   (A) 60 (B) 72 (C) 84 (D) 90 (E) NOTA

5. What is the smallest positive integer with exactly 12 positive integral factors which is not divisible by 3?

   (A) 136 (B) 140 (C) 160 (D) 220 (E) NOTA

6. Which of the following is equal to 311_8?

   (A) 111001001_2 (B) 1010101_2 (C) 11001001_2 (D) 1001001_2 (E) NOTA

7. How many positive prime numbers are divisors of 222,222?

   (A) 3 (B) 4 (C) 5 (D) 6 (E) NOTA

8. If 543_6 is equal to 179_n, what is n?

   (A) 9 (B) 10 (C) 11 (D) 12 (E) NOTA

9. Which of the following numbers is congruent to 1 (mod 3)?

   (A) 330 (B) 331 (C) 332 (D) 333 (E) NOTA

10. Find the largest positive integer, n, such that 3^n divides 311 factorial?

    (A) 103 (B) 152 (C) 153 (D) 155 (E) NOTA

11. Find the sum of the smallest 100 counting numbers that are not perfect squares.

(A) 3,080 (B) 5,720 (C) 6,105 (D) 6,250 (E) NOTA

12. When the digits of a positive two-digit integer are reversed, the resulting number is 36 more than the original number. Find the difference when the tens digit of the original number is subtracted from the units digit of the original number.

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

13. What is the sum of the digits of the base 9 representation of 2001?

(A) 16 (B) 17 (C) 18 (D) 20 (E) NOTA

14. What is the sum of the positive integral factors of 84?

(A) 112 (B) 128 (C) 224 (D) 432 (E) NOTA

15. If N is a positive integer and $N \equiv 2 \pmod 3$ and $N \equiv 1 \pmod 2$, what is the remainder when N is divided by 6?

(A) 5 (B) 3 (C) 2 (D) 1 (E) NOTA

16. What is the smallest positive integer that is a multiple of 13 and one more than a multiple of 7?

(A) 39 (B) 52 (C) 65 (D) 78 (E) NOTA

17. Which of the following four numbers is relatively prime with all of the other three?

(A) 221 (B) 1,001 (C) 1,728 (D) 2,737 (E) NOTA

18. Find the sum of the first 100 even positive integers that are not multiples of 4.

(A) 10,000 (B) 10,100 (C) 15,150 (D) 20,000 (E) NOTA

19. If $10a \equiv 1 \pmod{13}$, what is $17a$ congruent to (mod 13)?

(A) 1 (B) 3 (C) 9 (D) 12 (E) NOTA

20. The 4-digit number $6A6B$ is divisible by 72. What is the sum of the possible values of A?

(A) 2 (B) 7 (C) 9 (D) 11 (E) NOTA

21. What is the product of the four smallest positive prime numbers that are each congruent to 2 (mod 5)?

(A) 8,806 (B) 11,186 (C) 24,346 (D) 59,126 (E) NOTA

22. If $3x \equiv 4 \pmod 5$, and $5x \equiv 6 \pmod 7$, which of the following could be x?

(A) 19 (B) 34 (C) 53 (D) 630 (E) NOTA

23. Find the sum of all the positive even factors of 1,728.

(A) 4,999 (B) 5,040 (C) 5,041 (D) 5,044 (E) NOTA

24. What is the smallest positive integer which is one less than a multiple of each of the integers 2 through 10?

(A) 209 (B) 839 (C) 629 (D) 2519 (E) NOTA

25. What is the sum of the 10 smallest positive perfect cubes?

(A) 2,916 (B) 3,000 (C) 3,025 (D) 3,850 (E) NOTA

26. The sum of the first n counting numbers is equal to S where S is a multiple of 183. What is the smallest possible value for n?

(A) 60 (B) 61 (C) 182 (D) 183 (E) NOTA

27. Find sum of the products of each pair of twin primes where each number in the pair is less than 50.

(A) 2,279 (B) 3,178 (C) 4,478 (D) 4,621 (E) NOTA

28. In the Battle of the Tentacled Eye, 20 humans killed a total of 242 aliens. One of the humans, Pat B., killed more aliens than any of the other humans. What is the smallest number of aliens that Pat could have killed?

(A) 12 (B) 13 (C) 14 (D) 15 (E) NOTA

29. What is the tens digit of 7^{707}?

(A) 0 (B) 4 (C) 7 (D) 9 (E) NOTA

30. Which of the following numbers is divisible by 99?

(A) 5,256 (B) 7,018 (C) 18,623 (D) 32,571 (E) NOTA

31. What is the smallest possible positive difference between two integers whose product is 9,984?

(A) 2 (B) 4 (C) 6 (D) 8 (E) NOTA

32. What is the least common multiple of 297, 481, and 672?

(A) 32,032 (B) 31,999,968 (C) 864,864 (D) 63,999,936 (E) NOTA

33. Find the product of the positive integral divisors of 40.

(A) 40^4 (B) 40^6 (C) 40^8 (D) 40^{16} (E) NOTA

34. The sum of the first N positive perfect squares is a multiple of 41. What is the smallest possible value of N?

(A) 19 (B) 20 (C) 40 (D) 41 (E) NOTA

35. What is the remainder when 5^{301} is divided by 8?

(A) 1 (B) 3 (C) 5 (D) 7 (E) NOTA

36. What is the smallest positive integer that is a multiple of 4 and has no digit greater than 1 when expressed in base 5?

(A) 36 (B) 56 (C) 152 (D) 156 (E) NOTA

37. If $M \equiv 2 \pmod 4$ and $N \equiv 8 \pmod{16}$, what is the remainder when the product of M and N is divided by 32?

(A) 0 (B) 8 (C) 16 (D) 24 (E) NOTA

38. N is a positive integer with no prime factor greater than 3. How many numbers could N be if N has less than 10 positive integral factors?

(A) 23 (B) 24 (C) 25 (D) 27 (E) NOTA

39. If M is the least common multiple of the first 20 counting numbers, how many positive integers are factors of M?

(A) 960 (B) 1,120 (C) 1,200 (D) 1,728 (E) NOTA

40. What is the remainder when 337,500,000 is divided by 128?

(A) 0 (B) 32 (C) 64 (D) 96 (E) NOTA