Mu Alpha Theta National Convention: Denver, 2001
Sequences & Series Topic Test – Theta Division

1. The common difference of the arithmetic sequence 2, 7, 12, … is

   (A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

2. What is the common ratio of the geometric sequence 3, 6, 12, …?

   (A) 2 (B) 3 (C) 6 (D) 9 (E) NOTA

3. What is the sum of the first 28 odd positive integers?

   (A) 4656 (B) 210 (C) 784 (D) 812 (E) NOTA

4. Find the sum of the first 30 even natural numbers.

   (A) 1800 (B) 900 (C) 1830 (D) 930 (E) NOTA

5. $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 =$

   (A) 77 (B) 56 (C) 87 (D) 91 (E) NOTA

6. A prime of the form $2^{2^n} + 1$ where *n* is a nonnegative integer is called a **Fermat Prime** (Pierre de Fermat conjectured that all integers of this form are prime, a claim Leonhard Euler showed to be false). Find the sum of the four smallest Fermat Primes.

   (A) 346 (B) 282 (C) 25 (D) 65816 (E) NOTA

7. $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}} =$

   (A) 3 (B) -2 (C) 5 (D) 7 (E) NOTA

8. A sequence is defined explicitly by $a_n = 4n + 7(-1)^n$. What is the value of $a_2 + a_4$?

   (A) 66 (B) 10 (C) 38 (D) 27 (E) NOTA

9. A storeowner is setting up an impressive display of spinach cans to help with sales. The cans are to be placed on a stack with 8 levels, each level having one more can than the one above it. Half of the cans in stock will be used in his display, with 3 cans on the top level. How many cans does he have in stock?

   (A) 52 (B) 64 (C) 112 (D) 104 (E) NOTA

10. Evaluate $\sum_{n=1}^{k^2} n$.

(A) $\frac{\left(\sqrt{k}+1\right)\sqrt{k}}{2}$ (B) $\left(\frac{k(k+1)}{2}\right)^2$

(C) $\frac{k(k+1)(2k+1)}{6}$ (D) $\frac{k^4+k^2}{2}$ (E) NOTA

11. Find the ordered pair (a, b, c) so that 5, a, b, c, 21 forms an arithmetic sequence.

(A) (6, 13, 20) (B) (12, 13, 14) (C) (9, 13, 17) (D) (9, 15, 20) (E) NOTA

12. A private university hires Richard for $80,000 a year. He receives an annual increase in salary of $1,500. What will his salary be during his fifteenth year at the university?

(A) $99,500 (B) $163,500 (C) $357,500 (D) $101,000 (E) NOTA

13. The sum of fifty terms of an arithmetic series is 4025. If the first term is 7, what is the 49th term?

(A) $\frac{505}{7}$ (B) 160 (C) 151 (D) $\frac{147}{2}$ (E) NOTA

14. Three numbers are in a geometric sequence. The sum of the three numbers is 147 and their product is 21952. Find the sum of the squares of the three numbers.

(A) 10955 (B) 13377 (C) 8417 (D) 11649 (E) NOTA

15. Two arithmetic sequences have the same first and last term. Their sums differ by the sum of their first and last term. How many more terms does the sequence with the larger sum have?

(A) 3 (B) 2 (C) 1 (D) 0 (E) NOTA

16. Find the 51st term of the arithmetic sequence $3x$, $6x + 1$, $9x + 2$, …

(A) $153x + 50$ (B) $103x + 50$ (C) $51 + 156x$ (D) $49 + 101x$ (E) NOTA

17. A man sends out four letters on Saturday, with instructions to the recipients to write letters to two friends on the following Saturday, asking them to do likewise. There are no duplications and no one breaks the chain. How many total letters have been sent, up to and including the sixth Saturday?

(A) 252 (B) 2730 (C) 5460 (D) 504 (E) NOTA

18. The 10^{th} term of an arithmetic sequence is 27. The 20^{th} term is 12. Find the 40^{th} term.

(A) $-\frac{3}{2}$ (B) -18 (C) 15 (D) 18 (E) NOTA

19. Find the number of terms in the finite geometric series $6+2+\frac{2}{3}+\ldots+\frac{2}{387420489}$.

(A) 20 (B) 19 (C) 18 (D) 17 (E) NOTA

20. For the recursive sequence $a_n = a_{n-1} + n^2$, where $a_0 = 0$, find a_{20}.

(A) 1540 (B) 44100 (C) 3890 (D) 2870 (E) NOTA

21. The sum of the first ten terms of an arithmetic sequence with first term *a* and common difference *b* is 1150. The sum of the next ten terms of the same sequence is 3550. How long is the hypotenuse of a right triangle having legs of length *a* and *b*?

(A) 41 (B) 13 (C) 25 (D) 10 (E) NOTA

22. The fifth term of an arithmetic sequence is 4 and the *x*th term is 104, where $x > 5$. Given that the common difference of this sequence is an integer, how many possible values are there for *x*?

(A) 8 (B) 4 (C) 18 (D) 9 (E) NOTA

23. Due to its soaring popularity, the cast of the hit sitcom *Acquaintances* demand that they receive a 50% raise in their salary for each year the show stays on the air. If a particular cast member earns one million dollars per year in the first year, what is his salary in the fifth year? Express your answer in hundreds of dollars.

