

Alpha Individual Test
FAMAT State Convention 2002

The abbreviation NOTA
denotes
"None of These Answers."

1. $\sqrt{5 - \sqrt{5 - \sqrt{5 - \sqrt{5 - \sqrt{\dots}}}}} =$

- A. $\frac{\sqrt{21}-1}{2}$ B. $\frac{\sqrt{21}+1}{2}$
C. $\sqrt{5} - 0.5$ D. $\sqrt{5} + 0.5$
E. NOTA

2. Which is equivalent to $4^x \bullet 2^x$?

- A. 8^{x^2} B. 2^{6x}
C. 8^{2x} D. 2^{3x} E. NOTA

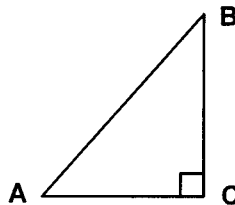
3. If $\frac{\sin(2x)}{\sin(x)} = \frac{a}{b}$ for $0 < x < \frac{\pi}{2}$ then which is an expression for $\cos x$?

- A. $\frac{2a}{b}$ B. $\frac{a}{2b}$
C. $\frac{b}{2a}$ D. $\frac{2b}{a}$ E. NOTA

4. If $\sin \theta = a$ and $\cos \theta = b$ for $0 < \theta < \frac{\pi}{2}$ then which is NOT true?

- A. $a = \sqrt{1 - b^2}$
B. $b = \sqrt{1 - a^2}$
C. $\theta = \text{Tan}^{-1}\left(\frac{a}{b}\right)$
D. $\theta = \text{Arc sec}\left(\frac{1}{b}\right)$
E. NOTA

5. In right triangle ABC, AC=5 cm and BC is decreasing at the rate of 0.2 cm per minute. If BC=10 cm at noon and 9.8 cm at exactly 12:01, when will $m\angle A$ be 54° ? Round to the nearest second. 12:01:05 denotes 5 seconds after 12:01 PM.



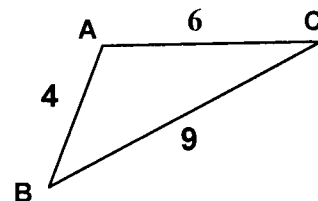
- A. 12:15:35
B. 12:15:59
C. 12:34:25
D. 12:34:41
E. NOTA

6. A line L with equation $x + Ay = B$ is perpendicular to a line through $(-1, 6)$ and $(0, 8)$ and point $P(-4, k)$ is on L. If $B = 101$ then give the value of k.

- A. 2 B. 48.5 C. 52.5
D. 105 E. NOTA

7. For the figure shown, give the value of $\cos A$. (AB=4, AC=6, BC=9)

- A. $\frac{4}{9}$
B. $\frac{-23}{24}$
C. $\frac{-24}{29}$
D. $\frac{-29}{48}$
E. NOTA



8. The graph of $y = \sin x \cos x$ is a sine curve. What is its period?

- A. $\frac{\pi}{2}$ B. π C. $\frac{3\pi}{2}$ D. 2π E. NOTA

9. The equation $2 \cos^2 \theta - \cos \theta = 0$, for $0 < \theta < 2\pi$ has solutions $\frac{a\pi}{6}, \frac{b\pi}{6}, \frac{c\pi}{6}$ and $\frac{d\pi}{6}$ where $a < b < c < d$. Give the value of $b+d$.

- A. 12 B. 13
C. 14 D. 19 E. NOTA

10. The hypotenuse of a right triangle is equal to $\sqrt[4]{5 + 2\sqrt{6}}$ and one leg has length $\sqrt[4]{3}$. If the length of the other leg is K , then give the value of K^6 .

