Solutions: Alpha Team, FAMAT State Convention 2002

1. Use the law of cosines to get AB: $36+64-2(48)\cos 100=(AB)^2$ solves to $AB\approx 10.80139922$.
   Use the law of sines, then to find angle B: $\frac{\sin B}{6}=\frac{\sin 100}{AB}$ to get $B\approx 33.16449$ degrees.
   The sum is approximately **43.97**.
2. $\cos B=\frac{5}{20}$ so $B\approx$**75.5** degrees.
3. Since $3x(2-x)=0$ the roots are x=0 and x=2, vertex is at x=1 or (1, 3) and the maximum is 3. The endpoints of the interval are f(0)=0 and f(3)= -9 so the minimum is -9. The range is therefore [-9, 3] so $a+b=$ **-6**.
4. cos(0)+cos(180)=0, and cos(1)+cos(179)=0, and so on. Also, the same principle works for pairs 181 and 359,182 and 358, etc. This leaves cos361+cos362+cos360. This sum rounds to **3**.
5. Let point B, the vertex in quadrant I be (2a, a). Since this point is one the curve, substitute to get $a=4-(2a)^2$ which solves to $\frac{-1+\sqrt{1-4(-16)}}{2(4)}$ by use of the quadratic formula, knowing the answer is positive. This gives $B(\frac{-1+\sqrt{65}}{4},\frac{-1+\sqrt{65}}{8})$. Use the Pythagorean Theorem to get the distance from the origin to B and then double this for the diagonal which is approximately **3.9** .
6. $200=100e^{A}$ gives $A=\ln 2$. $100e^{-1}=B$ which gives $B=\frac{100}{e}$. $100e^{C+1}=300$ so $C=\ln 3-1$
   $A+B+C$ to the nearest tenth is **37.6** .
7. $f(g(x))=2e^{\ln x+2}=2(e^{\ln x}\bullet e^{2})$ and so $\ln A\bullet x=2x\bullet e^{2}$ and $A=e^{2}$.
   $g(f(B-1))=g(2e^{B-1})=\ln(2e^{B-1})+2$. $\ln 2+\ln e^{B-1}+2=\ln 2+B-1+2$. Setting this equal to ln2 gives B= -1. $\frac{e^2}{e^{-1}}=e^3$.
8. The side of the small triangle is $\frac{1}{6}(3\cos\theta)=\frac{1}{2}\cos\theta$. The area of the small triangle is $\frac{\sqrt{3}}{4}(side^2)$ which gives $\frac{\sqrt{3}}{4}(\frac{1}{4}\cos^2\theta)$. Setting this equal to $\frac{1}{48}\sqrt{3}$ gives $\cos^2\theta=\frac{1}{3}$.
   Since the angle is in quadrant I, we get (in radians) the angle is approximately **0.955**.
9. Since the distance between the center and the focus is 3, and the center to the endpoint of the minor axis is 5, c=3 and b=5. Use $a^2-25=9$ to get $a^2=34$ and the equation of the ellipse is $\frac{x^2}{25}+\frac{(y-1)^2}{34}=1$. When y=1 we get x=5 in quadrant I. The answer is **5**.
10. The sum of the roots is -B/A and the product is C/A so the sum of the roots in the first equation is 2 and the product is -1 which gives $f(x)=k(x^2-2x-1)$ and since the y-intercept is 4, k=4 to get $f(x)=4x^2-8x-4$. So B= -8. In g the sum of the roots is 10, the product is 22, which gives $g(x)=K(x^2-10x+22)$ and to make B= -8 we get $\frac{8}{10}(x^2-10x+22)$ which gives D= $\frac{88}{5}$ or **17.6**.
11. Since $\sin(a+\frac{\pi}{2})=\cos a$ we get $\frac{\cos a}{\cos a}=1$ and $\tan\theta=1$ which in quadrant I is $\frac{\pi}{4}$.
12. Statement 3: if the lateral faces are equilateral triangles, the height would be 0. Statement 4: by the triangle inequality theorem, the third side must be between 1 and 9, noninclusive, so 8 is not in the solution set. Statement 5: the sum must be divisible by 180 since for an n-gon the angles add to $180(n-2)$. The answer is 1+11= **12**.
13. Using the law of sines $\frac{\sin C}{6}=\frac{\sin 20}{4}$ which gives $\sin C=0.75$. Since this is the ambiguous case of SSA, we can get $C\approx 48.59$ or 131.41. Use the obtuse angle, since this makes AC least. This gives B to be approximately 18.59 degrees. Using the law of cosines $36+16-2(24)(\cos 18.59)=x^2$ gives answer **2.6**.

14. $100 = 2a_9 + 6$, and then $2a_8 + 6 = 47$ and then $a_7 = 7.25$ and $a_6 = 0.625$ and since $a_5 < 0$ so k=5. Likewise, $2(100) + 6 = a_{11}$ and $2(206) + 6 = a_{12}$ and then $a_{13} = 842$. So r=13. The sum is **18**.

15. $\cos x = \dfrac{dot\,product}{product\ of\ the\ magnitudes} = \dfrac{15-48}{5 \bullet 13}$ and so $x \approx 121°$. $mi + nj = 20(\frac{3}{5}i + \frac{4}{5}j)$ so m=12 and n=16. $z = 15 - 48 = -33$. The sum is **116**.