(A) 160000 (B) 113906 (C) 50625 (D) 75937 (E) NOTA

24. What is the sum of all 5-digit palindromes? Express your answer in scientific notation.

(A) 4.54×10^7 (B) 4.13×10^7 (C) 4.01×10^7 (D) 4.95×10^7 (E) NOTA

25. Write $.3\overline{12}_5$ as a base 10 fraction.

(A) $\frac{38}{55}$ (B) $\frac{103}{330}$ (C) $\frac{103}{40}$ (D) $\frac{79}{120}$ (E) NOTA

26. A cube is inscribed in a sphere of radius 4. Another sphere is inscribed inside the cube and a second cube is inscribed in this sphere. If this pattern continues, what is the surface area of the 13^{th} sphere?

(A) $\frac{2^6\pi}{3^{12}}$ (B) $\frac{2^5\pi}{3^{11}}$ (C) $\frac{2^4\pi}{3^{10}}$ (D) $\frac{2^3\pi}{3^9}$ (E) NOTA

27. Find the sum of all integers between 100 and 2000 inclusive that are not divisible by 3.

(A) 332850 (B) 1996030 (C) 1331400 (D) 664650 (E) NOTA

28. A ball is dropped from a height of two meters, and bounces $\frac{3}{4}$ of its previous height on each bounce. What is the total distance (in meters) the ball travels (up and down) before coming to rest?

(A) 14 m (B) 12 m (C) 10 m (D) 8 m (E) NOTA

29. If *n* is odd and a_n is an arithmetic sequence with positive terms, find the minimum possible value of $\frac{a_{\frac{n+1}{2}}}{a_3+a_{n-2}}+\frac{a_3+a_{n-2}}{a_1+a_n}+\frac{a_1+a_n}{a_{\frac{n+1}{2}}}$.

(A) 4 (B) $\frac{7}{2}$ (C) 3 (D) $\frac{5}{2}$ (E) NOTA

30. $\frac{4}{5}+\frac{8}{25}+\frac{12}{125}+\ldots=$

(A) 3 (B) $\frac{12}{5}$ (C) $\frac{4}{3}$ (D) $\frac{3}{2}$ (E) NOTA

31. Starting with a circle of radius 1 m, a second circle is formed whose radius in meters is numerically equal to the area (in m²) of the previous circle. If this process is continued, what is the circumference (in meters) of the 24^{th} circle?

(A) $2\pi^{2^{25}-1}$ (B) $2\pi^{2^{24}-1}$ (C) $2\pi^{2^{24}}$ (D) $2\pi^{2^{23}}$ (E) NOTA

32. Patricia and Sean decide to play a game where they take turns rolling two fair, six-sided dice. If the sum of the spots is 7, that person wins. If Patricia rolls first and play continues until someone wins, what's the probability she will win this game?

(A) $\frac{1}{6}$ (B) $\frac{1}{210}$ (C) $\frac{6}{11}$ (D) 1 (E) NOTA

33. Find constants *A* and *B* such that $\frac{1}{x^2-9}=\frac{A}{x-3}+\frac{B}{x+3}$ and use this to evaluate $\sum_{n=4}^{\infty}\frac{1}{n^2-9}$.

(A) $\frac{49}{20}$ (B) $\frac{49}{120}$ (C) $\frac{29}{20}$ (D) $\frac{29}{120}$ (E) NOTA

34. What is the next term of the harmonic sequence beginning: $\frac{24}{5}$, 4, $\frac{24}{7}$

(A) $\frac{12}{5}$ (B) 3 (C) $\frac{22}{7}$ (D) $\frac{28}{9}$ (E) NOTA

35. Find the sum of the 100 smallest natural numbers *n* such that 2^n-1 is divisible by 7.

(A) 5050 (B) 10100 (C) 20200 (D) 15150 (E) NOTA

36. Evaluate: 110 – 112 + 114 – 116 + … + 1002 – 1004 + 1006 – 1008.

(A) -1008 (B) -898 (C) -900 (D) -1218 (E) NOTA

37. A recursive sequence is defined by $a_n=3a_{n-1}+4a_{n-2}-12a_{n-3}$ where $a_0=2$, $a_1=5$, and $a_2=13$. What's the remainder when a_{800} is divided by 10?

(A) 2 (B) 5 (C) 7 (D) 3 (E) NOTA

38. The sum of the first *n* terms of a sequence is $5n-\frac{2}{n}$. Determine the fourth term of the sequence, assuming *n* = 1 represents the first term of the sequence.

(A) $\frac{14}{3}$ (B) 5 (C) $\frac{31}{6}$ (D) $\frac{16}{3}$ (E) NOTA

39. Evaluate $1+\frac{\binom{50}{1}}{4}+\frac{\binom{50}{2}}{4^2}+\ldots+\frac{\binom{50}{50}}{4^{50}}$.

(A) $2^{100}\binom{100}{49}$ (B) $1+50\left(\frac{5}{4}\right)^{50}$ (C) $\frac{5^{50}}{4^{50}}$ (D) $4^{50}\binom{100}{50}$ (E) NOTA

40. Evaluate: $\sum_{n=5}^{\infty}\log_2\left(1-\frac{1}{n}\right)$

(A) $3-\log_2\left(\frac{5}{3}\right)$ (B) $2+\log_2\left(\frac{5}{3}\right)$ (C) 3 (D) 2 (E) NOTA