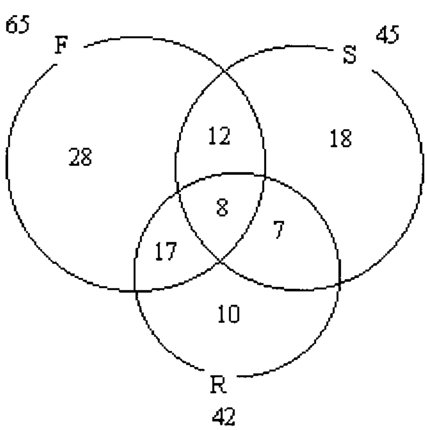
1	D	Cardinality of aleph-null, means that a set can be put into a 1-1 correspondence with the positive integers. I) yes one way to order the integers is 0, 1, -1, 2, -2, 3, -3 …. etc II) yes $\begin{array}{ccccc} \frac{1}{1} & \frac{2}{1} & \frac{3}{1} & \frac{4}{1} & \cdots \\ \frac{1}{2} & \frac{2}{2} & \frac{3}{2} & \frac{4}{2} & \cdots \\ \frac{1}{3} & \frac{2}{3} & \frac{3}{3} & \frac{4}{3} & \cdots \\ \frac{1}{4} & \frac{2}{4} & \frac{3}{4} & \frac{4}{4} & \cdots \end{array}$ to count positive rationals go in this order1/1, 1/2, 2/1, 1/2, 1/3, ~~2/2~~, 3,1, 4/1 etc (skip duplicates). To do all rationals, proceed like (I) 0, 1/1, -1/1, 1/2, -1/2 etc III) no
2	A	There are 25 such integers. To count all sets, use the multiplication rule with 2 choices (include or not included) for each position. Thus 2 * 2 * 2 * … * 2 (25 times) 2^{25}
3	E	Using the info given, complete the following Venn diagram.  I. 12 play football and soccer but not rugby (TRUE) II. 10 play only rugby (TRUE) III. 16 play football and rugby but not soccer (FALSE)
4	D	A partition of P is a collection of non-empty sets that are all pair-wise disjoint whose union = P Since each set is not empty, the intersection with P must also be not empty D is false
5	C	This is an example of Demorgan's law
6	C	For a relation R to be reflexive, it must be true that for all x $x R x$ C is false, since a line can't be perpendicular to itself

<table>
<tr><td>7</td><td>B</td><td>7. Universe of discourse "my children"; a=fat; b=gluttons; c=healthy; d=sons; e=taking exercise.
1) D-> ~A
2) C→ E
3) B → A
4) ~D → ~E
B → A → ~D → ~E → ~C
All gluttons, who are children of mine, are unhealthy.</td></tr>
<tr><td>8</td><td>B</td><td>8. A(1,3) = A(0,A(1,2))
A(1,2) = A(0,A(1,1))
A(1,1)=A(0,A(1,0))
A(1,0)=A(0,1)
A(0,1)=1+1=2
A(1,0)=2
A(1,1)=A(0,2)
A(0,2)=2+1=3
A(1,1)=3
A(1,2)=A(0,3)
A(0,3)=3+1=4
A(1,2)=4
A(1,3)=A(0,4)
A(0,4)=4+1=5
A(1,3)=5</td></tr>
<tr><td>9</td><td>C</td><td>The truth table for p→ q
<table><tr><th>p</th><th>q</th><th>p</th><th>→</th><th>q</th></tr><tr><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>F</td><td>T</td><td>T</td></tr><tr><td>F</td><td>F</td><td>F</td><td>T</td><td>F</td></tr></table>
The truth table for $\neg p \vee q$
<table><tr><th>p</th><th>q</th><th>~p</th><th>∨</th><th>q</th></tr><tr><td>T</td><td>T</td><td>F</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>T</td><td>T</td><td>T</td></tr><tr><td>F</td><td>F</td><td>T</td><td>T</td><td>F</td></tr></table></td></tr>
<tr><td>10</td><td>D</td><td>A says that there is at least one such person (x), but doesn't address that fact that there is supposed to be ONLY one. Choice D, adds that if there is another such person (z), then he is identical to the first (z=x)</td></tr>
</table>

