FAMAT State Convention- DISCRETE MATHEMATICS 2002

1 Which of the following sets have a cardinality of aleph-null $(\aleph_0)$

I. The Integers
II. The Rational Numbers
III. The Real Numbers

A) I only B) II only C) III only D) I and II E) NOTA

2 The Power Set of a set *S*, denoted by $P(S)$, is the set of all subsets of the set *S*. What is the cardinality of the Power Set of positive even integers less than 51.

A) 2^{25} B) 25! C) ${}_{50}C_{25}$ D) ${}_{50}P_{25}$ E) NOTA

3 Consider the following data for 120 mathematics students at a high school concerning the three different sports: football, soccer and rugby. 65 play football, 45 play soccer, 42 play rugby, 20 pla football and soccer, 25 play football and rugby, 15 play soccer and rugby and 8 play all three sports. Which of the following are true?

I. 12 play football and soccer but not rugby
II. 10 play only rugby
III. 16 play football and rugby but not soccer

A) I only B) II only C) III only D) I and III E) NOTA

4 Let the sets P_i, where *i* is an positive integer $\leq n$, be a finite partition of the set *P*. Which of the following is FALSE?

A) $\bigcup_{i=1}^{n} P_i = P$ B) $\bigcap_{i=1}^{n} P_i = \phi$ C) $\bigcup_{i=1}^{n} P_i \cup \bigcap_{i=1}^{n} P_i = P$

D) There exists an *i*, such that $P_i \cap P = \phi$ E) NOTA

5 If p and q are logical propositions, then the statement

$\neg p \vee \neg q \leftrightarrow \neg(p \wedge q)$ is an example of a

A) contradiction.
B) Distributive Law
C) Demorgan's Law
D) Domination Law
E) NOTA

6 Which of the following relations is not reflexive?

A) $\leq$ (less than or equal) on the set of integers
B) $\subseteq$ (subset) on a collection *C* of sets
C) $\perp$ (perpendicular) on the set of lines in a plane.
D) $\|$ (parallel) on the set of lines in a plane.
E) NOTA

7 Consider the following syllogism created by Lewis Carroll. What conclusion did he reach?

(1) All my sons are slim;
(2) No child of mine is healthy who takes no exercise;
(3) All gluttons, who are children of mine, are fat;
(4) No daughter of mine takes any exercise.

A) No gluttons, who are children of mine, are unhealthy
B) All gluttons, who are children of mine, are unhealthy.
C) All children of mine who are healthy are gluttons
D) All my fat sons are daughters.
E) NOTA

8 Consider the following function:

$$A(m,n)=\begin{cases} n+1 & \text{if } m=0 \\ A(m-1,1) & \text{if } m\neq 0, \text{ but } n=0 \\ A\big(m-1,A(m,n-1)\big) & \text{if } m\neq 0 \text{ and } n\neq 0 \end{cases}$$

Evaluate $A(1,3)$

A) 3
B) 5
C) 7
D) 1024
E) NOTA

9 The logical proposition $p\to q$ is equivalent to

A) $q\to p$ B) $\neg p\to\neg q$ C) $\neg p\vee q$ D) $p\vee\neg q$ E) NOTA

10 Let $L(x,y)$ be the statement "x loves y", where the universe of discourse for both x and y is the se of all people in the world. Use quantifiers to express:

There is exactly one person whom everybody loves.

A) $\exists x\big(\forall yL(y,x)\big)$

B) $\forall y\big(\exists xL(y,x)\big)$

C) $\forall x\Big(\forall yL(y,x)\wedge\exists z\Big(\big(\forall wL(w,z)\big)\to z=x\Big)\Big)$

D) $\exists x\Big(\forall yL(y,x)\wedge\forall z\Big(\big(\forall wL(w,z)\big)\to z=x\Big)\Big)$

E) NOTA

11 Solve for x. $\begin{bmatrix}8&4\\4&4\end{bmatrix}\cdot x+\begin{bmatrix}9&4\\2&8\end{bmatrix}=\begin{bmatrix}2&5\\4&8\end{bmatrix}$

A) $\begin{bmatrix}-2&\frac{9}{4}\\ \frac{1}{2}&-\frac{1}{2}\end{bmatrix}$ B) $\begin{bmatrix}-\frac{9}{4}&\frac{1}{4}\\ \frac{11}{4}&-\frac{1}{4}\end{bmatrix}$ C) $\begin{bmatrix}-\frac{7}{8}&\frac{1}{4}\\ \frac{1}{2}&0\end{bmatrix}$ D) Equation is not solvable for x E) NOTA

12 What is the determinant of

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 6 & -1 \\ 5 & 1 & 0 \end{bmatrix}$$

A) 21 B) 0 C) -1 D) -81 E) NOTA

13 Which of the following matrices is not in row-echelon form?

A) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 0 & 1 & 3 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 4 & 7 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix}$

E) NOTA

14 How many committees of 3 can be formed from eight people?

A) 512 B) 336 C) 56 D) 27 E) NOTA

15 Two cards are drawn from a standard 52-card deck with no replacement. What is the probability that the **second** card drawn is a King?

A) $\frac{1}{13}$ B) $\frac{13}{52 \cdot 52}$ C) $\frac{13 \cdot 12}{52 \cdot 52}$ D) $\frac{12}{52}$ E) NOTA

16 In Chuck-a-luck you pick a number from 1 to 6. Three dice are then rolled. If the number you pick comes up on all three dice, you get \$3. If the number comes up on exactly two dice you get \$2. If the number comes up on exactly one die, you get \$1. If the number doesn't come up on any of the dice, you pay \$1. What is the expected value of your winnings rounded to the nearest cent?

