# Famat state Competition - Calculus - 2002

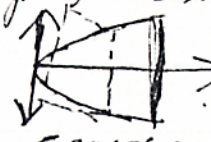
d 1. $\int \sin x\,dx + \frac{1}{3}\int e^{3x}\cdot 3\,dx = -\cos x + \frac{e^{3x}}{3} + C$

d 2.

b 3. $e^{xy}[xy' + y] + 3y' = 0 \quad (0, \frac{4}{3})$ $1[\frac{4}{3}] + 3y' = 0$ $3y' = -\frac{4}{3}$ $y' = -\frac{4}{9}$

b 4. $\frac{4}{8}[\sin(.5) + 2\sin 1 + 2\sin(1.5) + 2\sin(2) + \sin(2.5)]$

c 5. $s'(2)$ does not exist since $s(2)$ does not exist.
$g'(2)$ does not exist $\lim_{x\to 2} g(s(x))$ does not exist,

c 6. 

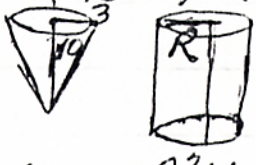
$A = bh \quad A = 8y^2$ $8\int_0^{16} x\,dx = 8\Big|_0^{16} \frac{x^2}{2} = 4(16)^2 = 1024$

e 7. $\int \frac{\cos x \sin x}{-\sin^2 x}\,dx = -\int \frac{\cos x}{\sin x}\,dx = -\int \cot x\,dx = -\ln|\sin x| + C$

d 8. $y' = \frac{6}{x} - \frac{1}{4} \quad x = 24$ $\frac{5' +}{24}$ $6\ln 24 - 6$

b 9. $A = \frac{1}{2}(10)^2 \sin\theta \quad A = 50\sin\theta \quad \frac{dA}{d\theta} = 50\cos\theta$
$\theta = \pi/6 \quad d\theta = \frac{\pi}{60}$ $50\cos\frac{\pi}{6}\cdot\frac{\pi}{60} = 50\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\pi}{60}\right) \doteq 2.267$

b 10. $\int_1^3 (t^2 - t - 6)\,dt = \Big|_1^3 \frac{t^3}{3} - \frac{t^2}{2} - 6t = (9 - 9/2 - 18) - (1/3 - 1/2 - 6) =$
$(-13\tfrac{1}{2}) - (6\tfrac{1}{6}) = -13\tfrac{3}{6} + 6\tfrac{1}{6} = -7\tfrac{1}{3}$

a 11. 

$\frac{r}{h} = \frac{3}{10}$ $V = \frac{\pi}{3}\left(\frac{9h^2}{100}\right)h \quad V = \frac{3h^3\pi}{100}$
$10r = 3h$ $\frac{dV}{dt} = \frac{9\pi h^2}{100}\frac{dh}{dt}$ at $h = 10$ $\frac{dV}{dt} = 9\pi$
$r = \frac{3h}{10}$

$V_{cyc} = \pi R^2 H$
$\frac{dV}{dt} = \pi R^2 \frac{dH}{dt} \quad 9\pi = \pi r^2 \frac{dH}{dt} \quad 9 = r^2 \frac{dH}{dt} \quad \frac{9}{r^2} = \frac{dH}{dt}$

d 12. 

$2\pi\int_0^2 x(8 - x^3)\,dx = 2\pi\Big|_0^2 4x^2 - \frac{x^5}{5} = 2\pi\left(16 - \frac{32}{5}\right) = \frac{96\pi}{5}$

b 13. $\lim_{x\to\infty} \frac{2|x|}{(x-1)^2} = 0 \quad \lim_{x\to-\infty} \frac{2|x|}{(x-1)^2} = 0 \quad y = 0$
$x = 1$

c 14. $\sec^2(\tan(\tan(2x)))\cdot\sec^2(\tan(2x))\cdot\sec^2(2x)\cdot 2$
$(0,0) \quad m_{tan} = 2 \quad 2x - y = 0$

b 15. $y\,dy = \tan(x)\,dx \quad \frac{y^2}{2} = \ln|\sec x| + C \quad 2 = \ln|+C \quad C = 2$
$\frac{y^2}{2} = \ln|\sec x| + 2 \quad y^2 = 2\ln|\sec x| + 4 \quad y = \sqrt{2\ln|\sec x| + 4}$
$y(1) \doteq 2.287$

