Mu Alpha Theta National Convention: Denver, 2001

Probability Topic Test – Theta Division

1. Consider rain and wind as independent events. The probability of rain is $\frac{1}{2}$, and the probability of wind is $\frac{1}{2}$. What is the probability of both rain and wind?

   (A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$ (E) NOTA

2. In conditions of problem 1 above, what is the probability of either rain or wind (or both)?

   (A) 1 (B) $\frac{7}{16}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$ (E) NOTA

3. On planet Morg a fair 5-sided die can be constructed. If two such dice are labeled with the numbers 1-5 and tossed, what is the probability the sum of the numbers displayed is 6?

   (A) $\frac{1}{12}$ (B) $\frac{1}{5}$ (C) $\frac{3}{10}$ (D) $\frac{4}{25}$ (E) NOTA

4. The probability the Mariners win any baseball game is $\frac{2}{3}$. What is the probability they win a seven-game series in exactly seven games? To win a seven-game series, a team must win four of the seven games. The series ends once this condition is met, even if all seven games have not been played.

   (A) $\frac{1}{7}$ (B) $\frac{10}{21}$ (C) $\frac{320}{2187}$ (D) $\frac{160}{729}$ (E) NOTA

5. Wally's Widget Works sends out a shipment of 72 individually packaged widgets, 6 of which are defective. If Ike the Inspector opens four packages, what is the probability at least one defective widget is found?

   (A) $\frac{137}{852}$ (B) $\frac{347}{1491}$ (C) $\frac{489}{1633}$ (D) $\frac{10}{71}$ (E) NOTA

6. Two cards are drawn from a standard 52-card deck. What is the probability the cards are of the same color (black or red)?

   (A) $\frac{10}{17}$ (B) $\frac{26}{51}$ (C) $\frac{25}{51}$ (D) $\frac{8}{17}$ (E) NOTA

7. Jim has four blue shirts, two red shirts, and three green shirts. He also owns two pairs of jeans, one pair of khakis, and three pairs of shorts. Fred has four red shirts, six brown shirts, and seven white shirts, as well as four pairs of shorts and six pairs of sweats. Jim and Fred each choose an outfit at random; what is the probability their choices are identical (in terms of color of shirts and style of legwear)?

(A) $\frac{1}{4}$ (B) $\frac{8}{765}$ (C) $\frac{1}{85}$ (D) $\frac{14}{765}$ (E) NOTA

8. Eight students are seated in a row; what is the probability Mary and Susan sit next to each other?

(A) $\frac{1}{120}$ (B) $\frac{3}{120}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) NOTA

9. Five people at a conference are seated at a circular table. After lunch, they reconvene, sitting around the same table. What is the probability they are seated in the same order around the table after lunch as they were before lunch, assuming their seating order to be random?

(A) $\frac{1}{6}$ (B) $\frac{1}{120}$ (C) 1 (D) $\frac{1}{24}$ (E) NOTA

10. The probability of an American winning a medal in any one event at a track and field meet is 2/3. If a meet has 10 events, what is the probability an American wins a medal in every event?

(A) $\frac{1{,}024}{59{,}049}$ (B) $\frac{381}{45{,}934}$ (C) $\frac{2{,}048}{177{,}147}$ (D) $\frac{8{,}191}{106{,}288}$ (E) NOTA

11. A number is chosen at random from among the 30 smallest natural numbers. What is the probability it is a prime number?

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$ (C) $\frac{3}{10}$ (D) $\frac{1}{6}$ (E) NOTA

12. The probability of Josh having an asthma attack during a tennis match is $\frac{1}{2}$. If he has such an attack his probability of winning is $\frac{1}{8}$, otherwise it is $\frac{6}{7}$. What is the probability of Josh winning any tennis match?

(A) $\frac{55}{112}$ (B) $\frac{7}{15}$ (C) $\frac{97}{198}$ (D) $\frac{1}{2}$ (E) NOTA

13. Given that $f(x)=7x$, if a real value of *x* is chosen at random between -2 and 2 inclusive, what is the probability that $f(x)>0$?

(A) $\frac{1}{3}$ (B) 1 (C) $\frac{1}{7}$ (D) $\frac{1}{2}$ (E) NOTA

14. On Tom's drive to work, each traffic light he must wait at costs him two minutes. His drive passes eight traffic lights. Without any traffic lights the drive would take him two minutes. His probability of having to wait at any given light is $\frac{1}{3}$. What is his probability of reaching work exactly ten minutes after departure?

