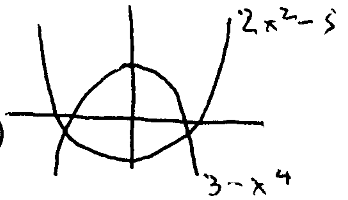
Calc Apps States 2002 Solutions

1) D



$2x^2-5=3-x^4 \Rightarrow x=\pm\sqrt{2}$

$A=\int_{-\sqrt{2}}^{\sqrt{2}}\left[(3-x^4)-(2x^2-5)\right]dx = \dfrac{176\sqrt{2}}{15}$ (D)

2) $\dfrac{dP}{dt}=kP \Rightarrow P=Ce^{kt}$

1999 $(t=0)$
2000 $(t=1)$
1984 $(t=-15)$

B $1.5\times10^6 = Ce^{k(0)} \Rightarrow C=1.5\times10^6$

$2.4\times10^6 = 1.5\times10^6\, e^{k(1)} \Rightarrow k\approx .47$

$P(-15)\approx 1300$ (B)

3) $10\ \text{km/hr}\approx 2.78\ \text{m/s}$

A $v=\int a\,dt = at+v_0$

$d=\int v\,dt=\frac{1}{2}at^2+v_0t+d_0$

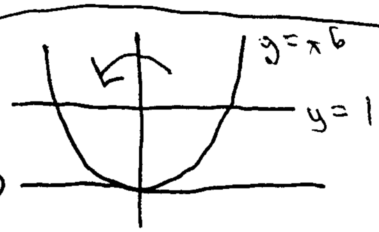
$0=at+2.78$

$20=2.77t+\frac{1}{2}at^2$

simultaneously solve for t

$t\approx 14.4$ (A)

4) D



$V=2\pi\int_0^1 x(1-x^6)\,dx = \dfrac{3\pi}{4}$ (D)

5) $\dfrac{dx}{dt}=k(x-25) \Rightarrow x=25+Ce^{kt}$

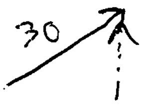
C

$100=25+Ce^{k(0)}$
$\Rightarrow C=75$
$95=25+75e^{k(3)}$
$\Rightarrow k\approx -.0$

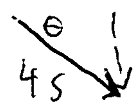
$90=25+75e^{-.023t} \Rightarrow t\approx 6$ (C)

6) E

F_1:



F_2:



$30\sin 50° = 45\sin\theta$

$\Rightarrow \theta\approx 31°$

7) $d=\frac{1}{2}at^2+v_0t+d_0$ (see #3)

E $50=\frac{1}{2}(-9.8)t_1^2+100 \Rightarrow t_1=3.2\ s$

$0=\frac{1}{2}(-9.8)t_2^2+((-9.8)(3.2)+0)t_2+50 \Rightarrow t_2=1.3$

$t_1-t_2=1.9\ s$ (E)

8) B $R=30\ \Omega$

$-\dfrac{1}{R^2}\dfrac{dR}{dt}=-\dfrac{1}{R_1^2}\dfrac{dR_1}{dt}-\dfrac{1}{R_2^2}\dfrac{dR_2}{dt}$

$\dfrac{dR}{dt}=.6$ (

9) $V=.9=\pi r^2h \Rightarrow h=\dfrac{.9}{\pi r^2}$

C $SA=2\pi r^2+2\pi rh=2\pi r^2+\dfrac{2(.9)}{r}$

$SA'=0 \Rightarrow r\approx .523$

$d\approx 1.05\ m$

10) A

$f(0)=1$
$f'(0)=0$
$f''(0)=-4$
$f^{(3)}(0)=0$
$f^{(4)}(0)=16$

$p(x)=1-2x^2+\dfrac{2x^4}{3}$ (A)

11) $y = r\sin\theta$ $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{\ldots \sin\ \theta\cos\theta}{2\sin\theta\cos^2\theta - \sin^3\theta}$

$x = r\cos\theta$

A

$\dfrac{dy}{dx}$ @ $\theta = \pi/4$ $\dfrac{dy}{dx} = 3$ (A)

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12) $\dfrac{1.25}{2}\left(\ln(3^2+5) + 2\ln(1.75^2+5) + 2\ln(.5^2+5) + 2\ln(.75^2+5) + \ln(2^2+5)\right)$

A ≈ 9.85 (A)

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13) $x_{n+1} = x_n - \dfrac{e^{x_n - 1} - 3}{e^{x_n - 1}}$

A

$x_0 = 1$

$x_1 = 3$

$x_2 \approx 2.41$ (A)

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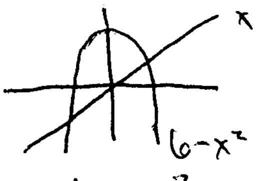
14)

