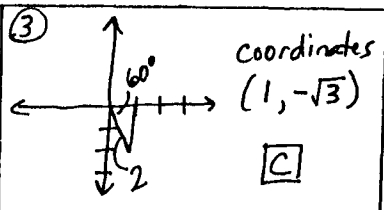
TRIGONOMETRY SOLUTIONS - FAMAT State Convention 2001

(1) $\sin\theta + \cos\theta$
$= \frac{4}{5} + \frac{3}{5}$
$= \frac{7}{5}$ [A]

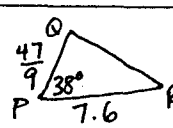
(2) $\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\tan\theta}{\sec^2\theta}$
$= 2\cdot\frac{\sin\theta}{\cos\theta}\cdot\cos^2\theta = 2\sin\theta\cos\theta$
$= \sin 2\theta$ [A]

(3) 

coordinates $(1, -\sqrt{3})$ [C]

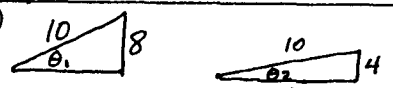
(4) $\tan\frac{\pi}{2}$, $\tan\frac{3\pi}{2}$ undefined
$(\frac{\pi}{4}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{4}) \cup (\frac{5\pi}{4}, \frac{3\pi}{2}) \cup (\frac{3\pi}{2}, \frac{7\pi}{4})$ [E]

(5) $\cos(3x) =$
$4\cos^3 x - 3\cos x$
$4 - 3 = 1$ [A]

(6) 

Area $= \frac{1}{2}ab\sin C$
$= \frac{1}{2}(\frac{47}{9})(7.6)\sin 38°$
$\doteq 12.21$ 12 [A]

(7) II, III, IV, V are odd functions
4 [B]

(8) 

$\theta_1 - \theta_2 = \arcsin(.8) - \arcsin(.4)$
$= .515778\ldots \doteq .516$ [C]

(9) $(44 \text{ rad})(\frac{180 \text{ degrees}}{\pi \text{ rad}})$
$\doteq 2521.01°$
coterminal with $1.01°$ QI [A]

(10) $\sum_{n=1}^{2004} \sin^n(90n) = \sin 90 + \sin^2 180 + \sin^3 270 + \sin^4 360 + \ldots$
$= \underbrace{1+0+-1+0}_{\text{repeat 500 times}} \quad +1 = 0+1 = 1$
$\sum_{n=1}^{2001} \cos^n(90n) = \underbrace{0+1+0+1}_{\text{repeat 500 times}} \quad +0 = 2(500)+0$
$= 1000$
$1000+1 = 1001$ [D]

(11) $\cos\frac{45}{2} = \sqrt{\frac{1+\cos 45}{2}}$
$= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$
$x=2,\ y=4$
$10y + x^3 = 40 + 8 = 48$ [D]

(12) $r = \frac{m}{2}\theta$
graph of a spiral [E]

(13) I. per $= \frac{\pi}{B} = \frac{\pi}{1} = \pi$
II. per $= \frac{2\pi}{B} = \frac{2\pi}{2} = \pi$
III. per $= \frac{2\pi}{B} = \frac{2\pi}{2} = \pi$
I, II, and III [D]

(14) $\frac{(27\text{cis}300)(256\text{cis}320)}{(243\text{cis}320)(64\text{cis}300)}$
$= \frac{27\cdot 256}{243\cdot 64} = \frac{4}{9}$ [A]

(15) $g\sec\theta = \frac{\cos^2\theta + (1-\sin\theta)^2}{\cos\theta(1-\sin\theta)} = \frac{\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta}{\cos\theta(1-\sin\theta)}$
$= \frac{2-2\sin\theta}{\cos\theta(1-\sin\theta)} = \frac{2}{\cos\theta} = 2\sec\theta$
$g\sec\theta = 2\sec\theta$
$g = 2$ [D]

(16) $\frac{\sqrt{5}}{4} = \frac{\cot x}{1-\cot x}$
$\sqrt{5} - \sqrt{5}\cot x = 4\cot x$
$\cot x = \frac{\sqrt{5}}{4+\sqrt{5}}$
$\tan x = \frac{4+\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}+5}{5}$
$= \frac{12\sqrt{5}+15}{15}$ $A+B+C = 12+5+15 = 32$ [D]

(17) $7^2 = 4^2 + 6^2 - 2(4)(6)\cos x$
$49 = 16 + 36 - 48\cos x$
$\cos x = \frac{1}{16}$
$\sec x = 16$ [E]