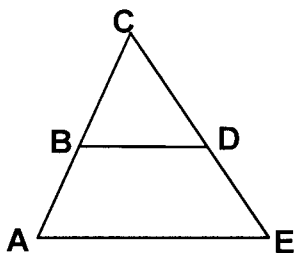
- A. 8 B. $5\sqrt{5}$
C. $2\sqrt{2}$ D. 125 E. NOTA

11. In $\triangle ABC$ with side lengths 5, 6 and 3, find the length of the altitude to the longest side.

- A. $\frac{\sqrt{15}}{6}$ B. 5
C. $\frac{4\sqrt{14}}{3}$ D. $\frac{2\sqrt{14}}{3}$ E. NOTA

12. In $\triangle ACE$, B is the midpoint of \overline{AC} and D is the midpoint of \overline{CE} . \overline{BD} has length $\cos \theta$ for $0 < \theta < \frac{\pi}{2}$ and $AE = \sin \theta$. Give the value of $\theta^2 - \theta$ to the nearest thousandth place.

- A. -0.249
B. -0.169
C. 0.119
D. 0.396
E. NOTA

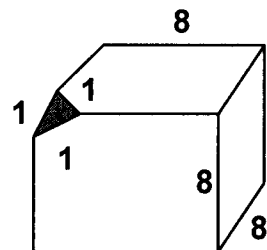


13. If $f(x-1) = \sqrt{2x+4}$ then find $f(3)$.

- A. $\sqrt{10}$ B. $2\sqrt{3}$
C. $\sqrt{6}$ D. $\sqrt{10} + 1$ E. NOTA

14. The surface area of the solid shown is $\frac{A+B\sqrt{C}}{4}$. If the face created by the truncation of one vertex of the cube is an equilateral triangle of side length 1, find the value of $A+B+C$. $B\sqrt{C}$ is in simplest radical form and the surface area includes all seven faces.

- A. 1541
B. 1537
C. 1534
D. 1532
E. NOTA



15. A ship sets off on a bearing (clockwise off of the north) of 120 degrees, at 30 mph. The current is 5 mph on a bearing of 40 degrees. With the current affecting the ship's speed, at what rate will it be traveling, to the nearest tenth of a mph?

- A. 25.1 mph B. 29.5 mph
C. 30.5 mph D. 31.3 mph
E. NOTA

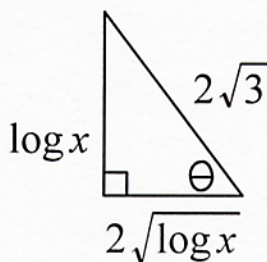
16. Triangle ABC has sides in the ratio of 5:12:13. If $\angle A$ is the smallest angle, and $\angle C$ is the largest angle, then what is the ratio of $\frac{\sin B}{\sin A}$?

- A. $\frac{12}{5}$ B. $\frac{12}{13}$
C. $\frac{13}{5}$ D. $\frac{13}{12}$ E. NOTA

17. If $\cos a = c$ for $\frac{\pi}{2} < a < \pi$, and $\sin b = d$ for $0 < b < \frac{\pi}{2}$, then which is an expression for $\cos(a+b)$?

- A. $\frac{\sqrt{1-d^2}}{d} - \frac{\sqrt{1-c^2}}{c}$
B. $\frac{\sqrt{1-d^2}}{c} + \frac{\sqrt{1-c^2}}{d}$
C. $c\sqrt{1-d^2} - d\sqrt{1-c^2}$
D. $c\sqrt{1-d^2} + d\sqrt{1-c^2}$
E. NOTA

18. For the right triangle shown, with $x > 1$ give the value of $\cos \theta$.



- A. $\frac{\sqrt{3}}{6}$ B. $\frac{\sqrt{2}}{6}$
C. $\frac{\sqrt{6}}{3}$ D. $\frac{\sqrt{6}}{2}$ E. NOTA

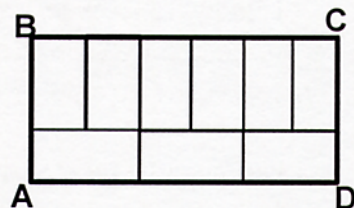
19. A dog fell into a storage barrel of water. It was cylindrical (the barrel, not the dog) with an inner diameter of 8 feet and an inner height of 10 feet. Before the dog fell in, the barrel was half full of water, and after he submerged totally, the water level rose to 8 feet high (inner height). The dog was then rescued*, pulled out, and the water level dropped to 4.5 feet high (water swallowed and in dog's coat). What was the volume of the dog, BB (before barrel)?

