Alpha Bowl Questions: FAMAT State Convention 2002

1. In $\triangle ABC$ shown, $m \angle B = x$ degrees and $AB = y$ cm.
   To the nearest hundredth place, find the value of $x + y$.

2. A circle of radius 10 has $\triangle ABC$ inscribed with $AB$ a diameter of the circle. If $BC = 5$, then give the measure of $\angle CBA$ to the nearest tenth of a degree.

3. For $f(x) = 6x - 3x^2$ over domain $[0, 3]$, the range is $[a, b]$. Find the value of $a + b$.

4. Consider the sequence given by $a_n = \cos(n \theta)$. Note, $n$ represents degrees.
   To the nearest tenth place, give the value of $\sum_{n=0}^{362} a_n$.

5. The graph of the parabola with equation $f(x) = 4 - x^2$ is reflected over the x-axis. A rectangle $ABCD$ is bounded by the two graphs, such that the vertices of the rectangle are on the graphs as shown. The rectangle has width $(AB)$ that is twice its length and the center of the rectangle is on the origin. Give the length of the diagonal $BD$, to the nearest tenth.

6. For $f(x) = 100e^x$ let $f(A) = 200$, $f(-1) = B$ and $f(c + 1) = 300$. Find the value of $A + B + C$, to the nearest tenth.

7. Let $f(x) = 2e^x$ and $g(x) = \ln(x) + 2$. If $f(g(x)) = 2x \cdot A$ and $g(f(B - 1)) = \ln(2)$ give the exact value of $\frac{A}{e^B}$.

8. A equilateral triangle is divided by three line segments into four smaller congruent equilateral triangles. The perimeter of the large triangle $ABC$ is $3 \cos \theta$ for $0 < \theta < \frac{\pi}{2}$. The area of one of the smaller triangles (shaded) is $\frac{1}{48} \sqrt{3}$. Give the value of $\theta$ in radians to the nearest thousandth place.

9. An ellipse has center $(0, 1)$, foci $(0, 4)$ and $(0, -2)$ and minor axis of length 10. If point $(A, 1)$ lies on the ellipse and $A > 0$ then give the exact value of $A$.

10. The graph of $f(x) = Ax^2 + Bx + C$ has roots $1 \pm \sqrt{2}$ and y-intercept $-4$. The graph of $g(x) = Dx^2 + Bx + E$ has roots $5 \pm \sqrt{3}$. Give the value of $D$.

11. If $\frac{\cos a}{\sin b} = \tan \theta$ for $0 < a < \frac{\pi}{2}$ and $0 < b < \frac{\pi}{2}$. If $0 < \theta < 2\pi$ and $a + \frac{\pi}{2} = b$ then give the least possible radian value of $\theta$.

12. Begin with P=0. If a statement below describes a situation which is possible, add to P the value in parentheses after the statement. An isosceles right triangle has a hypotenuse which is an integer. (1)
   A triangle has sides 3, 4 and 5, and angles of $30^\circ$, $60^\circ$ and $90^\circ$. (3)
   A regular hexagonal pyramid of height $\sqrt{3}$ has lateral faces that are equilateral triangles. (5)
   A triangle $ABC$ has integral length sides of lengths 4, 5 and $x$, such that $8 < x < 12$. (7)
   The sum of the interior angles of a regular n-gon is $990^\circ$. (9)
   A convex pentagon has five diagonals. (11) Give the final value of P.

13. In triangle $ABC$, $AB = 6$ and $m \angle A = 30^\circ$ and $BC = 4$. Give the least possible value of $AC$ to the nearest tenth.

14. Let $a_n = 2a_{n-1} + 6$ and $a(10) = 100$. If $a(k)$ is the greatest value of the sequence that is less than zero, and $a(r)$ is the least value of the sequence that is greater than 600, give the value of $k + r$.

15. Consider vectors $A$ and $B$ given by $3i + 4j$ and $5i - 12j$ respectively. Let $x$ degrees be the measure of the angle between $A$ and $B$, rounded to the nearest degree. Let the vector of length 20 with the same direction as $A$ be $m \hat{i} + n \hat{j}$ and let the dot product $A \cdot B$ be $z$. Give the value of $x + m + n + z$. 