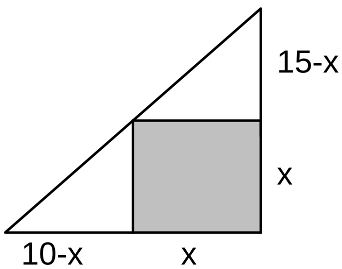
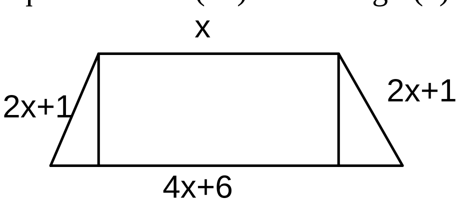
Quadrilaterals Solutions
FAMAT State Convention 2002

1. C With the given information, $m\angle ADE = 50$, so $m\angle BAC = 130$.
$m\angle C + m\angle BAC + m\angle B = 180$
$32 + 70 + m\angle B = 180, m\angle = 78$,
$m\angle EDC + m\angle EBC = 130 + 78 = 208$

2. C Let x=length of a side of the square. Since triangles are similar by AA, set up the proportion $\frac{15-x}{x} = \frac{x}{10-x}$, $x = 6$, area is 36.



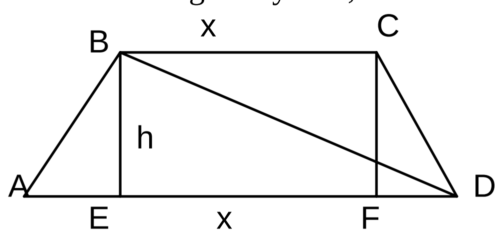
3. D CD=x,AB= $4x+6$ legs $= 2x+1$, $9x+8 = 62. x = 6$, bases are 6 and 30. Use Pythagorean theorem to find height. Using the triangle with hypotenuse 13, base of triangle is 12 so height is 5. Area equals median (18) times height(5)=90.



4. B Draw altitude from B to $\overleftrightarrow{CD}$, since $m\angle C = 60, BE = \frac{7}{2}\sqrt{3}, CE = \frac{7}{2}, ED = \frac{9}{2}$. Use Pythag to find $BD^2 = 57$. Draw altitude from A to $\overleftrightarrow{CD}$. Since altitudes are congruent, $AF = \frac{7}{2}\sqrt{3}, DF = \frac{7}{2}$. Use Pythag in triangle ACF to find $AC^2 = 169. 169 - 57 = 112$.

5. A AB+BN+NM+AM=36
CD+CN+NM+MD=36, adding these two equations and using the the facts AD=AM+MD,BC=BN+NC,
AB+CD+BC+2NM+AD=72
AB+CD+BC+AD =52;
subtracting these two equations gives
2NM=20 which makes NM=10

6. C Let the shorter base = x. Then drawing the altitudes from the upper base vertices, let the base of the rectangle =x. AE =FD= $\frac{8-x}{2}$. BD=7. Use a system using the Pythagorean Theorem. $3^2 + h^2 = \left(\frac{8-x}{2}\right)^2$ and $h^2 + \left(x + \frac{8-x}{2}\right)^2 = 7^2$. Solving each for h^2 and solving the system, $x = 5$.



7. B $m\angle B = m\angle D, 4x + 15 = 6x - 27$, $x = 21$ making $m\angle B = 99$. $\angle A$ is the supplement of $\angle B$ making $m\angle A = 81$

8. A Longest altitude is drawn to shortest side $4\sqrt{3}$. That altitude is opposite a 60 degree angle, so is $\sqrt{3} \bullet \frac{1}{2} \bullet 6\sqrt{2}$.

9. A Quadrilateral formed is a rectangle. Using the midline formula, shorter side is $\frac{1}{2} \bullet 6 = 3$. Longer side is $\frac{1}{2} \bullet 10 = 5$, Area =15.

Quadrilaterals Solutions
FAMAT State Convention 2002

10. E 44 Triangle AXB is isosceles so $m\angle XAB = m\angle XBA = 22$. $\angle BXC$ is an exterior angle of that triangle so is the sum of the two remote interior angles.

