Complex Numbers
FAMAT State Convention 2002

PLEASE NOTE THE FOLLOWING FOR THIS TEST: (1) $i = \sqrt{-1}$, the unit imaginary number. (2) If z is a complex number, then $\bar{z}$ is the conjugate of z, and $|z|$ is the absolute value of z. (3) Whenever a complex number is given in the standard form, $a + bi$, it is assumed that "a" and "b" are real numbers. (4) Choice E) NOTA is meant to denote "None of these Answers".

1) Evaluate $(2-3i)^4$.

   A) 119-120i B) $-119 + 120i$ C) 119+120i D) $-119 - 120i$ E) NOTA

2) Evaluate the determinant and solve for "x".

$$\begin{vmatrix} x & 2 & 1 \\ 0 & i & 1 \\ -1 & 2 & i \end{vmatrix} = i - 2$$

   A) -1 B) 0   C) 1   D) 2   E) NOTA

3) Evaluate $\left| 2z + \bar{z} \right|$ given that z = 5 + 8i.

   A) 15   B) 17   C) 23   D) 34   E) NOTA

4) Which of the following is a solution to $x^3 + 5x^2 + 10x + 12 = 0$.

   A) 3   B) $1 + i\sqrt{3}$ C) $1 - 2i\sqrt{3}$ D) $-1 + i\sqrt{3}$ E) NOTA

5) Evaluate the following: $\displaystyle\prod_{n=1}^{5} ni$

   A) 120i B) _15i_   C) 120   D) 15   E) NOTA

6) Given the polynomial $h(x) = 9x^3 - 2ix^2 + 3x - 6$, find $h(-i)$.

   A) 6-8i B) 6+8i C) $-6 + 8i$ D) $-6 - 8i$ E) NOTA

7) Find the Cartesian coordinates of the polar coordinate $\left(5, \dfrac{7\pi}{6}\right)$.

   A) $\left(\dfrac{-5\sqrt{3}}{2}, \dfrac{-5}{2}\right)$ B) $\left(\dfrac{-5\sqrt{2}}{2}, \dfrac{-5}{2}\right)$ C) $\left(\dfrac{-5\sqrt{3}}{3}, \dfrac{-5}{3}\right)$ D) $\left(\dfrac{-3\sqrt{2}}{2}, \dfrac{-3}{2}\right)$ E) NOTA

8) Evaluate the following: $\displaystyle\sum_{n=1}^{9}\left(cis\left(\frac{\pi}{2}\right)\right)^{n}$

   A) $-i$ B) -1 C) 1 D) i E) NOTA

9) Find $\left|\,6-4i\,\right|$.

   A) $\sqrt{13}$ B) $2\sqrt{13}$ C) 2 D) 10 E) NOTA

10) Express $\dfrac{1}{(2+i)(7-i)}$ as a complex number in a + bi form.

   A) $15+5i$ B) $\dfrac{1}{15}+i\dfrac{1}{15}$ C) $\dfrac{3}{50}+i\dfrac{1}{50}$ D) $\dfrac{3}{50}-i\dfrac{1}{50}$ E) NOTA

11) How many of the following statements are true? *(Note: z ≠ 0 + 0i)*

    I)   If z is a complex number, then $(z)(\bar{z})$ is <u>always</u> a real number.

    II)   If z is a complex number, then $z+\bar{z}$ is <u>always</u> an imaginary number.

    III)   The sum of the absolute values of the three cube roots of 8 is 6.

    IV)   If z is a complex number, then $(z)\left((\bar{z})^{-1}\right)=z^{2}$

   A) 0 B) 1 C) 2 D) 3 E) NOTA

12) Evaluate $2e^{\frac{i\pi}{6}}$.

   A) $-\sqrt{3}-i$ B) $-\sqrt{3}+i$ C) $\sqrt{3}-i$ D) $\sqrt{3}+i$ E) NOTA

13) Which of the following is equivalent to i^{2002}.

