Alpha Applications
FAMAT State Convention 2002

Note: "E) NOTA" denotes none of the above answers is correct, **i, j, k** denote the standard unit vectors.

1) The points (-1,-6), (2,9), and (3,22) lie on a parabola given by $y = ax^2 + bx + c$. Find $a + b + c$.

    A) –1   B) 0   C) 1   D) 2   E) NOTA

2) The region R, given by the inequalities below, is rotated about the y-axis to form solid S. Find the volume of S.

    A) 25π

    B) $\frac{125\pi}{3}$

    C) $\frac{250\pi}{3}$

    D) 125π

    E) NOTA

$$R = \begin{cases} y \le 5 + x \\ y \le 5 - x \\ y \ge 0 \end{cases}$$

3) An HDS worker is trying to move a very heavy box from, as usual, point A to point B. Point A is located at (2,7) and Point B is located at (-2,4). Given that the force required to move the box is 17 Newtons, find the amount of work done in moving the box in a straight line from A to B (Round to the nearest hundredth).

    A) 0.29 N•ft   B) 3.40 N•ft   C) 11.00 N•ft   D) 85.00 N•ft   E) NOTA

4) Tommy Boy is playing with his new balloon toy. Strangely enough, his new balloon is exactly a sphere. If he blows the balloon up from 288π to 2304π cubic units, find the change in surface area from the original to the new size.

    A) 100π B) 243π C) 432π D) 517π E) NOTA

5) A circle, with center at (3,4), contains the points (3,8) and (7,4) and can be expressed as $(x\text{-}a)^2 + (y\text{-}b)^2 = r^2$. Find $a + b + r$.

    A) –3   B) 0   C) 11   D) 13   E) NOTA

6) Function F has domain [-2,5] and range [-1,7]. What is the domain and range of F(x-2) + 1?

    A) D:[0,7],R:[0,8]   C) D:[-4,7],R:[0,8]   E) NOTA
    B) D:[0,7],R:[-2,6]   D) D:[-4,7],R:[-2,6]

7) Find the obtuse angle between the vectors $\mathbf{P} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{Q} = -5\mathbf{i} + \mathbf{j}$ (Round to the nearest tenth of a degree).

    A) 76.8° B) 92.0° C) 112.4° D) 193.2° E) NOTA

8) Evaluate $\lim\limits_{x \to 0}\left(\dfrac{\sin(x)}{x}\right)$

    A) -1   B) 0   C) 1   D) ∞ E) NOTA

9) Find the sum of the two smallest roots to the equation $x^4 - 5x^3 - 7x^2 + 41x - 30 = 0$.

    A) -2 B) 0   C) 1   D) 6   E) NOTA

10) A sock is filled with 15 marbles of which 4 are red, 3 are green, 5 are blue, and 3 are black. Three marbles are drawn at random without replacement. If it is known that the first marble was blue, and the second green, what is the probability that the third marble drawn is either red or green?

    A) $\dfrac{6}{13}$ B) $\dfrac{6}{15}$ C) $\dfrac{7}{15}$ D) $\dfrac{7}{13}$ E) NOTA

11) The height of a rocket, launched at time $t = 0$, by $h(t) = 11t - \dfrac{2}{3}t^2 + 2$, find the time when the rocket returns to the original launch height? (Note: h(t) represents height as a function of a time "t".)

    A) 2   B) 8.25   C) 16.5   D) 33   E) NOTA

12) The graph of the equation $y = \dfrac{x^3 - 2x^2 - 5x + 6}{x^3 + 2x^2 - 5x - 6}$ has asymptotes given by the following: $y = a$, $x = b$, $x = c$, $x = d$. Find $a + b + c + d$.

    A) -1 B) 0   C) 1   D) 2   E) NOTA

13) Which of the following best describes the figure traced by the polar graph $r = a + 2b\cos(\theta)$ where $a, b \in \Re$

    A) Cardioid   B) limacon   C) line   D) lemniscate   E) NOTA

14) The linear function ∂ contains the points (3, 18) and (-2, 17). Find ∂ (0).

    A) 0   B) 17   C) 17.4   D) 17.5   E) NOTA

15) Two ships both leave the port of Tampa, Florida at the same time, one headed for Cancun, Mexico, the other to Brownsville, Texas. Using the port of Tampa as the origin, suppose the coordinates for Cancun are given by (-18, -12) and the coordinates for Brownsville are given by (-21, 7). Assuming the ships travel in a straight line to their destination, what is the acute angle formed by the intersection of their paths at the Port of Tampa? (Ignore the curvature of the earth, and round your answer to the nearest whole degree).

    A) 48° B) 52° C) 53° D) 56° E) NOTA

16) The Student Senate is composed of 35 senators from around the school. The Select Committee on Election Reform is to be composed of 4 senators, selected at random. Assuming that the order in which they are selected is unimportant, how many different committees can be formed?