(A) $\frac{448}{2{,}187}$ (B) $\frac{4{,}980}{21{,}773}$ (C) $\frac{1{,}120}{6{,}561}$ (D) $\frac{6{,}530}{21{,}773}$ (E) NOTA

15. A bag contains seven blue marbles and three green marbles. When two marbles are removed simultaneously, what is the probability they are different colors?

(A) $\frac{4}{9}$ (B) $\frac{7}{15}$ (C) $\frac{2}{5}$ (D) $\frac{5}{9}$ (E) NOTA

16. Ten fair coins are tossed. What is the probability exactly 4 faces display heads?

(A) $\frac{91}{512}$ (B) $\frac{105}{512}$ (C) $\frac{63}{256}$ (D) $\frac{81}{512}$ (E) NOTA

17. One fair coin is tossed nine consecutive times. What is the probability no two heads or tails appear consecutively?

(A) $\frac{1}{128}$ (B) $\frac{1}{256}$ (C) $\frac{3}{64}$ (D) $\frac{3}{256}$ (E) NOTA

18. A dartboard is made up of 3 concentric circles of radii 2, 5, and 10 inches. A dart thrown will strike the board in a random location. What is the probability it will strike within the innermost circle?

(A) $\frac{1}{4}$ (B) $\frac{\pi}{25}$ (C) $\frac{\pi}{16}$ (D) $\frac{1}{25}$ (E) NOTA

19. Mike and Jim are pirates on a ship with a crew of nine. Four crewmembers must be sent out in a rowboat to an island, and lots are drawn to see who must go. What is the probability neither Mike nor Jim will be forced to go?

(A) $\frac{5}{18}$ (B) $\frac{1}{9}$ (C) $\frac{7}{12}$ (D) $\frac{7}{18}$ (E) NOTA

20. Eight students will be placed randomly in ten desks in a row. What is the probability no two empty desks will be adjacent?

(A) $\frac{1}{3}$ (B) $\frac{2}{11}$ (C) $\frac{4}{5}$ (D) $\frac{3}{11}$ (E) NOTA

21. Twenty chocolate chips considered to have negligible volume are mixed randomly into 32 ounces of cookie dough. The dough is then randomly divided into eight four-ounce cookies. If one cookie is selected at random what is the probability (to the nearest ten-thousandth) it will have zero chips?

(A) 0.0448 (B) 0.0139 (C) 0.0406 (D) 0.0692 (E) NOTA

22. A plane crash has one survivor, considered to have negligible size. He can survive 36 hours in cold ocean water without having to eat his disgusting survival rations. He is somewhere within a search radius of 4 miles. The Coast Guard can search 2 square miles per 24-hour day. What is the probability the survivor is found before he has been forced to eat his rations?

(A) $\frac{1}{3}$ (B) $\frac{1}{\pi}$ (C) $\frac{\pi}{6}$ (D) $\frac{3}{16\pi}$ (E) NOTA

23. When one of Paul's "friends" receives an invitation to a party he is throwing, their probability of attending is 1/2. If Paul invites 15 people, what is the probability at least 10 attend?

(A) $\frac{307}{2048}$ (B) $\frac{77}{512}$ (C) $\frac{309}{2048}$ (D) $\frac{155}{1024}$ (E) NOTA

24. Five blue socks and five red socks are in drawer *A*. Three blue socks and seven red socks are in drawer *B*. One sock is drawn at random from drawer *A* and placed in drawer *B*. If two socks are then drawn at random from drawer *B*, what is the probability the two form a matching pair?

(A) $\frac{29}{55}$ (B) $\frac{3}{11}$ (C) $\frac{28}{55}$ (D) $\frac{13}{25}$ (E) NOTA

25. Carlton chooses the vertices of a triangle randomly from a set of 18 points in space, no three of which are collinear. Brian makes the same type of random choice. What is the probability they end up with the same triangle?

(A) $\frac{1}{815}$ (B) $\frac{1}{816}$ (C) $\frac{1}{817}$ (D) $\frac{1}{818}$ (E) NOTA

26. Sean is stuck on the last problem of his test. At the end of each one-minute interval, he has a $\frac{1}{2}$ probability of suddenly realizing the answer. What is the probability he has the answer after five and a half minutes?

(A) $\frac{242}{243}$ (B) $\frac{63}{64}$ (C) $\frac{15}{16}$ (D) $\frac{31}{32}$ (E) NOTA

27. If $f(x)=3x^2$ and a real value of x is chosen at random between 0 and 3 inclusive, what is the probability that $f(x)<2$?