11	B	$\begin{bmatrix}8 & 4\\4 & 4\end{bmatrix}\cdot x+\begin{bmatrix}9 & 4\\2 & 8\end{bmatrix}=\begin{bmatrix}2 & 5\\4 & 8\end{bmatrix}$ subtract $\begin{bmatrix}2 & 5\\4 & 8\end{bmatrix}$ from both sides $\begin{bmatrix}8 & 4\\4 & 4\end{bmatrix}\cdot x=\begin{bmatrix}-7 & 1\\2 & 0\end{bmatrix}$ calculate inverse $\begin{bmatrix}\frac{1}{4} & -\frac{1}{4}\\-\frac{1}{4} & \frac{1}{2}\end{bmatrix}$. Remember Multiplication is not commutative, Inverse MUST appear on left side of each expression. Answer is $\begin{bmatrix}\frac{1}{4} & -\frac{1}{4}\\-\frac{1}{4} & \frac{1}{2}\end{bmatrix}\cdot\begin{bmatrix}2 & 5\\4 & 8\end{bmatrix}=\begin{bmatrix}-\frac{9}{4} & \frac{1}{4}\\\frac{11}{4} & -\frac{1}{4}\end{bmatrix}$
12	D	$\begin{bmatrix}2 & 1 & 3\\4 & 6 & -1\\5 & 1 & 0\end{bmatrix}\begin{matrix}2 & 1\\4 & 6\\5 & 1\end{matrix}$ $2\cdot6\cdot0+1\cdot-1\cdot5+3\cdot4\cdot1-5\cdot6\cdot3-1\cdot-1\cdot2-0\cdot4\cdot1$ $0+-5+12-90+2+0=-95+14=-81$
13	A	a matrix in echelon form if i) all zero rows, if any, are on the bottom of the matrix ii) each leading nonzero entry is the only nonzero entry in its column Thus A is not
14	C	${}_8C_3=\frac{8!}{3!5!}=\frac{8\cdot7\cdot6}{3\cdot2\cdot1}=8\cdot7=56$
15	A	Since no information is given about the "first" card, it can be ignored when calculating probability. The answer is then $\frac{4}{52}=\frac{1}{13}$
16	A	To calculate "expected value" you must calculate the probabilities and then multiply them by the payout. The chance of getting the same number on all dice is $\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{216}$ getting the same number twice xNN, NxN, NNx $xNN=\frac{5}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}=\frac{5}{216}$ so multiply by three for all $\frac{15}{216}$ getting one N is $\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}=\frac{25}{216}$ there are three ways Nxx, xNx, and xxN so total is $\frac{75}{216}$ The remainder of the ways are losses $1-\left(\frac{1}{216}+\frac{15}{216}+\frac{75}{216}\right)=1-\frac{91}{216}=\frac{125}{216}$ expected value is $\left(3\cdot\frac{1}{216}\right)+\left(2\cdot\frac{15}{216}\right)+\left(1\cdot\frac{91}{216}\right)-\left(1\cdot\frac{125}{216}\right)=-.08$

17	C	$6300 = 2^2 3^2 5^2 7 \to 3 \cdot 3 \cdot 3 \cdot 2 = 54$
18	D	2^{-1} means the number that you multiply by 2 to get the identity 2*1 = 2, 2*2=4, 2*3=6, 2*4=8=1(mod7) so $2^{-1} = 4$
19	C	the sum of the roots of a polynomial of degree n, is the opposite of the coefficient of the term of degree n-1 divided by the coefficient of the term of degree n 284/60 = 71/15
20	D	Q1 will be true when ever B OR C is true thus T T T F T T T F A4 Q2 will be false when A is false AND when both B and C are false T T T T T F F F A1 Q3 will be true when both A and B are true (just the first 2) T T F F F F F F A3 Q4 will be true when A is true and either B or C (just first 3 T T T F F F F F A1 So D

21	A	[x] the greatest integer function also known as the FLOOR function returns the integer to the LEFT. Thus –5.
22	C	$\sum_{i=1}^{4}\sum_{j=1}^{3} ij$ i j ij 1 1 1 1 2 2 1 3 3 **6** 2 1 2 2 2 4 2 3 6 **12** 3 1 3 3 2 6 3 3 9 **18** 4 1 4 4 2 8 4 3 12 **24** add the bold 60
23	D	$\frac{9!}{3!2!2!2!} = 7560$
24	C	circular removes 1, flippable divides by 2 so: 6! / 2 6*5*4*3 360
25	A	there are 36 possible rolls, six of them add up to 7 (1-6, 2-5, 3-4,4-3,2-5,1-6) so $\frac{6}{36} = \frac{1}{6}$
26	A	Each node is related to itself so the relation must be reflexive < is not reflexive (not C) 1 is related to all, so it is not a multiple (no B) and not > (no D) All the initial nodes that are related do divide the terminal nodes A x divides y
27	B	25. A finite connected graph is Eulerian (has a Euler circuit) if and only if each vertex has even degree) A) no. the top left vertex has degree 3 (3 connecting edges) B) Yes all vertices have even degree. Working left to right, top to bottom degrees 2 – 4 - 2 4 - 6 - 2 2 – 4- 2 C) no the top right vertex has degree 3

28	C	$33x \equiv 38 \pmod{280}$ gcd(33,280)=1, so there is a unique solution consider $33x \equiv 1 \pmod{280}$ apply Euclidean algorithm 280 = 8*33 + 16 33 = 2*16 + 1 back solve 1 = 33 – 2 * 16 16 = 280 – 8 * 33 so 1 = 33 – 2*(280 – 8*33) = 33 – 2*280 + 16*33 = 17*33 – 2*280 (1 is a linear combination of 33 and 280) 1 = 17*33 – 2*280 so 33 * 17 = 1 (mod 280) multiply both sides by 38 33 * (17*38) = 38 (mod 280) x = 17*38 = 646 (mod 280) = 86 (mod 280) x = 86 so answer is 8+6=14
29	A	Use the Chinese remainder theorem using $x \equiv 2 \pmod 3$ and $x \equiv 4 \pmod 5$, there is a unique solution (mod 3*5=15) add 5 to 4 to get 4, 9 and 14, then test in the first equation (all mod 15) 4 is congruent to 1 NO good 9 is congruent to 0 NO good 14 is congruent to 2 this is good consider $x \equiv 4 \pmod 5$ and $x \equiv 6 \pmod 7$ there is a unique solution mod 15*7=105 add 15's to 14 14, 29, 44, 59, 74, 89, 104 and test in both equations 104 mod 5 = 4 good 104 mod 7 = 6 good answer is 104 sum of digits is 5
30	C	$a = 8316$ and $b = 10920$ Use Euclidean algorithm 10920 = 1 * 8316 + 2604 8316 = 3*2604 + 504 2604 = 5*504 + 84 504 = 84 * 6 GCD (8316,10920) = 84 sum of the digits is 12