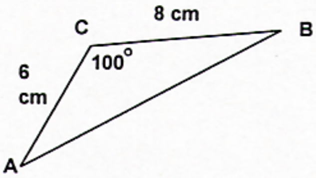
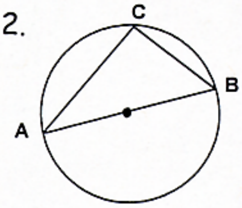
Alpha Bowl Questions: FAMAT State Convention 2002

1. In ΔABC shown, $m\angle B = x$ degrees and $AB = y$ cm.
To the nearest hundredth place, find the value of $x + y$.

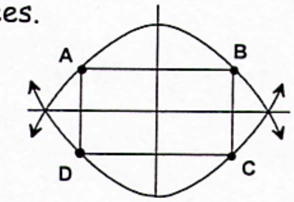


2. 

A circle of radius 10 has ΔABC inscribed with $\overline{AB}$ a diameter of the circle. If BC = 5, then give the measure of $\angle CBA$ to the nearest tenth of a degree.

3. For $f(x) = 6x - 3x^2$ over domain $[0, 3]$, the range is $[a, b]$.
Find the value of $a + b$.

4. Consider the sequence given by $a_n = \cos(n^\circ)$. Note, n represents degrees.
To the nearest tenth place, give the value of $\sum_{n=0}^{362} a_n$.

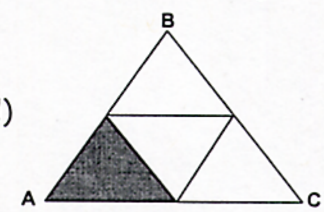


5. The graph of the parabola with equation $f(x) = 4 - x^2$ is reflected over the x-axis. A rectangle ABCD is bounded by the two graphs, such that the vertices of the rectangle are on the graphs as shown. The rectangle has width (AB) that is twice its length and the center of the rectangle is on the origin. Give the length of the diagonal $\overline{BD}$, to the nearest tenth.

6. For $f(x) = 100e^x$ let $f(A) = 200$, $f(-1) = B$ and $f(c+1) = 300$. Find the value of $A + B + C$, to the nearest tenth.

7. Let $f(x) = 2e^x$ and $g(x) = \ln(x) + 2$. If $f(g(x)) = 2x \bullet A$ and $g(f(B-1)) = \ln(2)$ give the exact value of $\frac{A}{e^B}$.

8. A equilateral triangle is divided by three line segments into four smaller congruent equilateral triangles. The perimeter of the large triangle ABC is $3\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The area of one of the smaller triangles (shaded) is $\frac{1}{48}\sqrt{3}$. Give the value of θ in radians to the nearest thousandth place.



not drawn to scale

9. An ellipse has center (0, 1), foci (0, 4) and $(0, -2)$ and minor axis of length 10.
If point (A, 1) lies on the ellipse and $A > 0$ then give the exact value of A.

10. The graph of $f(x) = Ax^2 + Bx + C$ has roots $1 \pm \sqrt{2}$ and y-intercept -4. The graph of $g(x) = Dx^2 + Bx + E$ has roots $5 \pm \sqrt{3}$. Give the value of D.

11. If $\frac{\cos a}{\sin b} = \tan\theta$ for $0 < a < \frac{\pi}{2}$ and $0 < b < \frac{\pi}{2}$. If $0 < \theta < 2\pi$ and $a + \frac{\pi}{2} = b$ then give the least possible radian value of θ.

12. Begin with P=0. If a statement below describes a situation which is <u>possible</u>, add to P the value in parentheses after the statement. An isosceles right triangle has a hypotenuse which is an integer.(1)
A triangle has sides 3, 4 and 5, and angles of $30^\circ, 60^\circ$ and 90°. (3)
A regular hexagonal pyramid of height $\sqrt{3}$ has lateral faces that are equilateral triangles. (5)
A triangle ABC has integral length sides of lengths 4 , 5 and x, such that $8 < x < 12$. (7)
The sum of the interior angles of a regular n-gon is 990°. (9)
A convex pentagon has five diagonals. (11) **Give the final value of P.**

13. In triangle ABC, $AB = 6$ and $m\angle A = 30^\circ$ and $BC = 4$. Give the least possible value of AC to the nearest tenth.

14. Let $a_n = 2a_{n-1} + 6$ and $a(10) = 100$. If $a(k)$ is the greatest value of the sequence that is less than zero, and $a(r)$ is the least value of of the sequence that is greater than 600, give the value of $k + r$.

15. Consider vectors A and B given by $3i + 4j$ and $5i - 12j$ respectively. Let x degrees be the measure of the angle between A and B, rounded to the nearest degree. Let the vector of length 20 with the same direction as A be $mi + nj$ and let the dot product $A \bullet B$ be z. Give the value of $x + m + n + z$.