Complex Numbers – Solutions/Answers
FAMAT State Convention 2002

1) B. $(2-3i)(2-3i)(2-3i)(2-3i) = (-5-12i)(-5-12i) =$ -119 + 120i.

2) B. $\begin{vmatrix} x & 2 & 1 \\ 0 & i & 1 \\ -1 & 2 & i \end{vmatrix} = x(ii-2) - 2(0+1) + 1(0+i) = -2-3x+i = i-2$. Thus –2 – 3 x = -2. x = 0.

3) B. $\bar{z} = 5-8i$. Thus 15+8i. Abs.Val = $\sqrt{15^2+8^2}$ = 17. Thus, B.

4) D. Factor to (x+3)(x^2 + 2x + 4) = 0. Quadratic formula for the "x^2…" term yields $-1 \pm i\sqrt{3}$. D.

5) A. (1i)(2i)(3i)(4i)(5i) = 5!i^5 = 120i = A.

6) C. $9(-i)^3 - 2i(-i)^2 + 3(-i) - 6 = -6 + 8i$ = C.

7) A. $x = r\cos\Theta = 5\cos 210 = \frac{-5\sqrt{3}}{2}$ $\quad y = r\sin\Theta = 5\sin 210 = \frac{-5}{2}$. Thus $\left(\frac{-5\sqrt{3}}{2}, \frac{-5}{2}\right)$ = A.

8) D. Note that $cis\left(\pi/2\right) = i$. The sum becomes

$$\sum_{n=1}^{9} i^n = i^1 + i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + i^9 = i - 1 - i + 1 + i - 1 - i + 1 + i = i.$$

9) B. $\left| 6-4i \right| = \sqrt{6^2 + (-4)^2} = \sqrt{52} = 2\sqrt{13}$.

10) D. $\frac{1}{(2+i)(7-i)} = \frac{1}{15+5i} = \frac{15-5i}{(15+5i)(15-5i)} = \frac{3-i}{50}$

11) C. (I) True, used in #10 to rationalize denominator. (II) False, always a REAL number. (III) True. Use DeMoivre's theorem for roots to find the cube roots. Note, their sum is six. (IV) Multiply by the conjugate of $\bar{z}$, which is z. But denominator is not eliminated, it changes to a real. Thus I and III are true, so a total of 2 statements are true. Thus C.

12) D. $2e^{\frac{i\pi}{6}} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \sqrt{3} + i$, D.

13) A. $i^{2002} = i^{2000} i^2 = 1(-1) = -1$

14) A. Factor into $\left(\left(x^2+x+1\right)^2\right)$. The exponential "2" serves to merely repeat the roots. Reduce the function to $x^2 + x + 1$, which solves to $-\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$. This is two repeated imaginary roots, A.

15) D. $\frac{2cis(\pi)}{cis(\pi/2)} = \frac{2\cos(\pi) + i\sin(\pi)}{\cos(\pi/2) + i\sin(\pi/2)} = \frac{2(-1)}{i} = 2i, D$

16) B. Find the axis-of symmetry using $x = \frac{-b}{2a}$. The axis of symmetry contains the minimum value of $f(s)$. In order for $f(s)$ to have two imaginary roots, the y-value at the minimum must be greater than zero. $\therefore x = \frac{5}{6}; f\left(\frac{5}{6}\right) = \frac{-25}{12} + \lambda > 0.$ Thus $\lambda > {}^{25}/_{12}$.

17) C. Apply the quadratic formula: $\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(2)(-12)}}{2(2)} = -1 \pm \sqrt{7}$

18) C. If $f(x) = \frac{6+x}{x}, x \neq 0$, then $f(\lambda) = \frac{6+\lambda}{\lambda}, x \neq 0 = 1-2i$; Thus $6 + \lambda = \lambda(1-2i)$. Thus $\lambda = 3i$.

19) B. $5cis(240°) = 5(\cos 240) + i\sin 240)) = 5\left(\frac{-1}{2} + \frac{-i\sqrt{3}}{2}\right) = \left(\frac{-5}{2} + \frac{-5i\sqrt{3}}{2}\right)$

20) B. A is given as the set of all things that are *not* complex numbers. Since the complex numbers are comprised of the set or real numbers AND the set of imaginary numbers, to not be a member of A, the element being tested cannot be either real and/or imaginary. I, II, III, IV do not fit these properties, thus B, none are members of set A.

21) B. In order to yield an imaginary result, one of the options must attempt to take the square root of a negative number. Only choice B, which inputs (7) results in this, thus B.

22) C. In order to have two real roots for a quadratic it must, by definition, cross the x-axis exactly twice.

23) C. *g(2,1) and f(1,1)* serve as the input parameters for the final *f* function, thus they must be evaluated first. g(2,1) = -(2)(1)(*i*) = -2*i*. f(1,1) = (1)*i*+ (1)(1) = 1 + *i*. Thus *f(g(2,1), f(1,1))* = (-2*i*)(*i*) + (-2*i*)(1+*i*) = 4-2*i*.

24) D. $\frac{3+i}{2-i} = \frac{3+i}{2-i}\left(\frac{2+i}{2+i}\right) = \frac{5+5i}{5} = 1+i$

25) A. $\sum_{1}^{7}(-1)^n(-i)^n = i - 1 - i + 1 + i - 1 - i = -1$

26) B, by definition.

27) E. None of the given sets are contained within the imaginary set. Rationals are a subset of reals. Complex is the collection of reals and imaginaries. There is no set known as the “hypothetical”. Therefore, NOTA, E.

28) D. – (3*i* + 2)(2*i* – 1)(3 + *i*) = – (3*i* + 2)(-5+5i)=25+5i. 25+5 = 30.

29) B. $\frac{i-1}{i} = 1+i$ and $\frac{-2}{i-1} = 1+i$. Thus the common ratio is 1+*i*. The series becomes the summation $i + -1 + i + -2 + -2 - 2i + -4i + 4 - 4i + 8 = 7 - 8i$

30) A. cis(30°) = $\cos(30) + i\sin(30) = \frac{\sqrt{3}}{2} + \frac{i}{2}$. This lies in quadrant I, thus A.