Calculus Applications
FAMAT State Convention 2002

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. A swimming pool is going to be constructed in the shape of the region enclosed by the graphs of $y = 3 - x^4$ and $y = 2x^2 - 5$. How much land should be marked off? (Answer in square units)

A) $\frac{88\sqrt{2}}{15}$
B) $176\sqrt{2}$
C) $\frac{176}{5}$
D) $\frac{176\sqrt{2}}{15}$
E) NOTA

2. The rate of growth of bacteria is proportional to the amount of bacteria in a population. This is given by the differential equation $\frac{dP}{dt} = kP$, where P is the bacteria population, t is time, and k is a constant. Two measurements of a specific sample were taken in the years 1999 and 2000 with populations of 1.5 million and 2.4 million respectively. How much bacteria was around in the year 1984? (Answer in two significant figures)

A) 900
B) 1,300
C) 140,000
D) 270,000
E) NOTA

3. A man driving his car at 10 km/hr suddenly sees a white cat jump in the road 20 meters away. He then slams on his brakes and stops the car just before hitting the cat. Assuming constant deceleration, in what amount of time did the car travel the 20 meters?
(Answer to the nearest tenth of a second)

A) 14.4
B) 15.9
C) 16.0
D) 28.8
E) NOTA

4. A flowerpot is going to be made in the shape of the rotation of the region bounded by $y = x^6$ and $y = 1$ about the y-axis. How much soil will the pot be able to hold? (Answer in cubic units)

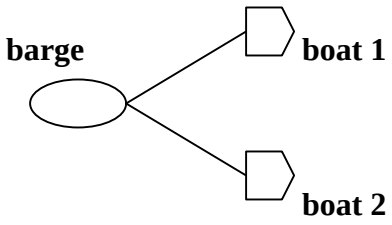
A) $\frac{24\pi}{7}$
B) $\frac{14\pi}{9}$
C) $\frac{3\pi}{2}$
D) $\frac{3\pi}{4}$
E) NOTA

5. Through experimentation it has been found that an object cools according to the following differential equation: $\frac{dx}{dt} = k(x - a)$ where x is the temperature of the object, t is the time it has been cooling, and a is the ambient temperature. Suppose a ball bearing is taken out of boiling water and placed in an air-conditioned room at 25° C. It takes three minutes for the ball bearing to cool to 95° C. How long does it take for the ball bearing to cool to 90° C? (Answer to the nearest minute)

A) 3
B) 4
C) 6
D) 10
E) NOTA

6. Two boats are towing a barge that requires 50 Newtons of force to keep moving. One boat is providing 30 Newtons at an angle of 50° North of East. The other can provide 45 Newtons maximum. What is the minimum angle South of East that the second boat must travel along so that the barge keeps going east? (Answer to the nearest degree)

A) 34°
B) 37°
C) 49°
D) 57°
E) NOTA



7. A basketball falls from 100 meters above the ground. How much more time does the ball take to fall the first 50 meters than the second 50 meters? (Answer to the nearest tenth of a second, acceleration due to gravity = 9.8 m/s^2)

A) 1.3
B) 1.5
C) 2.2
D) 4.5
E) NOTA

8. When two resistors R_1 and R_2 are connected in parallel, the total resistance R is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. How fast is the total resistance increasing when R_1 and R_2 are increasing at rates of 1 Ω/s and 1.5 Ω/s respectively and R_1 = 50 Ω and R_2 = 75 Ω?

A) 0.1 Ω/s
B) 0.6 Ω/s
C) 1.25 Ω/s
D) 2.3 Ω/s
E) NOTA

9. A cylindrical gas tank is to hold 0.9 m^3 of fuel. What is the diameter of a tank that will minimize the cost of sheet metal used to construct the tank? (Answer to the nearest hundredth of a meter)

A) 0.52 m
B) 0.67 m
C) 1.05 m
D) 1.34 m
E) NOTA

10. Which of the following is the correct 4th degree Taylor polynomial for f(x) = cos (2x) centered at $x_0 = 0$?

A) $p(x) = 1 - 2x^2 + \frac{2x^4}{3}$

B) $p(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$

C) $p(x) = x^2 - \frac{x^4}{6}$

D) $p(x) = 4x^2 - \frac{4x^4}{3}$

E) NOTA

11. What is the slope of the tangent line to $r = \sin^2(\theta)$ when $\theta = \pi/4$?

A) 1

B) $\sqrt{2}$

C) $2\sqrt{3}$

D) 3

E) NOTA

12. Use four trapezoids to approximate the area bounded by $y = \ln(x^2 + 5)$, x = -3, x = 2, and y = 0. (Answer to the nearest hundredth of a square unit)

A) 9.85
B) 9.74
C) 9.72
D) 9.67
E) NOTA

13. Use Newton's method to find x_2 given $x_0 = 1$ for the equation $e^{x-1} = 3$. (Answer to the nearest hundredth)

A) 2.41
B) 2.57
C) 2.14
D) 3.00
E) NOTA

14. What are the coordinates of the centroid of the region bounded by $y = 6 - x^2$ and $y = x$?

A) (0 , 3)

B) (-1 , 2)

C) $\left(-\frac{1}{2}, 2\right)$

D) $\left(-\frac{3}{4}, \frac{5}{2}\right)$

E) NOTA

15. Use Simpson's rule with n = 4 to approximate the area between $y = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$ and y = 0 from x = -1 to x = 1. (Answer to the nearest hundredth of a square unit)

A) 0.65
B) 0.68
C) 0.95
D) 0
E) NOTA

16. A probability density function is to have the form $p(x) = \frac{k}{x^3}$ on the interval [1,∞). What is the value of *k*?

