Integration Test
FAMAT State Convention 2002

For all questions, the answer (E) NOTA means "None of the above" answers is correct.

1. If f is a continuous even function, and $\int_0^a f(x)dx = 3$, then $\int_{-a}^a 3f(x)dx =$

(A) -3 (B) 0 (C) 6 (D) 9 (E) NOTA

2. Approximate the value of $\int_1^4 \frac{1}{x}dx$ by the trapezoidal rule using n=3.

(A) 1.083 (B) 1.386 (C) 1.458 (D) 1.833 (E) NOTA

3. Find the average value of the function $f(x) = \frac{x^2+4}{x}$ over the interval [1,4].

(A) $\frac{5}{2} + \frac{8}{3}\ln 2$ (B) $\frac{15}{8} + 2\ln 2$ (C) 3.75 (D) 11.25 (E) NOTA

4. Find the area between the curves $y = x+1$ and $y = x^2 - 1$.

(A) $\frac{10}{3}$ (B) $\frac{9}{2}$ (C) $\frac{2}{3}$ (D) 0 (E) NOTA

5. Approximate the value of $\int_0^1 \sqrt{x}dx$ to 3 decimal places using the Midpoint Rule, with n=2.

(A) 0.683 (B) 0.707 (C) 1.667 (D) 1.366 (E) NOTA

6. If $F(x)=\int_0^{x^2}\frac{1}{3t^2-1}dt$, find $F'(x)$.

(A) $\frac{1}{3x^4-1}$ (B) $\frac{1}{3x^2-1}$ (C) $\frac{2x}{3x^2-1}$ (D) $\ln(3x^4-1)$ (E) NOTA

7. Find $\int\frac{2}{9+x^2}dx$.

(A) $2\arctan\frac{x}{3}+C$ (B) $\frac{2}{3}\arctan\frac{x}{3}+C$ (C) $2\ln(9+x^2)+C$

(D) $6\arctan x+C$ (E) NOTA

8. $\int_0^{\frac{\pi}{4}}\tan^2 x\,dx=$

(A) $\frac{1}{2}-\frac{\pi}{4}$ (B) $1-\frac{\pi}{4}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{3}$ (E) NOTA

9. $\int\frac{1}{1+e^x}dx=$

(A) $x-\ln(1+e^x)+C$ (B) $\arctan e^x+C$ (C) $\ln(1+e^x)+C$

(D) $\frac{\ln(1+e^x)}{e^x}+C$ (E) NOTA

10. Find $\int_1^3\frac{dx}{2x^2-8x+10}$.

(A) $-\frac{\pi}{4}$ (B) 0 (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$ (E) NOTA

11. If $y' = \frac{2x}{x^2 - 8}$ and if the point (3,1) is on the graph of y, find the value of y when $x = 2.$

(A) $-2 \ln 2$ (B) $2 \ln 2$ (C) $1 - 2 \ln 2$ (D) $1 + 2 \ln 2$ (E) NOTA

12. Find $\lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} (3(1 + \frac{3i}{n})^2 - 2(1 + \frac{3i}{n}))$

(A) 18 (B) 27 (C) 30 (D) 48 (E) NOTA

13. $\int_0^4 (16 - x^2)^{\frac{1}{2}} \, dx =$

(A) $\frac{\pi}{2}$ (B) π (C) 4π (D) 16π (E) NOTA

14. Find the volume of the solid whose base is the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and whose cross sections, taken perpendicular to the x-axis are all squares.

(A) 24π (B) 48π (C) 96 (D) 192 (E) NOTA

15. Which one of the following four functions is not integrable over [1,5]?

(A) $f(x) = |x^2 - 2|$ (B) $f(x) = \ln(x + 1)$ (C) $f(x) = 3 \arctan x$

(D) $f(x) = \begin{cases} 3x & x \le 2 \\ x - 5 & x > 2 \end{cases}$ (E) NOTA

16. Which one of the following four integrals can be used to determine the volume of a solid formed by revolving the region bounded by the curves $y = x^2$ and $y^2 = 8x$ about the y-axis?

