Complex Numbers
FAMAT State Convention 2002

PLEASE NOTE THE FOLLOWING FOR THIS TEST: (1) \( i = \sqrt{-1} \), the unit imaginary number. (2) If \( z \) is a complex number, then \( \bar{z} \) is the conjugate of \( z \), and \( |z| \) is the absolute value of \( z \). (3) Whenever a complex number is given in the standard form, \( a + bi \), it is assumed that “a” and “b” are real numbers. (4) Choice E) NOTA is meant to denote “None of these Answers”.

1) Evaluate \((2 - 3i)^4\).
   A) 119-120i   B) -119 + 120i   C) 119+120i   D) -119 – 120i   E) NOTA

2) Evaluate the determinant and solve for “x”.
   \[
   \begin{vmatrix}
   x & 2 & 1 \\
   0 & i & 1 \\
   -1 & 2 & i \\
   \end{vmatrix}
   = i - 2
   \]
   A) -1   B) 0   C) 1   D) 2   E) NOTA

3) Evaluate \(2z + \overline{z}\) given that \(z = 5 + 8i\).
   A) 15   B) 17   C) 23   D) 34   E) NOTA

4) Which of the following is a solution to \(x^3 + 5x^2 + 10x + 12 = 0\).
   A) 3   B) \(1 + i\sqrt{3}\)   C) \(1 - 2i\sqrt{3}\)   D) \(-1 + i\sqrt{3}\)   E) NOTA

5) Evaluate the following: \(\prod_{n=1}^{5}^{ni}\)
   A) 120i   B) 15i   C) 120   D) 15   E) NOTA

6) Given the polynomial \(h(x) = 9x^3 - 2ix^2 + 3x - 6\), find \(h(-i)\).
   A) 6-8i   B) 6+8i   C) -6 + 8i   D) -6 – 8i   E) NOTA

7) Find the Cartesian coordinates of the polar coordinate \(\left(5, \frac{7\pi}{6}\right)\).
   A) \(\left(-\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)\)   B) \(\left(-\frac{5\sqrt{2}}{2}, -\frac{5}{2}\right)\)   C) \(\left(-\frac{5\sqrt{3}}{3}, -\frac{5}{3}\right)\)   D) \(\left(-\frac{3\sqrt{2}}{2}, -\frac{3}{2}\right)\)   E) NOTA
8) Evaluate the following: \[ \sum_{n=1}^{9} \left( \frac{\pi}{2} \right)^n \]

A) \(-i\)  B) \(-1\)  C) 1  D) \(i\)  E) NOTA

9) Find \(|6 - 4i|\).

A) \(\sqrt{13}\)  B) \(2\sqrt{13}\)  C) 2  D) 10  E) NOTA

10) Express \(\frac{1}{(2+i)(7-i)}\) as a complex number in a + bi form.

A) \(15 + 5i\)  B) \(\frac{1}{15} + i \frac{1}{15}\)  C) \(\frac{3}{50} + i \frac{1}{50}\)  D) \(\frac{3}{50} - i \frac{1}{50}\)  E) NOTA

11) How many of the following statements are true? (Note: \(z \neq 0 + 0i\))

I) If \(z\) is a complex number, then \(\bar{z}^2\) is always a real number.
II) If \(z\) is a complex number, then \(z + \bar{z}\) is always an imaginary number.
III) The sum of the absolute values of the three cube roots of 8 is 6.
IV) If \(z\) is a complex number, then \(\overline{(z)}(\overline{(z)}^{-1}) = z^2\)

A) 0  B) 1  C) 2  D) 3  E) NOTA

12) Evaluate \(\frac{i\pi}{6}\).

A) \(-\sqrt{3} - i\)  B) \(-\sqrt{3} + i\)  C) \(\sqrt{3} - i\)  D) \(\sqrt{3} + i\)  E) NOTA

13) Which of the following is equivalent to \(i^{2002}\).