   A) 52360   B) 324632   C) 1256640   D) 1623160   E) NOTA

17) Find "m" such that the vectors P and Q are orthogonal given that $\mathbf{P} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$
   and $\mathbf{Q} = -2\mathbf{i} + \mathbf{j} - m\mathbf{k}$.

   A) $m = -4$ B) $m = -1$ C) $m = 1$ D) $m = 4$ E) NOTA

18) In the mid-1980's a research group attempted to recover the R.M.S. Titanic from the Atlantic Ocean floor. Witness accounts show the ship sank in two pieces, so the group focused their search on the stern (rear). Once the stern was located on the ocean floor, they labeled that point as the origin (0,0,0) of their coordinate system for the search field of the bow. After searching, the bow was found to be located at (800, -700, 0) based on the coordinate system. The surface ship, located directly above the origin at (0,0,5280), needs to know what angle to send a recovery vessel down to the bow's wreckage site. Find the angle between the straight line path down to the stern and the straight line path to the bow wreckage. (Ignore curvature of the earth, ocean current, and round to the nearest whole degree).

   A) 6°   B) 11°   C) 13°   D) 17°   E) NOTA

19) The deer play on a certain grazing range in Utopia. A renowned ecologist studied this population over time and determined that the total population of deer could be modeled by the function below (P(t) represents deer population at time "t"). Determine the carrying capacity of the land (the measure to which the population tends to approach with long amounts of time).
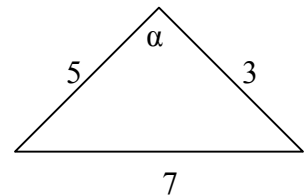
   A) 0
   B) 1
   C) 227
   D) ∞
   E) NOTA

$$P(t) = \frac{227}{1 - e^{-0.275t + 0.85}}$$

20) Which of the following best describes the vectors $\mathbf{u} = -6.5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = -8\mathbf{i} - 12\mathbf{j} + 8\mathbf{k}$.

   A) Perpendicular   B) parallel   C) skew   D) Neither   E) NOTA

21) Find tan(α) given the picture at right.

   A) $-\sqrt{3}$ B) $\frac{-\sqrt{2}}{2}$ C) $\sqrt{3}$ D) $\frac{\sqrt{3}}{3}$ E) NOTA

22) Find the arc length traversed by the tip of the four inch minute hand of a clock in 92 minutes. (Note: Round your answer to the nearest hundredth).

   A) 16.39   B) 32.78   C) 38.54   D) 77.07   E) NOTA

23) Find the distance between the polar points $(3, 60°)$ and $(-2,-30°)$ to the nearest tenth.

A) 1.4   B) 3.6   C) 4.8   D) 23.4   E) NOTA

24) Evaluate $\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$ given that $f(x+1) = 2x^2 + 6x + 3$.

A) -1   B) 3   C) $2x + 4$ D) $4x + 2$ E) NOTA

25) Which of the following defines the locus of points equidistant from a point and a line.

A) Parabola   B) Hyperbola C) Circle   D) Ellipse   E) NOTA

26) What is the minimum value of $3sin(2x-\pi) + 1$ on the interval $[0,\pi]$

A) $\frac{\pi}{4}$ B) $\frac{3\pi}{4}$ C) -2   D) 4   E) NOTA

27) Six marbles are placed in a sack; 2 red, 2 white, 2 blue. Two marbles are selected at random without replacement from the sack. Compute the probability that the second ball drawn is blue.

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{1}{6}$ D) $\frac{1}{9}$ E) NOTA

28) Find the coefficient of the 117th term in the expansion of $(a + b)^{150}$. (Note: $_nC_r$ denotes "n" objects combined "r" at a time; $_nP_r$ denotes "n" objects permuted "r" at a time.)

A) $_{150}C_{116}$ B) $_{150}C_{117}$ C) $_{150}P_{116}$ D) $_{150}P_{117}$ E) NOTA

29) Find the foci of the hyperbola given by the equation $\frac{(y+1)^2}{4}-\frac{(x-1)^2}{9}=1$

A) $(1,-1\pm\sqrt{13})$
B) $(-1,1\pm\sqrt{13})$
C) $(1,-1\pm\sqrt{5})$
D) $(1,1\pm\sqrt{5})$
E) NOTA

30) Given that $x=\frac{9}{5}y^2-\frac{10}{9}$, find all values for which $x = y$.

A) $\frac{10}{9}$ B) $\frac{5}{9}$ C) $\left\{\frac{10}{9},\frac{5}{9}\right\}$ D) $\left\{\frac{10}{9},-\frac{5}{9}\right\}$ E) NOTA