(A) $\pi\int_0^4 (\sqrt{y} - \frac{1}{8}y^2)^2\,dy$ (B) $\pi\int_0^2 (y - (\frac{1}{8}y^2)^2)\,dy$ (C) $\pi\int_0^2 (\sqrt{8y} - y)^2\,dy$

(D) $\pi\int_0^4 (y - (\frac{1}{8}y^2)^2)\,dy$ (E) NOTA

17. $\int_0^4 \frac{x^2+3x+2}{x+2}\,dx =$

(A) 12 (B) 8 (C) 5 (D) 4 (E) NOTA

18. Use inscribed rectangles with n=4 to estimate $\int_{-1}^{1} (x^2+1)\,dx$.

(A) 0 (B) $2\frac{1}{4}$ (C) $2\frac{2}{3}$ (D) $2\frac{3}{4}$ (E) NOTA

19. $\int_0^1 \frac{3^x}{3^x+1}\,dx =$

(A) $\frac{\ln 2}{\ln 3}$ (B) $\frac{\ln 4}{\ln 3}$ (C) 0 (D) $\ln 2$ (E) NOTA

20. $\int \frac{1}{x^2+2x-8}\,dx =$

(A) $\ln\left|x^2+2x-8\right|+C$ (B) $\frac{1}{6}\ln\left|\frac{x-2}{x+4}\right|+C$ (C) $\frac{1}{3}\arctan\frac{x+1}{3}+C$

(D) $-\frac{1}{3}\arctan\frac{x+1}{-3}+C$ (E) NOTA

21. An object begins at rest at the point (5,0) and moves along the x-axis with constant acceleration. If the velocity of the object after 3 seconds is 12 units per second, where will the object be located at 10 seconds?

(A) (45,0) (B) (125,0) (C) (205,0) (D) (405,0)

(E) NOTA

22. Find the Riemann sum for $f(x)=x^2-x$ over the interval [0, 6] using $x_0=0, x_1=3, x_2=4, x_3=6$ and $c_1=0, c_2=4, c_3=5.$

(A) 90 (B) 70 (C) 52 (D) 14

(E) NOTA

23. If $\int_{-2}^{1} f(x)\,dx=0$ and $\int_{0}^{1} f(x)\,dx=4$, then $\int_{-2}^{0}(f(x)+2)\,dx=$

(A) –4 (B) –2 (C) 0 (D) 6 (E) NOTA

24. $\int x\sqrt{1-x}\ dx =$

(A) $\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$ (B) $\frac{2-x}{2(1-x)^{\frac{1}{2}}} + C$ (C) $\frac{x^2}{2} - \frac{2x^{\frac{5}{2}}}{5} + C$

(D) $-\frac{2}{15}(1-x)^{\frac{3}{2}}(3x+2) + C$ (E) NOTA

25. Find $F'(x)$ given that $F(x) = \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \tan^4 t\, dt$.

(A) –8 (B) 0 (C) $\frac{27-5\sqrt{3}}{135}$ (D) $\frac{1-9\sqrt{3}}{5}$ (E) NOTA

26. Find the average value of the function $f(x) = \frac{1}{\sqrt{1-x^2}}$ from 0 to $\frac{1}{2}$.

(A) $\frac{\pi}{24}$ (B) $\frac{\pi}{12}$ (C) $\frac{\sqrt{3}}{3}$ (D) $\frac{\pi}{6}$ (E) NOTA

27. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2 3x \cos 3x\ dx =$

(A) $\frac{-\sqrt{2}}{36}$ (B) $\frac{-\sqrt{2}}{12}$ (C) $\frac{-\sqrt{2}}{4}$ (D) $\frac{3\sqrt{3}-2\sqrt{2}}{8}$ (E) NOTA

28. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{3x^3}{1+x^2}$ given the initial condition $y(0) = 1$.

(A) $3x^3 \arctan x + 1$ (B) $\frac{3}{2}\left(x^2 - \ln\left(1+x^2\right)\right) + 1$

(C) $3x - \frac{3}{2}\ln\left(1+x^2\right) + 1$ (D) $\frac{3}{2}x^2 + \frac{3}{2}\ln\left(1+x^2\right) + 1$ (E) NOTA

29. $\int_1^2 (5^3 - 3^2)\,dx =$

(A) $\frac{125}{\ln 5} - \frac{9}{\ln 3}$ (B) 241 (C) 116 (D) 98 (E) NOTA

30. $\int_0^1 10^{3x}\,dx =$

(A) $\frac{3}{\ln 10}$ (B) $\frac{1000}{3\ln 10}$ (C) $\frac{999}{\ln 10}$ (D) $\frac{333}{\ln 10}$ (E) NOTA