11. D Draw the square. Using one of the trapezoids formed, draw the altitude of the trap from the vertex of the octagon. Let this length be x. The triangle formed by drawing the altitude is isosceles right so the other leg of the triangle is x. The hypotenuse is 5, $x = \frac{5}{2}\sqrt{2}$. Side of the square is $10+5\sqrt{2}$, area = $150+100\sqrt{2}$

12. D If a quadrilateral is inscribed in a circle, opposite angles are supplementary. $m\angle = 100$.

13. A Only II is true for $\overline{CD} \parallel \overline{AB}$.

14. B DEAF is a parallelogram (opp sides parallel). $\angle B \cong \angle C$ by Isos Triangle Thm. $\angle A \cong \angle DFC \cong \angle BED$ - corresponding angles of parallel lines, $m\angle A = m\angle BED = m\angle EFG = m\angle DFC$ =x. $m\angle B = m\angle C = y$, $m\angle EDB = m\angle DC = z$, $x + y + z = 180, 2z + x = 180,$ solving this system, $y = z$, so BE=DE=AF. Since AB=7, AE+ED=7 and DF+AF=7. p=14

15. D Extend $\overleftrightarrow{ED}$ so it intersects $\overline{AB}$. altitude of triangle AEB= $2\sqrt{3}$, so EF=BC $-2\sqrt{3} = 4 - 2\sqrt{3}$.

16. E $10+10\sqrt{3}$ Draw both diagonals forming 30-60-90 triangles since diagonals in a rhombus are $\perp$ and bisect angles. Legs of triangles formed are 5 and $5\sqrt{3}$.

17. B Triangle EBC is right isosceles. Since hypotenuse $=5\sqrt{2}$, legs = 5. area of triangle CBE= $\frac{25}{2}$, area of trapezoid is $\frac{95}{2}$. So ratio is 5:19.

18. C Distance to midpoint of each side containing the vertex is 2, since there are two of these the distance is 4. Drawing the segment from the vertex to the midpoint of the other two sides forms 2 congruent right triangles with legs 4 and 2 making the hypotenuse $2\sqrt{5}$,there are 2 of these so that sum is $4\sqrt{5}$. 4+ $4\sqrt{5}$

19. D Area of square is $\frac{d^2}{2}$ = 72.

20. A is the only true statement

21. B Opposite angles in an inscribed quadrilateral are supplementary making $m\angle ABC = 112$. $\angle EBC$ is the supplement of $\angle ABC$.

22. C Since the perimeter is 28 and the bases are 8 and 12, the legs of the trapezoid have to be 4. Drawing the altitudes to form triangles, the leg that is not the altitude must be 2 so the triangles are 30-60-90 making the altitude $2\sqrt{3}$. The area is the median 10 times the height $2\sqrt{3} = 20\sqrt{3}$.

23. A Find the equations of the diagonals, then solve the system. Equation of $\overleftrightarrow{AC} = 5x + 2y = 5$, equation of $\overleftrightarrow{BD} = x = 4y = 11$. Point of intersection is $\left(\frac{21}{11}, -\frac{25}{11}\right)$. Sum is $-\frac{4}{11}$.

Quadrilaterals Solutions
FAMAT State Convention 2002

24. C EF=4 making FB=8. Use Pythagorean Theorem to find AF= $4\sqrt{13}$

25. E Using Pythag, AC=15 making the area of ABC=60, AD=12 making the area of triangle ACE=84. Sum is 144.

26. B Diagonals are perpendicular and bisect each other forming 4 congruent triangles. Hypotenuse = 25, let = 7 so other leg is 24. Area is product of diagonals divided by 2. $\frac{48 \bullet 14}{2} = 336$

27. C Triangle AEF is similar to triangle CEF by AA (sides are ||). So CE:EA=BC:AF. Since DF=4x, FA=3x, BC=7x (opposite sides congruent) making the ration 7:3.

28. A Altitudes are congruent, so the ratio of the areas is the ratio of the bases 15:18=5:6.

29. B Let the longer side equal x. The shorter side is $77 - x$ since perimeter is 154, the sum of 2 consecutive sides is 77. Set the areas equal $(77 - x)(12) = 10x$. Therefore $x = 42$. Since 42 is the longer side, its altitude is 10 so area is 420.

30. C Set $x + 4 = 2x - 1$ and $x = 5$. Making AB=AD=8 and BC=CD=9. Perimeter is 34.