(18) $\cot(x-y) = \frac{1+\tan x\tan y}{\tan x - \tan y}$
$= \frac{1+\frac{1}{a}\cdot\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}} = \frac{\frac{ab+1}{ab}}{\frac{b-a}{ab}}$
$= \frac{ab+1}{b-a}$ [E]

(19) values of $\sin x$ repeat every 2π (or 360°)
$\text{Prob} = \left(\frac{\arcsin.7 - \arcsin.6}{2\pi}\right)2$
multiply by 2 because there are 2 such intervals
$\doteq 6.042$ [D]

(20) $\sin^2 x + \sin^2 y = 1$
$\sin^2 x + \cos^2(\frac{\pi}{2} - y) = 1$
$x = \frac{\pi}{2} - y$
$x + y = \frac{\pi}{2}$
$\doteq 1.57079\ldots$
$\doteq 1.5708$ to 4 places
$5+7+0+8 = 20$ [C]

(21) $y = \frac{e^x - e^{-x}}{2}$
$x = \frac{e^y - e^{-y}}{2}$
$2x = e^y - e^{-y}$
$2x = e^y - \frac{1}{e^y}$
$2xe^y = e^{2y} - 1$
$e^{2y} - 2xe^y - 1 = 0$
$e^y = \frac{2x \pm \sqrt{4x^2+4}}{2}$
$= x + \sqrt{1+x^2}$
$e^y = x + \sqrt{1+x^2}$
$y = \ln(x + \sqrt{1+x^2})$ [B]

(22) $\lim_{x\to 0}\frac{\sin x}{x} + \lim_{x\to\infty}\frac{\cos x}{x}$
$= 0 + 0 = 0$ [B]

(23) $x^2 - y^2 = \sec^2\theta - \tan^2\theta$
$x^2 - y^2 = 1$
hyperbola [D]

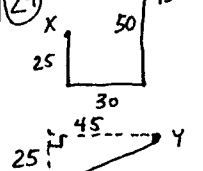
(24) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
$A^{-1} = \frac{1}{-\cos^2 x - \sin^2 x}\begin{bmatrix} \cos x & -\sin x \\ -\sin x & -\cos x \end{bmatrix}$
$= -1\begin{bmatrix} \cos x & -\sin x \\ -\sin x & -\cos x \end{bmatrix} = \begin{bmatrix} -\cos x & \sin x \\ \sin x & \cos x \end{bmatrix}$ [B]

(25) $\frac{x^2}{\frac{1}{4}} + \frac{y^2}{1} = 1$
Area $= \pi ab$
$= \pi(\frac{1}{2})(1) = \frac{\pi}{2}$
$\sin(\frac{\pi}{2}) = 1$ [D]

(26) $\frac{\sec x - 1}{\sec x + 1} = \frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x} + 1}$
$= \frac{1-\cos x}{1+\cos x} = \left(\pm\sqrt{\frac{1-\cos x}{1+\cos x}}\right)^2$
$= \tan^2(\frac{x}{2})$
$\frac{1}{2}$ angle identity for tan
$\frac{P}{Q} = \frac{1}{2}$
$P+Q = 1+2$
$= 3$ [A]

(27) $\csc^2 x(\cos^4 x - \sin^4 x - 1) =$
$\csc^2 x[(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) - 1] =$
$\csc^2 x(\cos^2 x - \sin^2 x - 1) = \csc^2 x(\cos^2 x - 1 - \sin^2 x)$
$= \csc^2 x(-\sin^2 x - \sin^2 x) = \csc^2 x(-2\sin^2 x)$
$= \frac{-2\sin^2 x}{\sin^2 x} = -2$ [A]

(28) $.\overline{2}_6 = \frac{2}{6} + \frac{2}{36} + \ldots$
$= \frac{\frac{2}{6}}{1-\frac{1}{6}} = \frac{2}{5}$
$.\overline{4}_8 = \frac{4}{8} + \frac{4}{64} + \ldots$
$= \frac{\frac{4}{8}}{1-\frac{1}{8}} = \frac{4}{7}$
$\arcsin(\frac{2}{5}) + \arcsin(\frac{4}{7}) \doteq 56.7$

(29) 

Ship 1 travels 120 miles at 20 mph = 6 hours
Ship 2 takes 5.5 hours to travel $\sqrt{25^2+45^2}$ miles
$= \sqrt{2650} = 5\sqrt{106}$
Ship 2 travels $\frac{5\sqrt{106}}{5.5}$ mph, or $\frac{10\sqrt{106}}{11}$ [C]

(30) $y = \sqrt{\csc x + y}$
$y^2 = \csc x + y$
$y^2 - y = \csc x$
$x = \text{Arccsc}(y^2 - y)$
$= \text{Arccsc}[(y)(y-1)]$
[A]