C



$6 - x^2 = x \Rightarrow x = \{-3, 2\}$

$A = \int_{-3}^{2}\left[(6-x^2) - x\right]dx = \dfrac{125}{6}$

$\left(-\frac{1}{2}, 2\right)$ (C

$\bar{x} = \dfrac{1}{A}\int_{-3}^{2} x(6 - x^2 - x)\,dx = -1/2$ $\bar{y} = \dfrac{1}{A}\int_{-3}^{2}\frac{1}{2}(6-x^2-x)(6-x^2+x)\,dx =$

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15) $\dfrac{1}{\sqrt{2\pi}} \cdot \dfrac{1/2}{3}\left(e^{-1/2} + 4e^{-(.5)^2/2} + 2e^0 + 4e^{-(.5)^2/2} + e^{-1/2}\right) \approx .68$ (B)

B

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16) $\int_1^{\infty}\dfrac{K}{x^3}dx = 1$ $\left.\dfrac{-K}{2x^2}\right|_{x=1}^{x=\infty} = 1 \Rightarrow K = 2$ (B)

B

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17) $E(x) = \int_0^{\pi/2} x\sin(2x)\,dx = \pi/4$ (E) using integration by parts

E

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18) $\int_2^c 4(x-2)^3dx = \left.(x-2)^4\right|_{x=2}^{x=c} \Rightarrow (c-2)^4 = 1/2 \Rightarrow c = 2 + \dfrac{1}{\sqrt[4]{2}}$ (

D

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19) $y_1 = 4 + (.5)(2(1)^2 - 4) = 3$

C $y_2 = 3 + (.5)(2(1.5)^2 - 4) = 3.75$

$y_3 = 3.75 + (.5)(2(2)^2 - 4) = 5.875$ (C)

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20) $\vec{J} = \left\langle \int_1^5 t^2dt, \int_1^5 3^t\,dt \right\rangle = \left\langle \dfrac{124}{3}, \dfrac{240}{\ln 3} \right\rangle$

D mag $= \sqrt{\left(\frac{124}{3}\right)^2 + \left(\frac{240}{\ln 3}\right)^2} \approx 222$ (D)

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21) $\dfrac{dV}{ds} = \dfrac{dV/dt}{ds/dt} = \dfrac{4\pi r^2\,dr/dt}{8\pi r\ dr/dt} = \dfrac{r}{2} = \dfrac{3}{2}$ (A)

A

22) graph of $(R\cos t, R\sin t)$ is circle of radius R
B centered at $(0,0)$ $\therefore$ surface area $= 4\pi R^2$ (B)

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23) $2P = Pe^{rt} \Rightarrow 2 = e^{.08t} \Rightarrow t \approx 8.7$
B

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24) $\vec{v}(t) = \vec{r}\,'(t) = \dfrac{t\cos t - \sin t}{t^2}\,\hat{\imath} + \dfrac{1}{t}\,\hat{k}$
C $\vec{v}(\pi) = -\dfrac{1}{\pi}\,\hat{\imath} + \dfrac{1}{\pi}\,\hat{k}$

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25) $d = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + (4x+7)^2}$ $\quad 2x + 2(4)(4x+7) = 0 \Rightarrow x = -28/17$
C $y = 4\left(\frac{-28}{17}\right) + 7 = 7/17 \Rightarrow \left(\frac{-28}{17}, \frac{7}{17}\right)$ (C)

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26)


A $A = \dfrac{w}{3}\left(\dfrac{w}{3}\sin(180-\theta)\right) + \dfrac{w^2}{9}\cos(180-\theta)\sin(180-\theta$
$A'(\theta) = 0 \Rightarrow \theta = 120°$ (A)

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27) I. true because $\vec{r}\,''(t) \neq \vec{0}$
D II. true because $\vec{r}\,'(t) \neq \vec{0}$
III. true $s = \int_0^t \sqrt{(2t)^2 + (e^t)^2 + \frac{1}{25} + 1}\,dt$

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28) $\dfrac{dP}{da} = 2 + 6e^{ab}$ (E)
E

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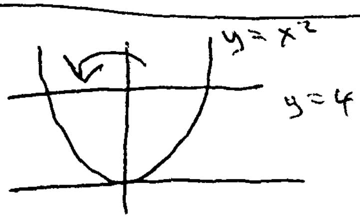
29)


$V = 2\pi\int_0^2 x(4-x^2)\,dx = 8\pi$ (C)
C

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30) $\bar{f} = \dfrac{1}{\pi/2 - \pi/4}\int_{\pi/4}^{\pi/2} \sin(2x)\,dx = 2/\pi$ (D)
D