   A) -1 B) $-i$ C) i D) 1 E) NOTA

14) Describe the nature of the solutions for the equation: $x^{4}+2x^{3}+3x^{2}+2x+1=0$

   A) 0 Real, 2 Repeated Complex Non-Real   C) 4 Real, 4 Complex Non-Real   E) NOTA
   B) 2 Real, 2 Complex Non-Real   D) 4 Real, 0 Complex Non-Real

15) Evaluate $\dfrac{2cis(\pi)}{cis\left(\pi/_2\right)}$

    A) –1   B) –i C) i D) $2i$ E) NOTA

16) For what values of "λ" will the function $f(s) = 3s^2 - 5s + \lambda$ have two complex non-real roots?
(Note: Assume $\lambda \in$ Reals)

    A) $\lambda > \dfrac{12}{25}$ B) $\lambda > \dfrac{25}{12}$ C) $\lambda > \dfrac{3}{5}$ D) $\lambda > \dfrac{5}{3}$ E) NOTA

17) Solve the following: $2x^2 + 4x - 12 = 0$.

    A) $7 \pm i$ B) $1 \pm \sqrt{7}$ C) $-1 \pm \sqrt{7}$ D) $\sqrt{7}$ E) NOTA

18) Given that $f(x) = \dfrac{6+x}{x}$, $x \neq 0$ and $f(\lambda) = 1 - 2i$, find λ.

    A) $-6i$ B) $-3i$ C) $3i$ D) $6i$ E) NOTA

19) Which of the following is equivalent to $5cis(240°)$?

    A) $\dfrac{-5}{3} - \dfrac{5\sqrt{3}}{3}i$ B) $\dfrac{-5}{2} - \dfrac{5\sqrt{3}}{2}i$ C) $\dfrac{-7}{2} - \dfrac{7\sqrt{3}}{2}i$ D) $\dfrac{-3}{2} - \dfrac{3\sqrt{3}}{2}i$ E) NOTA

20) If A = $\{x : x \notin$ Complex Numbers$\}$, which of the following could not be a member of A?

    I) $\sqrt{3}$ A) All are members of A

    II) $\dfrac{22}{7}$ B) None are members of A

    III) $-17.\overline{3}$ C) IV only

    IV) $\sqrt{-3}$ D) III & IV only

                                      E) NOTA

21) Which of the following yields an imaginary result given that $f(x) = \sqrt{16x-9}$ and $g(x) = \sqrt{-2x+5}$

    A) $f(g(2))$ B) $g(f(7))$ C) $f(g(0))$ D) $f(f(f(3)))$ E) NOTA

22) If $f(x)$ is a quadratic function with two non-equal real roots, how many times does the graph of $f(x)$ cross the x-axis? *(Note: Assume $x \in$ Reals)*

A) 0   B) 1   C) 2   D) Situation Impossible   E) NOTA

23) Given the two relations $f(x,y) = xi + yx$ and $g(v,w) = -vwi$. Find $f(g(2,1), f(1,1))$.

A) $2 + 2i$ B) $4 + 2i$ C) $4 - 2i$ D) $2 - 2i$ E) NOTA

24) Which of the following is equivalent to $\dfrac{3+i}{2-i}$?

A) $-1 - i$ B) $1 - i$ C) $-1 + i$ D) $1 + i$ E) NOTA

25) $\displaystyle\sum_{n=1}^{7} \left[\left((-1)^n \right)\left((-i)^n \right) \right]$

A) -1 B) $-i$ C) i D) 1 E) NOTA

26) Which of the following is the correct expansion for $cis(x)$?

A) $\cos(x) - i\sin(x)$ C) $i\cos(x)\sin(x)$ E) NOTA
B) $\cos(x) + i\sin(x)$ D) $\sin(x) + i\cos(x)$

27) Which of the following sets is a subset of the imaginary number set?

A) Reals   B) Complex   C) Rational   D) Hypothetical   E) NOTA

28) If $ai + b = -(3i + 2)(2i - 1)(3 + i)$. Find a + b.

A) -30 B) -25 C) 5   D) 30   E) NOTA

29) Find the sum of the first 7 terms of the geometric sequence i, $i - 1$, -2, …

A) $7 + 8i$ B) $7 - 8i$ C) $8 + 7i$ D) $8 - 7i$ E) NOTA

30) Which quadrant of the Argand plane contains the point $cis(30^\circ)$?

A) I   B) II   C) III   D) IV   E) NOTA