A) $\frac{1}{3}$
B) 2
C) $\frac{7}{3}$
D) $\frac{1}{2}$
E) NOTA

17. The expected value (mean) of a probability density function is defined as $E(x) = \sum_{i=1}^{n} x_i \cdot p(x_i)$.

Find the expected value of $p(x) = \sin(2x)$ for $0 < x < \frac{\pi}{2}$.

A) $\frac{\pi}{8}$
B) $\frac{5\pi}{8}$
C) $\int_0^x t\sin(2t)dt,\ 0 < x < \frac{\pi}{2}$
D) $\frac{2}{\pi}\int_0^x t\sin(2t)dt,\ 0 < x < \frac{\pi}{2}$
E) NOTA

18. The median of a probability density function is the value *c* such that
$p(a < x < c) = p(c < x < b) = \frac{1}{2}$ where [a,b] is the interval for which $p(x)$ is defined.
What is the median of $p(x) = 4(x-2)^3$ for 2 < x < 3 ?

A) 2.5
B) 2.75
C) $2 - \frac{1}{\sqrt[4]{2}}$
D) $2 + \frac{1}{\sqrt[4]{2}}$
E) NOTA

19. Euler's method provides a way to find numerical approximations to solutions of differential equations. For the general first order equation y' = F(x,y) the algorithm for the approximations is $y_{n+1} = y_n + h \cdot F(x_n, y_n)$ where n is the number of trials and h is the step size. Use Euler's method to approximate y(2) of $\frac{dy}{dx} = 2x^2 - y$ with y(1) = 4 and h = 0.5.

A) 3
B) 3.75
C) 5.875
D) 9.1825
E) NOTA

20. The impulse $\vec{J}$ due to a force $\vec{F}(t)$ at time *t* is related by the equation $\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt$.

Find the magnitude of the impulse from a force $< t^2, 3^t >$ from t = 1 to t = 5. (Answer to the nearest unit)

A) 5
B) 241
C) 243
D) 222
E) NOTA

21. The radius of a sphere is changing at a rate of 2.5 cm/s when the diameter is 6 cm.
Find $\frac{dV}{dS}$ where V is the volume of the sphere and S is the surface area of the sphere.

A) 1.5
B) 2
C) 2.5
D) 3
E) NOTA

22. What is the surface area of the surface created when the parametric curve defined by
x = R·cos(t) and y = R·sin(t) from t = 0 to t = 2π is revolved about the y-axis?

A) πR^2
B) $4\pi R^2$
C) $\frac{4}{3}\pi R^3$
D) $\frac{\pi}{R^2}$
E) NOTA

23. How long will it take an investment to double if it is compounded continuously at a rate of 8.00% ? (Answer to the nearest tenth of a year)

A) 5.6
B) 8.7
C) 0.6
D) 17.3
E) NOTA

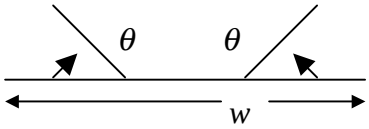
24. The position of a particle moving in space is given by the equation $\vec{r}(t) = \frac{\sin(t)}{t}\hat{i} - \hat{j} + \ln(t)\hat{k}$ where t > 0.
Find its velocity vector at t = π .

A) $\frac{1}{\pi}\hat{i} + \hat{j} + \frac{1}{\pi}\hat{k}$
B) $-\frac{1}{\pi}\hat{i} + \hat{k}$
C) $-\frac{1}{\pi}\hat{i} + \frac{1}{\pi}\hat{j}$
D) $\frac{1}{\pi}\hat{i} - \hat{k}$
E) NOTA

25. A roach crawls along the line determined by y = 4x + 7. At what point is it closest to the origin?

A) $\left(-\frac{7}{5}, \frac{7}{5}\right)$
B) $(0,7)$
C) $\left(-\frac{28}{17}, \frac{7}{17}\right)$
D) $\left(\frac{28}{17}, \frac{231}{17}\right)$
E) NOTA

26. A rain gutter is to be constructed of a metal sheet of length *l* and width *w* by bending up one third of the sheet on each side. What is the angle θ that maximizes the volume of water the gutter can hold?



A) 120°
B) 100°
C) 90°
D) 60°
E) NOTA

27. A rocket is shot from the origin and follows the path given by $\vec{r}(t) = \left\langle t^2, e^t - 1, \frac{t}{5} \right\rangle$ where t > 0.

Which of the following statements are true?

I. The rocket is always accelerating.
II. The rocket's path is continuous and smooth
III. A graph of path length vs. *t* would be strictly increasing.

A) I only.
B) II only.
C) I and II
D) I, II, and III
E) NOTA

28. Suppose that the amount of production, *P*, from a factory depends on the equation $P_{(a,b)} = 2a + e^{ab}$, where *a* and *b* are the number of workers in sectors **A** and **B** respectively. What is the rate of change of the production with respect to the number of workers in sector **A**?

A) $2 + e^b$
B) $2a + e^a$
C) $2 + e^{ab}$
D) 0
E) NOTA

29. Find the volume of the space bounded by the surface $z = x^2 + y^2$ and the plane z = 4.

A) 4π
B) $4\pi^2$
C) 8π
D) 10π
E) NOTA

30. Find the average value of the function f(x) = sin(2x) from x = $\pi/4$ to x = $\pi/2$.

A) $\frac{1}{\pi}$
B) $\frac{\sqrt{3}}{2}$
C) $\frac{\sqrt{2}}{2}$
D) $\frac{2}{\pi}$
E) NOTA