*no actual dogs were harmed in the making of this test.

- A. 24π cubic ft. B. 32π cubic ft.
C. 48π cubic ft. D. 192π cubic ft.
E. NOTA

20. The perimeter of the large rectangle $ABCD$ shown is 72. $ABCD$ is divided into 9 congruent rectangles as shown. What is the perimeter of one of these rectangles?

- A. 22
B. 24
C. 32
D. 48
E. NOTA



21. In $\triangle ABC$, $AB=12$, $BC=8$ and $m\angle A = 40^\circ$. Give the smallest possible degree measure of $\angle B$ to the nearest tenth.

- A. 34.6 B. 65.4
C. 74.6 D. 105.4 E. NOTA

22. If $\cos x + \cos y = \frac{1}{2}$ and $\sin x - \sin y = \frac{1}{3}$ find the value of $\cos(x+y)$.

- A. $\frac{13}{36}$ B. $-\frac{59}{72}$
C. $-\frac{13}{36}$ D. $\frac{59}{72}$ E. NOTA

23. A jar contains black and white beads only. The ratio of black beads to white beads is 8:3. If the jar contains 44 beads, how many black beads are in the jar?

- A. 4 B. 12
 C. 28 D. 32 E. NOTA

24. If $\log_2 x - 2 \log_2 3 = 4$ then find the value of $2\sqrt{x}$.

- A. 24 B. $16\sqrt{6}$
 C. $8\sqrt{6}$ D. 10 E. NOTA

25. If S is the set of points $Q(x, y)$ such that the distance from Q to the point $P(-1, 0)$ is equal to the distance from Q to the line $x = 3$. If the point $(K, 4)$ is in set S then $K = ?$

- A. $4\sqrt{2} - 1$ B. -3
 C. $4\sqrt{2} + 1$ D. -1
 E. NOTA

26. Point P moves along the graph of circle Q which has equation $x^2 + y^2 - 4x + 6y - 3 = 0$. P moves in one direction only and completes the circumference of the circle in one second. If $f(t)$ is defined as the distance from P to the point $(10, 12)$ for time $t \geq 0$. Find the minimum value of f .

- A. 17 B. 16
 C. 13 D. 1 E. NOTA

27. Two circles have centers on the x -axis, and have radii 8 and 10 respectively. If their centers are distance 16 apart, at what distance from the x -axis do the circles meet, to the nearest tenth place?

- A. 4.1 B. 3.9
 C. 3.7 D. 3.5 E. NOTA

28. $\triangle ABC$ has one side \overline{AB} such that A is on the origin, and B lies on the positive x -axis. Vertex C lies on the line $x - 4y = 2$, with C in Quadrant I. If vertex A is on the origin, and $AB = 10$, and the area of $\triangle ABC$ is 20, find the length of side \overline{AC} . Round your answer to the nearest tenth place.

- A. 4.0 B. 16.5
 C. 18.4 D. 20.6 E. NOTA

29. For $x < 0$, $\sqrt{2x} \cdot \sqrt{8x} =$

- A. $\sqrt{16x^2}$ B. $-4x$
 C. $4|x|$ D. $-4|x|$ E. NOTA

30. The line through the origin with angle of inclination 30° intersects with the graph of $y = 6x - 3x^2$ at $(0, 0)$ and (m, n) . Give the value of $27n$.

- A. $18\sqrt{3} - 3$ B. $6\sqrt{3} - 1$
 C. $\frac{\sqrt{3}}{6}$ D. $2\sqrt{22}$
 E. NOTA