A) −8 cents B) −5 cents C) 3 cents D) 12 cents E) NOTA

17 6300 has how many distinct positive integral divisors?

A) 7 B) 8 C) 54 D) 135 E) NOTA

18 Consider the group $G = \{1,2,3,4,5,6\}$ under multiplication modulo 7. Find 2^{-1}.

A) -2 B) $\frac{1}{2}$ C) 1 D) 4 E) NOTA

19 What is the sum of the roots of $60x^5 - 284x^4 + 319x^3 + 4x^2 - 81x + 18 = 0$?

A) −284 B) $\frac{3}{10}$ C) $\frac{71}{15}$ D) 284 E) NOTA

20 Consider the following incomplete truth table. The answers for each column are listed below. Match the proper set of answers with the column name.
For example if you think answers A2 belong in column Q4, the answer will be Q4=A2

			Q1	Q2	Q3	Q4
A	B	C	B ∪ C	A∩(B∪C)	A∩B	(A∩B)∪(A∩C)
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

A1	A2	A3	A4
T	T	T	T
T	T	T	T
T	T	F	T
F	F	F	F
F	F	F	T
F	F	F	T
F	F	F	T
F	T	F	F

A) Q1=A4, Q2= A1, Q3=A3, Q4=A2
B) Q1=A2, Q2= A4, Q3=A1, Q4=A3
C) Q1=A1, Q2=A3, Q3=A2, Q4=A4
D) Q1=A4, Q2=A1, Q3=A3, Q4=A1
E) NOTA

21 If [x] denotes the **greatest integer function of *x*,** what is the value of [-4.1]?
A) -5 B) -4 C) 4 D) ∞ E) NOTA

22 What is the value of $\sum_{i=1}^{4}\sum_{j=1}^{3} ij$

A) 7 B) 12 C) 60 D) 144 E) NOTA

23 Find the number of ways that nine toys can be divided among four children if the youngest child is to receive three toys and each of the others two toys.

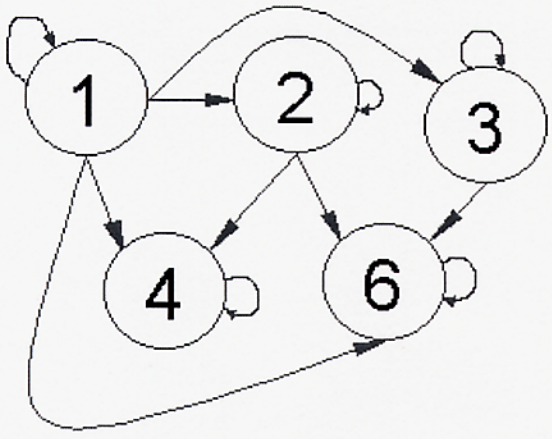
A) 36 B) 126 C) 3024 D) 7560 E) NOTA

24 How many ways can 7 keys be arranged on a keyring?
A) 5040 B) 720 C) 360 D) 7 E) NOTA

25 What is the probability of rolling a 7 using two standard fair dice?

A) $\frac{1}{6}$ B) $\frac{1}{7}$ C) $\frac{7}{12}$ D) $\frac{7}{11}$ E) NOTA

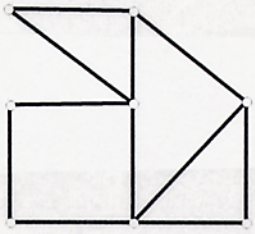
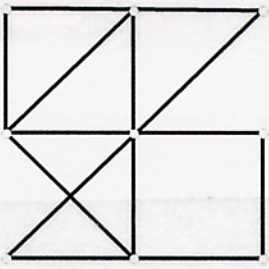
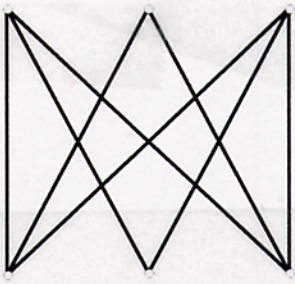
26 Consider the following directed multigraph, R:



If $R = \{(x,y) \mid x, y \text{ are vertices on graph}\}$, which of the following best describes the relation R?

A) x divides y B) x is a multiple of y C) x is less than y
D) x is greater than y E) NOTA

27 Which of the following multigraphs has a Eulerian circuit?

A)  B)  C) 

D) All have Eulerian circuits E) NOTA

28 If the solution, x, of the congruence relation $33x \equiv 38 \pmod{280}$ can be written in the form $a \cdot 10 + b$, then find $a+b$

A) 8 B) 9 C) 14 D) 16 E) NOTA

29 Let x be the smallest positive solution to the following three congruence equations:
$x \equiv 2 \pmod 3$ $x \equiv 4 \pmod 5$ $x \equiv 6 \pmod 7$
If x can be written in the form $100 \cdot a + 10 \cdot b + c$, find $a+b+c$.

A) 5 B) 11 C) 14 D) 17 E) NOTA

30 Let $a = 8316$ and $b = 10920$. If x is the greatest common divisor of a and b, which can be written in the form $10 \cdot a + b$, find $a+b$.

A) 3 B) 6 C) 12 D) 17 E) NOTA