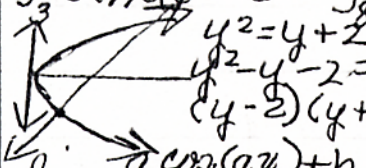
d 16. $s'(x) = \frac{1}{2}x - \sin x \quad s''(x) = \frac{1}{2} - \cos x \quad \cos x = \frac{1}{2}$
$\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

a 17. $2[f(g(x))] \cdot f'(g(x)) \cdot g'(x)$

$2(2)(-1/2)(3) = -6$

a 18. $f(2)=2 \quad f'(x)=2x(x^4+3x^2-8) \quad f'(2)=80 \quad 80x-y=158$

d 19. $\int_3^8 f(x)\,dx = -5 \quad \int_8^3 [f(x)-g(x)]\,dx = 5-(-5)=10$

d 20. 

$y^2 = y+2$

$y^2-y-2=0$

$(y-2)(y+1)=0$

$\int_{-1}^{2}(y+2-y^2)\,dy = \left|_{-1}^{2} \frac{y^2}{2}+2y-\frac{y^3}{3}\right.$

$(2+4-8/3)-(1/2-2+1/3) = 8-3-1/2 = 4\,1/2$

a 21. $\lim_{x\to 0} \frac{a\cos(ax)+b}{3x^2} = \lim_{x\to 0} \frac{-a^2\sin(ax)}{6x} = \lim_{x\to 0} \frac{-a^3\cos(ax)}{6} = \frac{-32}{3}$ (if limit exists)

$a\cos(4x)+b=0$ at $x=0$ $\quad \therefore b=-4 \quad ab=-16$

$\frac{-a^3}{6} = \frac{-32}{3} \quad -a^3=64 \quad a=4$

b 22. $300\int_0^{10} 2^{-.3t}\,dt \quad u=-\frac{3}{10}t \quad du=-\frac{3}{10}dt \quad -1000\int_0^{-3} 2^u\,du = 1000\int_{-3}^{0} 2^u\,du =$

$\frac{1000}{\ln 2}\left|_{-3}^{0} 2^u = \frac{1000}{\ln 2}\left[1-\frac{1}{8}\right] = \frac{1000}{\ln 2}\cdot\frac{7}{8} = \frac{875}{\ln 2} \doteq 12\right.$

c 23. $(x,x^2) \quad (x,x) \quad (y,y) \quad (y,y) \quad f(y)=y-\sqrt{y} \quad f'(y)=1-\frac{1}{2\sqrt{y}} \quad \frac{1}{2\sqrt{y}}=1 \quad \frac{1}{2}=\sqrt{y} \quad 1/4=y$

a 24. $\int_{\pi/4}^{\pi/3} \frac{\sec^2 x}{2\tan x}\,dx \quad u=\tan x \quad du=\sec^2 x\,dx \quad \int_1^{\sqrt{3}} \frac{1}{2u}\,du \quad \left|_1^{\sqrt{3}} \frac{1}{2}\ln|u| = \frac{1}{2}\ln\sqrt{3}-\frac{1}{2}\ln 1\right.$

a 25. $x=y^3+y \quad 1=3y^2y'+y' \quad y'=\frac{1}{3y^2+1} \quad y'(30)=\frac{1}{28}$

$(30,3)\in S^{-1}$

b 26. $f(1)=2 \quad f'(x)=ax(e^{bx^2})\cdot 2bx + e^{bx^2}\cdot a \quad 2b+1=0$

$ae^b=2 \quad 0=ae^{bx^2}[2bx^2+1] \quad f'(1)=0 \quad 2b=-1 \quad b=-1/2$

$ae^{-1/2}=2 \quad a=2\sqrt{e} \quad ab=-\sqrt{e}$

e 27. $f''(x)=\frac{1}{x-2}$ for $x>2$ $\quad \therefore e$

$f''(x)>0$

d 28. $\frac{2}{\sqrt{1-\frac{1}{4}}} = \frac{2}{\sqrt{3/4}} = \frac{2}{\sqrt{3}/2} = \frac{4\sqrt{3}}{3}$

c 29. $x=\tan y \quad \pi\int_0^{\pi/4} (\tan y)^2\,dy$

a 30. $\int[1-2x\csc(x^2)\,dx] = x-\ln|\csc(x^2)-\cot(x^2)|+C$