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{6}}{9}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{1}{3}$ (E) NOTA

28. A game of poker is played in which five cards are dealt from a standard, 52-card deck. What is the probability (to four significant figures) that a given player is dealt a straight flush (five consecutive cards in the same suit, e.g. 2-3-4-5-6 of hearts or 10-J-Q-K-A of clubs)? Note that an ace cannot be the low card of a straight, nor can a straight "wrap around" the ends of the spectrum (e.g. A-2-3-4-5 and K-A-2-3-4 are not legal straights).

(A) $1.295\text{x}10^{-5}$ (B) $1.366\text{x}10^{-5}$ (C) $1.385\text{x}10^{-5}$ (D) $1.440\text{x}10^{-5}$ (E) NOTA

29. When one fair six-sided die is cast 3 times, what is the probability the numbers are in descending order? Equal numbers cannot be placed in descending order.

(A) $\frac{5}{54}$ (B) $\frac{1}{108}$ (C) $\frac{5}{432}$ (D) $\frac{1}{72}$ (E) NOTA

30. Bob and Jane will each arrive at the train station at random times between one and four o'clock. Bob will wait 15 minutes; Jane will wait 30 minutes. What is the probability they will be at the station at the same time?

(A) $\frac{1}{4}$ (B) $\frac{63}{181}$ (C) $\frac{65}{288}$ (D) $\frac{67}{288}$ (E) NOTA

31. On the face of one of five slips of paper an "X" is placed. The slips of paper are then placed face down on a table. James, John, Jack, and Joe are seated around the table. They will try to grab the slips of paper at an appointed time, and any distribution of the slips is equally likely, though all slips will be grabbed. The person with the "X" gets a million dollars. What is the probability Joe gets the million dollars?

(A) $\frac{71}{512}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{49}{240}$ (E) NOTA

32. Sally's extension school offers three subjects: English, Science, and History. A student may be enrolled in one, two, or three subjects. Eighteen are enrolled in Science and English, twelve in History and English, and ten in History and Science. 200 total students are enrolled, with 174 taking exactly one subject. If a student is chosen at random, what is the probability he is enrolled in all three subjects?

(A) $\frac{7}{200}$ (B) $\frac{1}{25}$ (C) $\frac{3}{100}$ (D) $\frac{1}{40}$ (E) NOTA

33. In a horse race, the odds are 17:1 against "Fat Tom" placing. In another horse race, the odds are 13:1 against "Big Daddy" placing. What is the probability that neither horse places?

(A) $\frac{14}{15}$ (B) $\frac{221}{252}$ (C) $\frac{31}{252}$ (D) $\frac{1}{15}$ (E) NOTA

34. The probability an event has occurred before a given time t is given by $P(t)=\frac{t^2}{t^2+1}$, where $t \geq 0$. What is the probability the event has not occurred by $t = 5$?

(A) $\frac{1}{25}$ (B) $\frac{1}{26}$ (C) $\frac{24}{25}$ (D) $\frac{25}{26}$ (E) NOTA

35. One of the smallest 200 natural numbers is chosen at random. What is the probability it has only 3 factors?

(A) $\frac{3}{50}$ (B) $\frac{2}{25}$ (C) $\frac{1}{10}$ (D) $\frac{3}{100}$ (E) NOTA

36. The probability of event *A* occurring if event *B* occurs is $\frac{4}{7}$. The probability event *B* occurs if event *A* occurs is $\frac{4}{5}$. The probability of both events occurring is $\frac{1}{3}$. What is the probability of neither event occurring?

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{5}{12}$ (D) $\frac{1}{3}$ (E) NOTA

37. In the conditions of problem 36 above, what is the probability event *A* occurs?

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{5}{12}$ (D) $\frac{1}{3}$ (E) NOTA

38. A real number, *A*, is chosen randomly between 5 and 10, inclusive. Another real number, *B*, is chosen randomly between 0 and 10. What is the probability that $A + B < 10$?

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) NOTA

39. An apartment building has 3 units with separate mailboxes. One letter is sent to each unit, but the mailman is in a hurry and thus places one letter in each box at random. What is the probability all 3 letters are placed in the correct boxes?

(A) $\frac{1}{120}$ (B) $\frac{1}{6}$ (C) $\frac{1}{36}$ (D) $\frac{1}{12}$ (E) NOTA

40. Three points are on vertices of an 8x8 grid of unit squares. Point *A* is in the upper left corner, point *B* is three units below and five units to the right of *A*, and point *C* is in the lower right corner (eight units below and eight units to the right of point *A*). If a path is drawn from *A* to *C* along the gridlines and can only be drawn going down or to the right, what is the probability it passes through point *B*?

(A) $\frac{174}{175}$ (B) $\frac{1{,}567}{6{,}435}$ (C) $\frac{1{,}568}{6{,}435}$ (D) $\frac{523}{2{,}145}$ (E) NOTA