A) \(-1\)  B) \(-i\)  C) \(i\)  D) 1  E) NOTA

14) Describe the nature of the solutions for the equation: \(x^4 + 2x^3 + 3x^2 + 2x + 1 = 0\)

A) 0 Real, 2 Repeated Complex Non-Real  C) 4 Real, 4 Complex Non-Real  E) NOTA
B) 2 Real, 2 Complex Non-Real  D) 4 Real, 0 Complex Non-Real
15) Evaluate \( \frac{2 \text{cis}(\pi)}{\text{cis}(\pi/2)} \)

A) \(-1\)  B) \(-i\)  C) \(i\)  D) \(2i\)  E) NOTA

16) For what values of \(\lambda\) will the function \(f(s) = 3s^2 - 5s + \lambda\) have two complex non-real roots? (Note: Assume \(\lambda \in \text{Reals}\))

A) \(\lambda > \frac{25}{12}\)  B) \(\lambda > \frac{12}{25}\)  C) \(\lambda > \frac{3}{5}\)  D) \(\lambda > \frac{5}{3}\)  E) NOTA

17) Solve the following: \(2x^2 + 4x - 12 = 0\).

A) \(7 \pm i\)  B) \(1 \pm \sqrt{7}\)  C) \(-1 \pm \sqrt{7}\)  D) \(\sqrt{7}\)  E) NOTA

18) Given that \(f(x) = \frac{6 + x}{x}, x \neq 0\) and \(f(\lambda) = 1 - 2i,\) find \(\lambda\).

A) \(-6i\)  B) \(-3i\)  C) \(3i\)  D) \(6i\)  E) NOTA

19) Which of the following is equivalent to \(5 \text{cis}(240^\circ)\)?

A) \(-\frac{5}{3} - \frac{5\sqrt{3}}{3}i\)  B) \(-\frac{5}{2} - \frac{5\sqrt{3}}{2}i\)  C) \(-\frac{7}{2} - \frac{7\sqrt{3}}{2}i\)  D) \(-\frac{3}{2} - \frac{3\sqrt{3}}{2}i\)  E) NOTA

20) If \(A = \{x : x \notin \text{Complex Numbers}\}\), which of the following could not be a member of \(A\)?

I) \(\sqrt{3}\)  II) \(\frac{22}{7}\)  III) \(-17.3\)  IV) \(\sqrt{-3}\)  A) All are members of \(A\)  B) None are members of \(A\)  C) IV only  D) III & IV only  E) NOTA

21) Which of the following yields an imaginary result given that \(f(x) = \sqrt{16x - 9}\) and \(g(x) = \sqrt{-2x + 5}\)

A) \(f(g(2))\)  B) \(g(f(7))\)  C) \(f(g(0))\)  D) \(f(f(3)))\)  E) NOTA
22) If \( f(x) \) is a quadratic function with two non-equal real roots, how many times does the graph of \( f(x) \) cross the \( x \)-axis? \( \text{(Note: Assume } x \in \text{Reals} \)\)

A) 0  B) 1  C) 2  D) Situation Impossible  E) NOTA

23) Given the two relations \( f(x,y) = xi + yx \) and \( g(v,w) = -vwi \). Find \( f(g(2,1), f(1,1)) \).

A) 2 + 2i  B) 4 + 2i  C) 4 – 2i  D) 2 – 2i  E) NOTA

24) Which of the following is equivalent to \( \frac{3+i}{2-i} \)?

A) \( -1 - i \)  B) \( 1 - i \)  C) \( -1 + i \)  D) \( 1 + i \)  E) NOTA

25) \( \sum_{n=1}^{7} \left[ (-1)^n \left[ (-i)^n \right] \right] \)

A) \(-1\)  B) \(-i\)  C) \(i\)  D) \(1\)  E) NOTA

26) Which of the following is the correct expansion for \( \text{cis}(x) \)?

A) \( \cos(x) - \sin(x) \)  B) \( \cos(x) + i\sin(x) \)  C) \( i\cos(x)\sin(x) \)  D) \( \sin(x) + i\cos(x) \)  E) NOTA

27) Which of the following sets is a subset of the imaginary number set?

A) Reals  B) Complex  C) Rational  D) Hypothetical  E) NOTA

28) If \( ai + b = -(3i + 2)(2i - 1)(3 + i) \). Find \( a + b \).

A) \(-30\)  B) \(-25\)  C) \(5\)  D) \(30\)  E) NOTA

29) Find the sum of the first 7 terms of the geometric sequence \( i, i - 1, -2, \ldots \)

A) \(7 + 8i\)  B) \(7 - 8i\)  C) \(8 + 7i\)  D) \(8 - 7i\)  E) NOTA

30) Which quadrant of the Argand plane contains the point \( \text{cis}(30^\circ) \)?

A) I  B) II  C) III  D) IV  E) NOTA